NOVEL GAUGING IN $\mathcal{N}=(2,2)$ SUPERSPACE

Dmitri Bykov

with S. Kutsubin
& A. Kuzovchikov

arXiv: 2512.24726

See also DB, U. Lindström, M. Roček, arXiv: 2605.18536

Steklov Mathematical Institute
ITMP (Moscow State University)

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Various SUSY Gauge fields

- 4D Matter = Chiral fields Z $\bar{D}_\alpha Z = 0 \quad \alpha=1,2$

[cf. Weinberg vol. 3]

Global U(1) symmetry $Z \mapsto e^{i\varphi} Z$

Gauging: $\mathcal{L}_{matter} = \bar{Z} e^{\boxed{V}} Z$ real gauge superfield V

Chiral

$Z \mapsto e^{\boxed{\Phi}} Z \,, \quad \bar{Z} \mapsto e^{\bar{\Phi}} \bar{Z} \,, \quad V \mapsto V - \Phi - \bar{\Phi}$

- 2D Matter = Chiral, Twisted chiral, Semichiral

$Z \qquad S \qquad X$

Global U(1) symmetry $Z \mapsto e^{i\varphi} Z \,, \quad S \mapsto S e^{i\varphi}$

Chiral   Twisted chiral

Gauging: $$Z \mapsto e^{\Phi} Z, \quad S \mapsto e^{\Lambda} S$$

"Large Vector Multiplet" [Lindström, Ryb, Roček, von Unge, Zabzine '2007]

Real vector fields $V_1, V_2, V_3 \sim V_{\mathbb{R}}, V_{\mathbb{C}}$

- New & Simple way of gauging [DB, Kuzovchikov, Kutsubin '2025]

  Real vector field $V_{\mathbb{R}}$, Semichiral (complex) field X

- Relation between the two: $V_{\mathbb{C}} \leftrightarrow X$ [DB, Lindström, Roček '2026]

Applications

- Novel gauging of generalized Kähler symmetries
- T-duality in superspace
- Quotients
- Gauging in Kähler geometry with central extensions (EX. $\mathbb{C}^n$) THIS TALK
- EX. η-deformed $\mathbb{CP}^{n-1}_\eta$ sigma model & its T-dual

[Demulder et.al. 2019]

[DB, Lüst 2020]

$\mathcal{N}=(2,2)$ Superspace in 2D

- * Dim. reduction of $\mathcal{N}=1$ in 4D
- * Kähler sigma models [Zumino '1979]

SUPERALGEBRA

Lightcone coords. $x^{\pm}$, Supercoords. $\theta^{\pm}, \overline{\theta}^{\pm}$

$$Q_{\pm} = \frac{\partial}{\partial\theta^{\pm}} + i\,\overline{\theta}^{\pm}\frac{\partial}{\partial x^{\pm}} \qquad \overline{Q}_{\pm} = -\frac{\partial}{\partial\overline{\theta}^{\pm}} - i\,\theta^{\pm}\frac{\partial}{\partial x^{\pm}}$$

$$Q_{\pm}^2 = \overline{Q}_{\pm}^2 = 0 \qquad \{Q_{\pm}, \overline{Q}_{\mp}\} = 0$$

$$\{Q_{\pm}, \overline{Q}_{\pm}\} = -2i\frac{\partial}{\partial x^{\pm}}$$

Superfields

Superderivatives $D_\pm = \frac{\partial}{\partial\theta^\pm} - i\bar\theta^\pm \frac{\partial}{\partial x^\pm}$ $\quad \bar D_\pm = -\frac{\partial}{\partial\bar\theta^\pm} + i\theta^\pm \frac{\partial}{\partial x^\pm}$

$D_\pm^2 = \bar D_\pm^2 = 0$ $\quad \{D_\pm, Q_{\dots}\} = \{\bar D_\pm, Q_{\dots}\} = 0$

Chiral $\bar D_+ Z = \bar D_- Z = 0$ [Wess, Zumino 1974]

Twisted chiral $\bar D_+ S = D_- S = 0$ [Gates, Hull Roček 1984]

Semi-chiral $\bar D_+ X = 0$ or $D_- Y = 0$ [Buscher Lindström Roček 1988]

General Lagrangian $L = \int d^4\theta \, K(Z,\bar Z, S, \bar S, X, \bar X, Y, \bar Y)$

Kähler geometry

* Only chiral & anti-chiral superfields $Z_A, \overline{Z}_A$

$$g_{A\bar{B}} = \frac{\partial^2 K}{\partial Z^A \partial \overline{Z}^{\bar{B}}} \qquad K(Z_A, \overline{Z}_A) = \text{Kähler potential}$$

$$L = g_{A\bar{B}}(Z, \overline{Z})\, \partial_\alpha Z^A \partial^\alpha \overline{Z}^B + i\, \overline{\psi}^B \not{D} \psi^A g_{A\bar{B}} + \dots$$

"Invariant" definitions

$$\left(g, \overset{\text{complex structure}}{J}, \omega = g \cdot J \right)$$

$$\omega := \frac{i}{2} g_{A\bar{B}}\, dZ^A \wedge d\overline{Z}^B, \quad d\omega = 0 \quad \Leftrightarrow \quad \underset{\text{Levi-Civita}}{\nabla} J = 0$$

Baby generalized Kähler geometry

* Bi-Hermitian geometry: J_+, J_- g Hermitian w.r.t. both

* Non-topological B-field $\int B_{\alpha\beta} dX^\alpha \wedge dX^\beta$

$$T := dB \rightsquigarrow \text{Torsion}$$

$$(\nabla \pm T) J_\pm = 0$$

[Gates, Hull
Roček 1984]

* $[J_+, J_-] = 0 \Rightarrow$ chiral Z_A & tw. chiral S_B fields

Ex. $SU(2) \times U(1)$ WZNW model [Roček, Schoutens
Sevrin 1991]

Full-fledged generalized Kähler geometry

- $[J_+, J_-] \neq 0$ [Sevrin, Troost 1996]

  ⇒ Add semi-chiral fields

- Math definitions [Hitchin 2003] [Gualtieri 2004, 2010]

- Equivalence with physics SUSY models [Lindström, Roček, v. Unge, Zabzine 2005]

Gauging

Henceforth use additive notation

* Only chiral fields: replace $Z+\bar{Z} \mapsto Z+\bar{Z}+V$

Gauge transfos: $Z \mapsto Z+\Phi$

chiral

$V \mapsto V-\Phi-\bar{\Phi}$

Gauged potential: $K(Z+\bar{Z}+V)$

BUT This only works for invariant Kähler potential $K(Z+\bar{Z})$

Non-invariant case

* Inv. of metric does not imply inv. of K

$$\mathcal{L}_v K = f(z) + \bar{f}(\bar{z}) \quad \sim \text{up to Kähler transf.}$$

Ex. Torus T^2 $\quad z \mapsto z + i\varepsilon_1 + \tau\varepsilon_2$

$\exists\ K_{1,2}$ inv. under ε_1 or ε_2 but NOT under both

$$K_1 = \frac{1}{2}(z+\bar{z})^2$$

* May be an obstruction to gauging [Hull, Karlhede, Lindström, Roček 1986]

Ex. $\mathbb{C} \times \mathbb{C}$ $\quad z_1, z_2$

4 parameters

General flat metric $\quad ds^2 = \sum_{i,j=1}^{2} a_{ij} \, dz_i \, d\bar{z}_j$

Isometries: $\quad z_j \mapsto z_j + i\,\varepsilon_j \qquad \varepsilon_j \in \mathbb{R}$

$\nexists$ invariant Kähler potential

Invariants: $\quad t_1 = z_1 + \bar{z}_1 \, , \quad t_2 = z_2 + \bar{z}_2$

3 parameters

Most general potential $\quad K = \sum_{i \le j} d_{ij} \, t_i t_j$

Math Interpretation

NB: Kähler ⊂ Symplectic

Class. mechanics: $(\mathcal{M}, \Omega)$ – symplectic, $d\Omega = 0$

$v_1 \dots v_N$ – Lie algebra of vector fields on $\mathcal{M}$

$\mathfrak{g}$ $\quad [V_A, V_B] = f_{AB}^{\ C} V_C$ $\qquad$ "moment map" ↙ (Hamiltonian)

$$\mathcal{L}_{V_A} \Omega = 0 \quad \Longrightarrow \quad V_A \mapsto H_A$$

$\hat{\mathfrak{g}}$ $\quad \{H_A, H_B\} = f_{AB}^{\ C} H_C + \mathcal{C}_{AB}$ $\qquad$ central extension

[Souriau, Kostant, 1960's]

Theorem $\mathfrak{g}$ Abelian $\Rightarrow$ $\mathcal{C}_{AB} \simeq 0 \leftrightarrow K$ invariant

[DB, Kuzovchikov, to appear]

But one can make K invariant: $\mathbb{C}\times\mathbb{C}$ continued

$$ds^2 = \sum_{i,j=1}^{2} a_{ij}\, dz_i\, d\bar{z}_j \qquad a_{ij} = \begin{pmatrix} a_1 & d - i\beta \\ d + i\beta & a_2 \end{pmatrix}$$

$$K = \frac{a_1}{2}(z_1+\bar{z}_1)^2 + \frac{a_2}{2}(z_2+\bar{z}_2)^2 + \alpha(z_1+\bar{z}_1)(z_2+\bar{z}_2) +$$

$$+ \boxed{i\beta(z_1+\bar{z}_1)(z_2-\bar{z}_2)} \;\leftsquigarrow\; \text{non-invariant piece}$$

tw. chiral

* Replace by $i\beta(z_1+\bar{z}_1)(z_2-\bar{z}_2-S_2+\bar{S}_2)$

* This is a gen. Kähler transform (i.e. total derivative)

But now inv. under $z_2 \mapsto z_2 + i\varepsilon_2$, $S_2 \mapsto S_2 + i\varepsilon_2$

How to gauge? Our proposal

[DB, Kutsubin, Kuzovchikov 2025]

* $(\overset{\text{real gauge superfield}}{V}, X \leftarrow \text{semi-chiral}) \qquad \bar{D}_+ X = 0$

Gauge transfos: $V \longmapsto V - \overset{\text{chiral}}{\Phi} - \bar{\Phi}$

$$X \longmapsto X + \Phi - \overset{\text{tw.chiral}}{\Lambda}$$

Introduce $V' = V + X + \bar{X} \;\Rightarrow\; V' \longmapsto V' - \Lambda - \bar{\Lambda}$

* Gauge field strength $F = D_-(V + X)$

$$F = \bar{F} = 0 \;\longleftrightarrow\; (V, X) \text{ pure gauge}$$

* FI term: $\xi \int d^4\theta\, V \simeq \xi \int d^4\theta\, V'$

Application: SUSY T-duality

[Gates, Hull, Roček, 1984]
[Roček, Verlinde 1992]

Assume a holomorphic isometry V leaving K invariant

$$\mathcal{L}_V K = 0$$

General model: $Z \mapsto Z + i\varepsilon$, $\varepsilon \in \mathbb{R}/\mathbb{Z}$

$$K = K(Z + \bar{Z}, \text{all other coords.})$$

① Gauging: $Z \mapsto Z + \overset{\text{chiral}}{\Phi}$, $V \mapsto \overset{\text{real}}{V} - \Phi - \bar{\Phi}$

$$K_g = K[Z + \bar{Z} + V, \ldots]$$

② Add Lagrange multiplier $V\cdot(S+\bar{S})$ twisted chiral

③ Set $Z=\bar{Z}=0$

④ Integrate out V: $\dfrac{\partial K}{\partial V}(V,\dots)=S+\bar{S}$

$$\Rightarrow V=V[S+\bar{S},\dots]$$

$$L_g=\int d^4\theta\,\Big(\underbrace{K_g[V,\dots]-V\cdot(S+\bar{S})}\Big)\Big|_{V=V[S+\bar{S},\dots]}$$

"T-Dual" potential depending on chiral & tw. chiral fields = Legendre transform of K

T-duality with central extension

$$\mathbb{C}\times\mathbb{C} \text{ model} \xrightarrow{\text{compactify}} \mathbb{C}^*\times\mathbb{C}^* \text{ model}$$

$z_1 \sim z_1 + 2\pi i \,, \quad z_2 \sim z_2 + 2\pi i$

$$K = \frac{a_1}{2}\left(z_1+\bar{z}_1\right)^2 + \frac{a_2}{2}\left(z_2+\bar{z}_2\right)^2 + \alpha\left(z_1+\bar{z}_1\right)\left(z_2+\bar{z}_2\right) + i\beta\left(z_1+\bar{z}_1\right)\left(z_2-\bar{z}_2-s_2+\bar{s}_2\right)$$

① Gauge:

$$K_g = \frac{a_1}{2}\left(z_1+\bar{z}_1+V_1\right)^2 + \frac{a_2}{2}\left(z_2+\bar{z}_2+V_2\right)^2 + \alpha\left(z_1+\bar{z}_1+V_1\right)\left(z_2+\bar{z}_2+V_2\right) + i\beta\left(z_1+\bar{z}_1+V_1\right)\left(z_2-\bar{z}_2-s_2+\bar{s}_2+X-\bar{X}\right)$$

② Add Lagrange multipliers $Y\cdot\left(V_2+X\right) + T\cdot V_1 + \text{c.c.}$

semi-chiral $D_-Y=0$

tw.chiral

③ Set $z_1 = z_2 = s_2 = 0$

④ Integrate out V_1, V_2

$x = X + \bar{X}\,,\ y = Y + \bar{Y}$

$$K^V = -\frac{1}{2\Delta}\left(a_1 y^2 - 2\alpha\, x y + a_2 x^2\right) + \frac{i}{\beta}\left(XY - \bar{X}\bar{Y}\right)$$

$\hookrightarrow \Delta = a_1 a_2 - \alpha^2$

* $\beta \mapsto 0$ limit "steepest descent"

$X:\ \bar{D}_+ Y = 0$ $\qquad$ recall $\qquad D_- Y = 0$ $\qquad \Rightarrow\ X, Y$

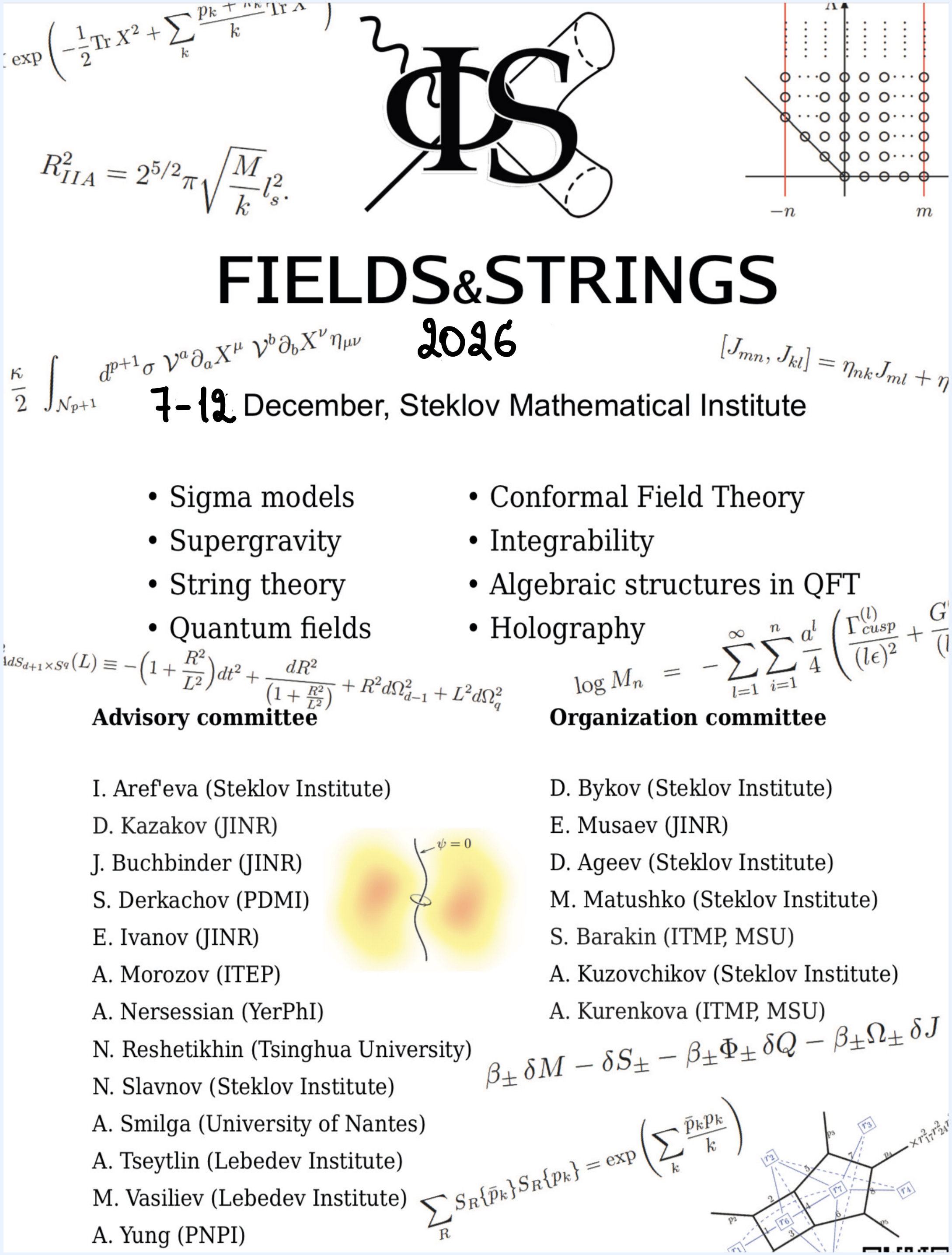
$Y:\ D_- X = 0$ $\qquad\qquad\qquad\quad \bar{D}_+ X = 0$ $\qquad$ tw. chiral

* Inversion of radii: set $\alpha = 0 \Rightarrow K^V = -\frac{1}{2a_2} y^2 - \frac{1}{2a_1} x^2 + \ldots$

$\Rightarrow$ T-duality!

Summary & Outlook

- A new method for gauging isometries in gen. Kähler geometry / $N=(2,2)$ sigma models
- Overcomes previously known obstructions to gauging
- Any obstructions remaining ?
- Gen. Kähler quotient construction ?
- Applications to integrable sigma models



FIELDS&STRINGS

2026

7-12 December, Steklov Mathematical Institute

$$R^2_{IIA} = 2^{5/2}\pi\sqrt{\frac{M}{k}}l_s^2.$$

- Sigma models
- Supergravity
- String theory
- Quantum fields
- Conformal Field Theory
- Integrability
- Algebraic structures in QFT
- Holography

$$\beta_\pm\,\delta M - \delta S_\pm - \beta_\pm \Phi_\pm\,\delta Q - \beta_\pm \Omega_\pm\,\delta J$$

$$\sum_R S_R\{\bar{p}_k\} S_R\{p_k\} = \exp\left(\sum_k \frac{\bar{p}_k p_k}{k}\right)$$

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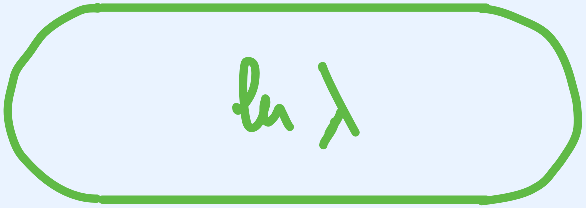
THANK

YOU!

Motivation

- Integrable (?) sigma models
- Non-homogeneous (η)-deformations of homogeneous target spaces [Klimčik 2002]
- Ex. "Sausage" model [Fateev, Onofri, Zamolodchikov 1993]

$$ds^2 = \frac{|dZ|^2}{(\lambda+|Z|^2)(\lambda^{-1}+|Z|^2)}$$



- RG flow: $\lambda = e^{t} \in (0,1)$

η-deformed $\mathbb{CP}^{n-1}_\eta$

- Def. of symmetric spaces – general theory

[Delduc
Magro '2013
Vicedo]

$$L = -\tfrac{1}{2}\kappa\Big((g^{-1}\partial_+ g)^{(1)}, \frac{1+\eta^2}{1-\eta R_g \circ P_1}(g^{-1}\partial_- g)^{(1)}\Big)$$

$g \in G/H$

↖ resembles the classical r-matrix

- Not very practical or explicit

[Lukyanov 2012]

- Stable under RG flow @ 1 loop

[Fateev, Litvinov 2018]

[Hoare, Levine, Tseytlin 2019]

- $\mathbb{CP}^1$ case: relation to (deformed) Gross-Neveu model

[DB 2020^+]

[DB, Pribytok 2023]

* $\mathbb{CP}^{n-1}_\eta$ - generalized Kähler for $n>2$

[Demulder, Hassler Piccinini, Thompson 2020]

$\Rightarrow$ Admits $N=(2,2)$ SUSY

[Litvinov 2019]

* T-dual $(\mathbb{CP}^{n-1}_\eta)^\vee$ - ordinary Kähler

[DB, Lüst 2020]

$$K^\vee = \sum_{j=2}^{n-1} i\eta\left(z_j \overline{z_{j-1}} - \overline{z}_j z_{j-1}\right) + \frac{1}{2\eta}\sum_{j=1}^{n} P\left(2\eta\left(t_j - t_{j-1}\right)\right)$$

$$P(t) = i\left(\mathrm{Li}_2\left(e^{it}\right) - \frac{t^2}{4}\right), \qquad t_j = z_j + \overline{z}_j$$

* How to T-dualize back to $\mathbb{CP}^{n-1}_\eta$?

[DB 2025 Kutsubin Kuzovchikov]

Will tell you today!