

Global analysis of J/ψ production in non-relativistic QCD with k_T -factorization

Andrei Prokhorov¹, Sergey Baranov², Artem Lipatov³, Maxim Malyshev³

¹DLNP JINR, Dubna

²LPI, Moscow

³SINP MSU, Moscow

QUARKS 2026

May 18-23 2026

Charmonia production: recent status and challenges

▶ Nonrelativistic QCD (NRQCD)

↪ NLO accuracy in collinear factorization

↪ good description of charmonia and bottomonia unpolarized data (separated datasets) [*Phys.Lett.B* 673, 197 (2009); *Phys.Rev.D* 84, 051501 (2011); *Phys.Rev.D* 83, 111503 (2011); *Phys.Rev.Lett.* 106, 172002 (2012); *Phys.Rev.D* 90, 074021 (2014); *Phys.Rev.D* 83, 114021 (2011)]

↪ Global fit for ψ' in [*Phys. Rev. D* 107, 034003 (2023)] (limitation $p_T > 7$ GeV), global fit for J/ψ and χ_c is absent

↪ polarization problem [*Phys.Rev.Lett.*108, 172002 (2012)]

▶ Potential NRQCD (pNRQCD)

↪ NLO accuracy in collinear factorization

↪ matching with NRQCD

↪ Good description of charmonia and bottomonia (selective fit) ([*Phys.Rev.D* 112, L011902 (2025)])

▶ Improved color evaporation model (iCEM)

↪ Predictions only for separated charmonia datasets

↪ impossibility to predict polarization

▶ Is it possible to achieve a good simultaneous description of charmonia data?

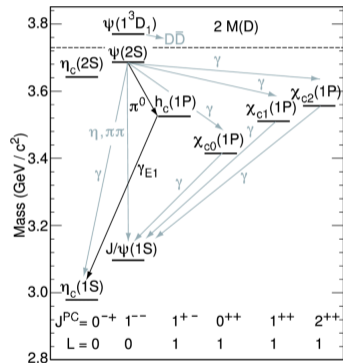
Introduction: Nonrelativistic QCD

- **Nonrelativistic QCD (NRQCD)** [*Phys.Rev.D 53,150 (1996)*],[*Phys.Rev.D 51,1125 (1995)*]:

$$\sigma(pp \rightarrow \psi + X) = \sum_n \sigma(pp \rightarrow c\bar{c}[{}^{2S+1}L_J^{(a)}] + X) \langle \mathcal{O}^\psi[n] \rangle$$

- $\sigma(pp \rightarrow c\bar{c}[{}^{2S+1}L_J^{(a)}] + X)$ cross-section of heavy quark pair production $Q\bar{Q}$ in Fock state $n = {}^{2S+1}L_J^{(a)}$ with definite spin S , orbital L и total J angular momenta and color representation a (color singlet (CS) [1] and color octet (CO) [8]) - can be calculated in the framework of perturbative QCD.
- **LDME** (nonperturbative matrix element) $\langle \mathcal{O}^\psi[n] \rangle$ describe the transition of intermediate state $c\bar{c}[n]$ into physical quarkonium via emission of soft gluons.
- Decomposition of wave functions of S-wave charmonia over the small parameter v of quark relative motion and Fock states:

$$|\psi\rangle = O(v^0)|c\bar{c}[{}^3S_1^{(1)}]\rangle + O(v^1)|c\bar{c}[{}^3P_J^{(8)}]g\rangle + O(v^2)|c\bar{c}[{}^3S_1^{(8)}]gg\rangle + \dots$$



Potential model and LDME's

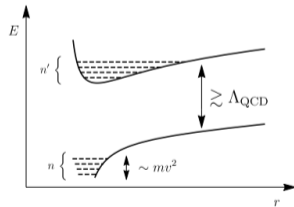
- ▶ Potential model for $c\bar{c}$ in color singlet (CS) state: $V^{CS}(r) \sim -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r$ (attraction + confinement)
- ▶ Schrodinger equation \rightarrow wave function: $\psi_{nlm}(r, \theta, \phi) = R_n(r) \times Y_{l,m}(\theta, \phi)$
- ▶ LDME: S-wave ($L = 0$): $\langle \mathcal{O}^{[2S+1]L_J} \rangle = 2N_c |R^2(0)| / 4\pi$
P-wave ($L = 1$): $\langle \mathcal{O}^{[2S+1]L_J} \rangle = 6N_c (2J + 1) |R'^2(0)| / 4\pi$

- ▶ LDME's could be extracted from decay width:

$$\Gamma(J/\psi \rightarrow ee) = \frac{8\pi e_c^2 \alpha^2}{3m_{J/\psi}^2} \times \left(1 - 2C_F \frac{\alpha_s}{\pi} + \text{rel.cor.}\right)^2 \mathcal{O}^{J/\psi} [{}^3S_1^{[1]}]$$

- ▶ Heavy quark spin symmetry (HQSS): $\langle \mathcal{O}^{\eta_c} [{}^1S_0^{[1]}] \rangle = \frac{1}{3} \langle \mathcal{O}^{J/\psi} [{}^3S_1^{[1]}] \rangle$

- ▶ Color octet state: repulsive potential $V^{CO}(r) \sim +\frac{1}{6} \frac{\alpha_s}{r} + \sigma r$



High energy factorization (k_T -factorization) approach

- ▶ In the region of high energies (small x) parton model assumptions about collinear factorization of parton distributions and partonic cross sections are violated. Along with contributions $\sim \alpha_s^n \ln^n \mu^2 / \Lambda_{\text{QCD}}^2$ arises contributions $\sim \alpha_s^n \ln^n 1/x$, which can be summarized up to all orders of perturbative theory with evolution equations **BFKL** (Balitsky-Fadin-Kuraev-Lipatov) or **CCFM** (Catani-Ciafaloni-Fiorani-Marchesini)

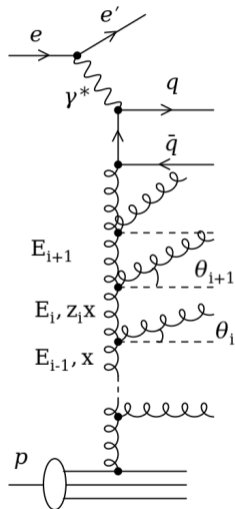
- ▶ In **k_T -factorization approach** [*Phys.Rep.* 100,1 (1983)], [*Sov.J.Nucl.Phys.* 53,657 (1991)], [*Nucl.Phys.B* 366,135 (1991)]:

$$d\sigma = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} d\mathbf{k}_{1T}^2 d\mathbf{k}_{2T}^2 \frac{d\phi_1}{2\pi} \frac{d\phi_2}{2\pi} f_1(x_1, \mathbf{k}_{1T}^2, \mu_F^2) f_2(x_2, \mathbf{k}_{2T}^2, \mu_F^2) d\hat{\sigma}^*(k_1, k_2, \mu_R^2),$$

where $f(x, \mathbf{k}_T^2, \mu_F^2)$ - TMD (transverse momentum dependent) parton distribution function in proton which obey the BFKL or CCFM evolution equation.

- ▶ LO in k_T -factorization approach can include the large piece of high-order corrections from collinear QCD approach (partially NLO + NNLO + ...) taking them into account in the form of TMD gluon distributions $f(x, \mathbf{k}_T^2, \mu_F^2)$ in the proton.
- ▶ **Gauge invariance of off-shell amplitudes** are achieved by using the effective vertex [*Nucl. Phys. B* 452,369 (1995)]

CCFM evolution equation and double counting problem



- ▶ **CCFM evolution equation** [*Nucl. Phys. B* 296, 49 (1988)], [*PLB* 234,339 (1990)] at leading logarithmic approximation:

$$f_g(x, \mathbf{k}_T^2, q^2) = f_g(x, \mathbf{k}_T^2, q_0^2) \Delta_s(q, q_0) + \int dz \int \frac{dq'^2}{q'^2} \Theta(q - zq') \times \\ \times \Delta_s(q, zq') \tilde{P}_{gg}(z, q', \mathbf{k}_T) f_g(x/z, \mathbf{k}'_T, q'^2),$$

- ▶ Emitted gluons obey the angular ordering conditions (due to color coherence effect):

$$q_{i+1} \quad q > z_n q_n > z_{n-1} q_{n-1} > \dots > q_1 > q_0,$$

↪ at $z \rightarrow 0$: no constraints on q_i - BFKL conditions

q_i ↪ at $z \rightarrow 1$: ordering on q_i - DGLAP conditions

- ▶ NLO corrections in k_T -factorization approach:

↪ more complicated calculation of off-shell amplitudes (extended set of Feynman diagrams)

↪ **Double counting problem** for LO and NLO: some contributions (gluon emissions) can be counted twice - from tree-level amplitude (NLO) and initial gluon radiation (from evolution of TMD gluon distributions) (LO)

LO+NLO[†] merging scheme

- ▶ **CS contributions:**

LO	$2 \rightarrow 2$	$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^3S_1^{[1]} \right] (p) + g(p_g)$
----	-------------------	--

- ▶ **CO contributions:**

LO	$2 \rightarrow 1$	$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p),$
NLO	$2 \rightarrow 2$	$g^*(k_1) + g^*(k_2) \rightarrow c\bar{c} \left[{}^1S_0^{[8]}, {}^3P_J^{[8]}, {}^3S_1^{[8]} \right] (p) + g(p_g)$

- ▶ **Double counting problem:** separation of LO and NLO contributions is needed (DCE cut)
 - ↔ k_T - transverse momenta of incoming gluon can serve as characteristic momentum of emitted gluons (as a result of evolution) with a good accuracy: $\langle k_T \rangle > \langle p_{gT}^{\text{emission}} \rangle$

- ▶ Merging scheme of LO + tree-level NLO (NLO[†]) (validated for χ_{cJ} production at central rapidities [Eur. Phys. J. C 84, 348 (2024)]):
 - ★ $2 \rightarrow 1$: without cuts
 - ★ $2 \rightarrow 2$: $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}|)$

- ▶ **Modified merging scheme** of LO + tree-level NLO (NLO[†]) at forward rapidities and low p_T (initial transverse momentum $\mathbf{k}_T^{\text{init}}$ of incoming gluons are important !):
 - ★ $2 \rightarrow 1$: without cuts
 - ★ $2 \rightarrow 2$: $|\mathbf{p}_{gT}| > \max(|\mathbf{k}_{1T}|, |\mathbf{k}_{2T}| + \overline{|\mathbf{k}_{2T}^{\text{init}}|})$

Description of charmonia family

- ▶ **ψ' production**: fit the unpolarized datasets: $p_T = 2 - 140$ GeV and $|y| < 4.5$ (n.d.f. = 721)
 - ↪ **ATLAS** 5, 7, 8, 13 TeV || **CMS** 5, 7, 13 TeV || **LHCb** 7, 13 TeV || **ALICE** 7, 8, 13 TeV

$\chi^2/n.d.f.$	A0	JH'2013-set2	JH'2013-set2 ($p_T > 6$)
Summary	2.15	7.29	2.34

- ↪ improvement in comparison with LO calculation only
- ↪ Results are presented in [[Phys.Rev.D 112, 114022 \(2025\)](#)]

- ▶ **χ_{cJ} production**: fit the unpolarized datasets: $p_T = 6 - 30$ GeV and $|y| < 0.75$ (n.d.f. = 21)
 - ↪ **ATLAS** 7 TeV || **CDF** 1.8 TeV

$\chi^2/n.d.f.$	A0	JH'2013-set2
Summary	1.8	0.78

- ↪ improvement in comparison with LO calculation only
- ↪ Results are presented in ([[Eur.Phys.J.C 84, 348 \(2024\)](#)])

J/ψ production

$$\frac{d\sigma^{J/\psi}}{dp_T} = \frac{d\sigma(c\bar{c}[{}^3S_1^{[1]}])}{dp_T} \langle \mathcal{O}^{J/\psi}[{}^3S_1^{[1]}] \rangle + \frac{d\sigma(c\bar{c}[{}^1S_0^{[8]}])}{dp_T} \langle \mathcal{O}^{J/\psi}[{}^1S_0^{[8]}] \rangle + \sum_J (2J+1) \frac{d\sigma(c\bar{c}[{}^3P_J^{[8]}])}{dp_T} \langle \mathcal{O}^{J/\psi}[{}^3P_0^{[8]}] \rangle + \frac{d\sigma(c\bar{c}[{}^3S_1^{[8]}])}{dp_T} \langle \mathcal{O}^{J/\psi}[{}^3S_1^{[8]}] \rangle + \frac{d\sigma(\psi' \rightarrow J/\psi X)}{dp_T} + \frac{d\sigma(\chi_{cJ} \rightarrow J/\psi X)}{dp_T}$$

► Direct J/ψ

↪ CS LDME from the meson decay width: $\langle \mathcal{O}^{J/\psi}[{}^3S_1^{[1]}] \rangle = 1.16 \text{ GeV}^3$

↪ HQSS: $\langle \mathcal{O}^{J/\psi}[{}^3P_J^{[8]}] \rangle = (2J+1) \langle \mathcal{O}^{J/\psi}[{}^3P_0^{[8]}] \rangle$ where $J = 0, 1, 2$

↪ only 3 free parameters (CO LDME's): $\langle \mathcal{O}^{J/\psi}[{}^1S_0^{[8]}] \rangle$, $\langle \mathcal{O}^{J/\psi}[{}^3P_0^{[8]}] \rangle$ and $\langle \mathcal{O}^{J/\psi}[{}^3S_1^{[8]}] \rangle$

► Feed-down $\psi(2S)$ and χ_{cJ} production

↪ LDME's from previous analysis

↪ problem: large uncertainties for χ_{cJ} LDME

► Fit:

↪ minimization of χ^2

↪ two sets of TMD gluon distributions: **A0** and **JH'2013-set-2**

J/ψ LDME's fitting procedure

► Stage I

↔ Fit the **unpolarized J/ψ datasets**: $p_T = 0 - 360$ GeV and $y < 4.5$

ATLAS 5, 7, 8, 13 TeV || **CMS** 5, 7, 13 TeV ||

LHCb 5, 7, 8, 13 TeV

↔ extraction of linear combination from **unpolarized J/ψ datasets**: example for A0 TMD

$$\langle \mathcal{O}^{J/\psi} [{}^3P_0^{[8]}] \rangle = 0.0116 - 0.0834 |R'(0)|_{\chi_{cJ}}^2 \text{ GeV}^5$$

$$\langle \mathcal{O}^{J/\psi} [{}^3S_1^{[8]}] \rangle = 0.00167 - 1.35 \langle \mathcal{O}^{\chi_{cJ}} [{}^3S_1^{[8]}] \rangle \text{ GeV}^3$$

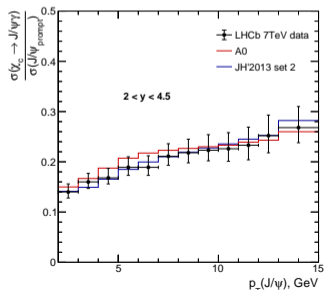
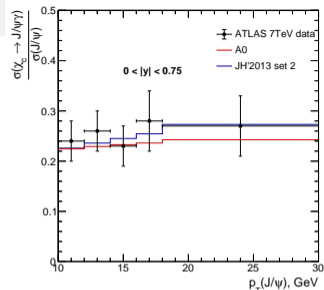
► Stage II

↔ refit the **unpolarized χ_{cJ} datasets**:

ATLAS 7 TeV || **CDF** 1.8 TeV || **LHCb** 7 TeV

↔ inclusion of $\sigma(\chi_{cJ} \rightarrow J/\psi\gamma)/\sigma(J/\psi)$ - now available due to J/ψ predictions

↔ extraction of χ_{cJ} LDME $\Rightarrow J/\psi$ LDME



Fit results

Fit details:

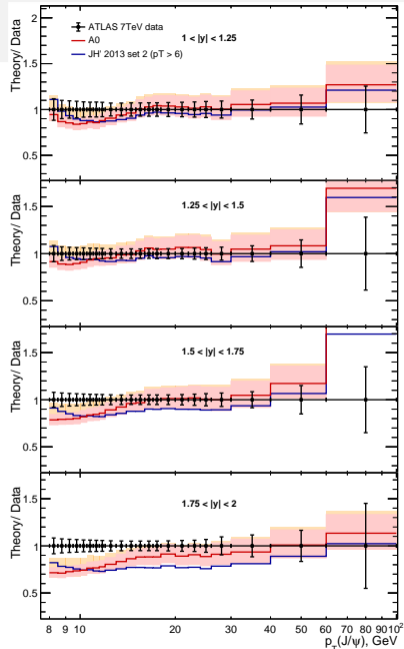
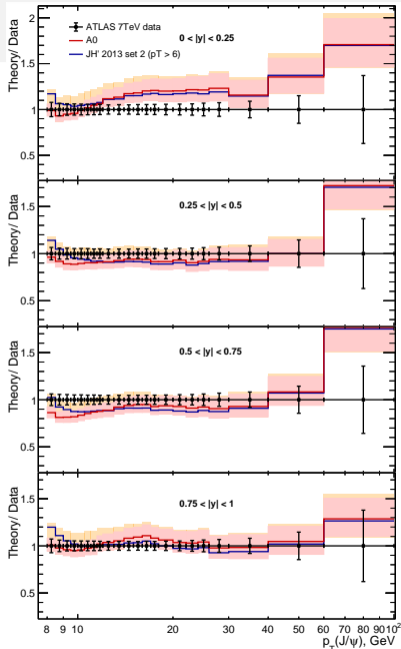
- Total n.d.f. = 983 (869 for $p_T > 6$ GeV) for J/ψ
- Total n.d.f. = 35 for χ_{cJ}
- χ^2 fit include only experimental unc.

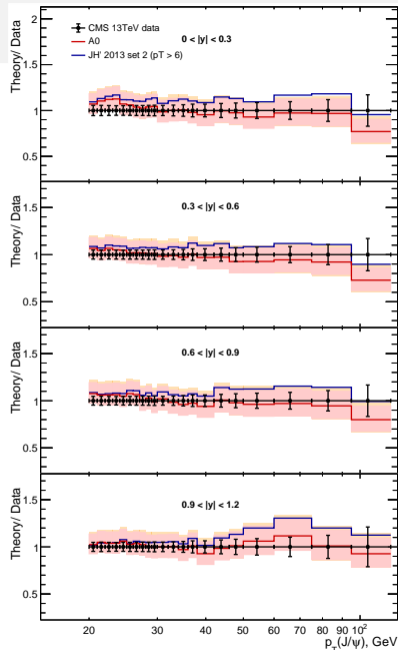
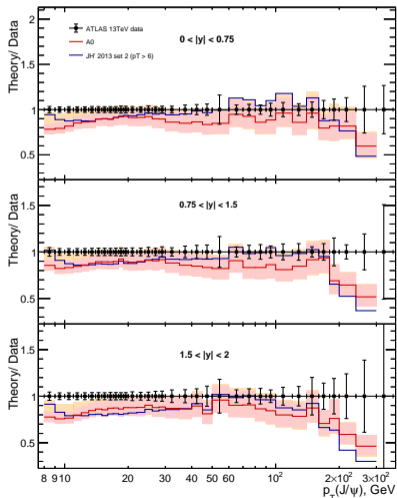
Theoretical uncertainties:

- Scale variation $\mu_R \rightarrow \mu_R/2, 2\mu_R$ with TMD A0±
- CO mass variation: $m_{CO} = m_{CS} + \Lambda_{QCD} \pm 50$ MeV
in CO→CS transition ${}^3P_1^{[8]} \rightarrow {}^3S_1^{[1]} + g$ and
 ${}^3S_1^{[8]} \rightarrow {}^3P_J^{[8]} (\rightarrow {}^3S_1^{[1]} + g_2) + g_1$

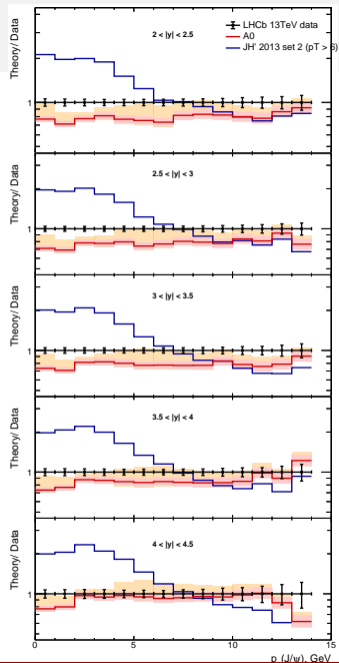
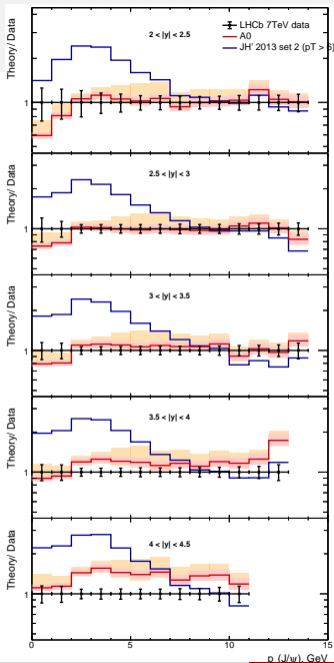
$\chi^2/n.d.f.$	A0	JH'2013 set 2 ($p_T > 6$)
ATLAS 5.02 TeV	0.36	1.1
ATLAS 7 TeV	3.5	4.2
ATLAS 8 TeV	0.96	1.1
ATLAS 13 TeV	9.6	7.7
CMS 5.02 TeV	2.4	6.8
CMS 7 TeV	1.9	0.53
CMS 13 TeV	0.75	2.6
LHCb 5 TeV	14.8	18.9
LHCb 7 TeV	4.6	5.3
LHCb 8 TeV	1.6	4.6
LHCb 13 TeV	15.6	5.9
Summary	4.47	4.02

LDME	A0	JH'2013 set 2 ($p_T > 6$)
$\langle \mathcal{O}^{J/\psi} [{}^3P_0^{[8]}] \rangle / \text{GeV}^5$	8.3×10^{-3}	14.6×10^{-3}
$\langle \mathcal{O}^{J/\psi} [{}^3S_1^{[8]}] \rangle / \text{GeV}^3$	11.6×10^{-4}	5.4×10^{-4}
$ R'(0) _{\chi_{c0}}^2 / \text{GeV}^5$	4.0×10^{-2}	5.6×10^{-2}
$\langle \mathcal{O}^{\chi_{c0}} [{}^3S_1^{[8]}] \rangle / \text{GeV}^3$	3.8×10^{-4}	3.8×10^{-4}

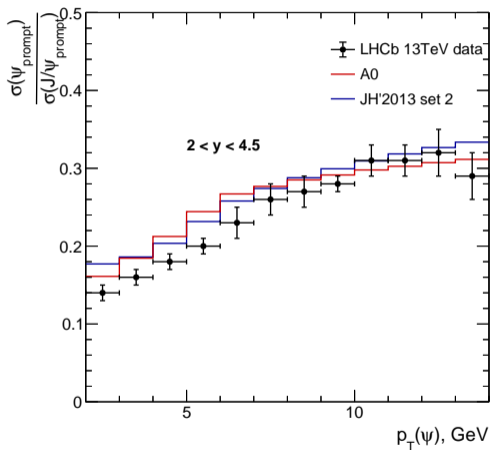
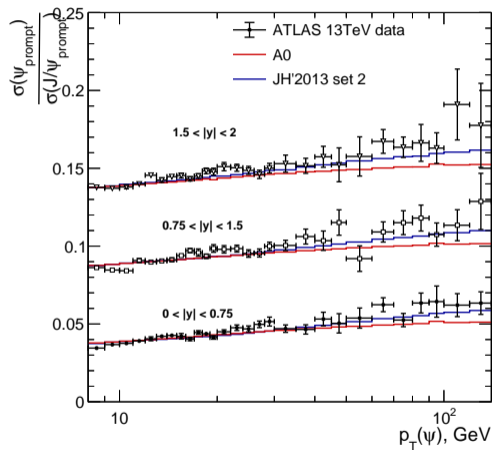




LHCb 7 and 13 TeV



Fraction of prompt ψ' and χ_{cJ} ALTAS and LHCb 13 TeV



Summary

- ▶ First attempt to carry out a simultaneous description of all available unpolarized data on ψ' , J/ψ , χ_c meson production in the entire kinematic region within high-energy factorization approach
- ▶ Tree-level NLO corrections allow to achieve a reasonable agreement with data in the whole kinematic range
- ▶ Results:
 - ↪ reproduce the differential cross sections
 - ↪ reproduce the fraction of prompt production $R_{\psi'/J/\psi}$ and $R_{\chi_{cJ}/J/\psi}$
 - ↪ A0 and JH'2013-set-2 gluon densities shows no significant difference at $p_T > 6$ GeV
 - ↪ JH'2013-set-2 overestimate data at low $p_T < 6$ GeV
 - ↪ LDME's for $^1S_0^{[8]}$ and $^3P_J^{[8]}$ could be determined as linear combination only for ψ' and J/ψ
- ▶ Comprehensive polarization analysis coming soon

This work was supported by Russian Science Foundation under grant № 25-22-00066,
<https://rscf.ru/en/project/25-22-00066/>