One-loop correction to the photon velocity in scalar Lorentz-violating QED.

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Introduction

Motivation

Violation of Lorentz Invariance may occur in theories of quantum gravity, noncommutative field theory, some approaches in string theory and others
Review: Addazi et al, 2022

Standard Model Extension (SME)

- Colladay and Kostelecky, 1998 : phenomenological consideration of an extension of the Standard Model with violation of Lorentz Invariance
- Kostelecky, Lane and Pickering, 2002 : calculation of infinite parts of one-loop diagrams in QED Sector of SME, proof of renormalizability
- ...
- Satunin, 2018 : A narrow subclass of QED SME – different max. velocities for photon $(1+c_\gamma)$ and electron $(1+c_e)$. Finite loop correction to photon dispersion relation: $k_0^2 = (1+2c_\gamma)\vec{k}^2 - \frac{e^2}{15\pi^2}\frac{(c_e-c_\gamma)^2\vec{k}^4}{m_e^2}$.

What happens for scalar particles instead of electrons?

Introduction: LIV Scalar QED

- Edwards and Kostelecky, 2018: a quadratic action for a complex (and Hermitian) scalar field in spacetime of arbitrary dimension, which includes all possible spin-independent Lorentz violation operators of arbitrary mass dimension
- Furtado, Filho and Asuncao, 2020: Infinite parts of one-loop diagrams for LIV only for photons
- Baeta Scarpelli, Felipe, Brito, and Petrov, 2022: Calculation of infinite parts of two-point diagrams in full LIV Scalar QED
- Altschul, Brito, Felipe, Karki, Lehum, and Petrov, 2023 : Calculation of infinite parts of three- and four-point scalar-vector vertex functions. Full one-loop renormalization of LIV Scalar QED

But how will the corrections behave in the simplest model, where the corrections are determined by a single component?

Extended quantum electrodynamics

The Lagrangian of the Scalar QED SME, C- and P- even subsector:
Edwards and Kostelecky, 2018

$$\mathcal{L} = (D_\mu \phi)^* (\eta^{\mu\nu} + c^{\mu\nu})(D_\nu \phi) - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} \kappa_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}.$$

$c^{\mu\nu}$ and $\kappa_{\mu\nu\lambda\rho}$ violate LI

$$c^{\mu\nu} = c_s \cdot \delta^i_\mu \delta^j_\nu \delta_{ij};$$
$$\kappa_{\mu\nu\lambda\rho} = c_\gamma \cdot \delta^i_\mu \delta^j_\nu \delta^k_\lambda \delta^l_\rho (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk}).$$

We consider two different maximal velocities:
$(1 + c_\gamma)$ – for the photon and
$(1 + c_s)$ – for scalars

$$\mathcal{L} = \partial_\sigma \phi^* \partial^\sigma \phi - c_s \partial_i \phi^* \partial^i \phi - m^2 \phi^* \phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} c_\gamma F_{ij} F^{ij} + ie(A^\sigma \phi \partial_\sigma \phi^* - A_\sigma \phi^* \partial^\sigma \phi) - c_s ie(A^i \phi \partial_i \phi^* - A_i \phi^* \partial^i \phi) + e^2 |A|^2 |\phi|^2 + c_s e^2 (A^i A_i) |\phi|^2$$

The simplest model

Photon propagator:

$$D_{\mu\sigma} = -i\frac{diag(1+c_\gamma, -1, -1, -1)}{k_0^2 - (1+c_\gamma)k_i^2}$$

Scalar propagator:

$$S(p) = \frac{i}{p_0^2 - (1+c_s)p_i^2 - m^2}$$

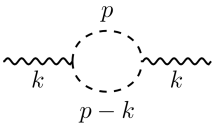
Vertex functions:

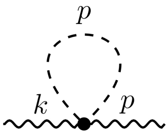
$$\Gamma_1^\sigma = ie\Big[(2p^\sigma - k^\sigma) - c_s(2p^i - k^i)\Big]$$

$$\Gamma_2^{\mu\nu} = 2ie^2(\eta^{\mu\nu} - c_s\eta^{ij})$$

Dispersion relation

$$s: \quad p_0^2 = m^2 + (1+c_s)(\vec{p})^2;$$
$$\gamma: \quad k_0^2 = (1+c_\gamma)(\vec{k})^2.$$





Photon polarization operator

$$\Pi(k) = -\frac{2e^2}{(4\pi)^2(\sqrt{1+c_s})^3}\int_0^1 dx\,(2x-1)(x-1)\cdot\left[\frac{2}{\epsilon} - \ln\Delta + \gamma - \ln(4\pi)\right]$$

On-shell renormalization scheme: $\Pi(0) = 0$.

$$\Pi(k) = \frac{2e^2}{(4\pi)^2(\sqrt{1+c_s})^3}\int_0^1 dx\,(2x-1)(x-1)\cdot\ln\left(1 + x(1-x)y\right)$$

where $y = \frac{k_i^2(c_s - c_\gamma)}{m^2}$

Photon propagator in LIV

Sum over one-particle reduced diagrams:

$$D_{00}^{loop} = -i\frac{1}{1-(1+c_\gamma)\Pi(k)} \cdot \frac{1+c_\gamma}{k_0^2 - k_i^2\Big(1+c_\gamma-(c_s-c_\gamma)\Pi(k)\Big)}; \qquad D_{0i}^{loop} = 0;$$

$$D_{ij}^{loop} = -i\frac{1}{1-\Pi(k)} \cdot \frac{g_{ij}}{k_0^2 - k_i^2\Big(1+c_\gamma-(c_s-c_\gamma)\Pi(k)\Big)}$$

$\frac{1}{1-\Pi(k)}$ and $\frac{1}{1-(1+c_\gamma)\Pi(k)}$ give the Landau pole.

Dispersion relation in LIV

$$k_0^2 = k_i^2\Big(1+c_\gamma-(c_s-c_\gamma)\Pi(k)\Big)$$

One-loop correction to the photon velocity

$$\Pi(k) = \frac{2e^2}{(4\pi)^2(\sqrt{1+c_s})^3}\int_0^1 dx\,(2x-1)(x-1)\cdot\ln\left(1+x(1-x)y\right)$$

where $y = \frac{k_i^2(c_s-c_\gamma)}{m^2}$

- $y > 0$: the photon polarization operator acquires a form:

$$\Pi(k) = \frac{2e^2}{(4\pi)^2(\sqrt{1+c_e})^3}\left(-\frac{4}{9}-\frac{4}{3y}+\frac{1}{3}\left(\frac{\sqrt{4+y}}{y^{3/2}}\right)(y-2)\ln\left(1-\frac{y}{\sqrt{y(4+y)}}\right)+\ldots\right.$$

- $-4 < y < 0$: the photon polarization operator acquires a form:

$$\Pi(k) = \frac{2e^2}{(4\pi)^2(\sqrt{1+c_s})^3}\left(-\frac{4}{9}-\frac{4}{3y}+\frac{1}{3}\left(\frac{4+y}{y}\right)^{3/2} arcth\left[-\sqrt{\frac{y}{4+y}}\right]\right)$$

One-loop correction to the photon velocity

Group photon velocity: $c_{ph} = |\frac{\partial k_0}{\partial k_i}|$

- $|y| \ll 1$:

$$\Pi \sim \frac{e^2}{480\pi^2(\sqrt{1+c_s})^{3/2}}|y|$$

$$k_0^2 = k_i^2\left(1 + c_\gamma - (c_s - c_\gamma)\frac{e^2}{480\pi^2(1+c_s)^{3/2}}\left|\frac{k_i^2(c_s - c_\gamma)}{m^2}\right|\right)$$

$$k_0^2 = k_i^2(1+c_\gamma) - \frac{k_i^4}{m_{eff}^2} \Rightarrow \qquad c_\gamma^{ph} \approx 1 + \frac{c_\gamma}{2} - \frac{3}{2}\frac{k_i^2}{m_{eff}^2}$$

Conclusion

- Thus, we have calculated corrections to the photon velocity in the theory with Lorentz violation in the model with preservation of SO(3) invariance.
- It is interesting to note that the resulting contribution comes with a negative sign, and the same result was obtained in the case of fermionic loops. In the future, it is also interesting to consider the contribution from vector particles, for example, from W bosons.

Thank you for your attention