Islands in the exterior of Schwarzschild black hole with a boundary

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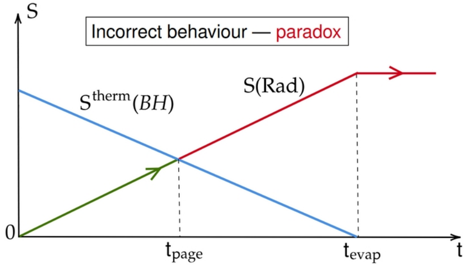
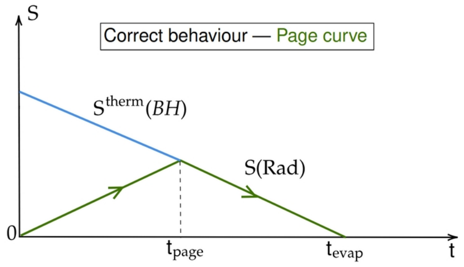
Introduction: information paradox and islands

Since Hawking's discovery that black hole radiates [Hawking '75], the information loss paradox has been one of the most intriguing issues in quantum gravity [Hawking '76]





Total system "Hawking radiation (R) + black hole (BH)" $\mathcal{H}_{\text{tot}} = \mathcal{H}_R \otimes \mathcal{H}_{\text{BH}}$

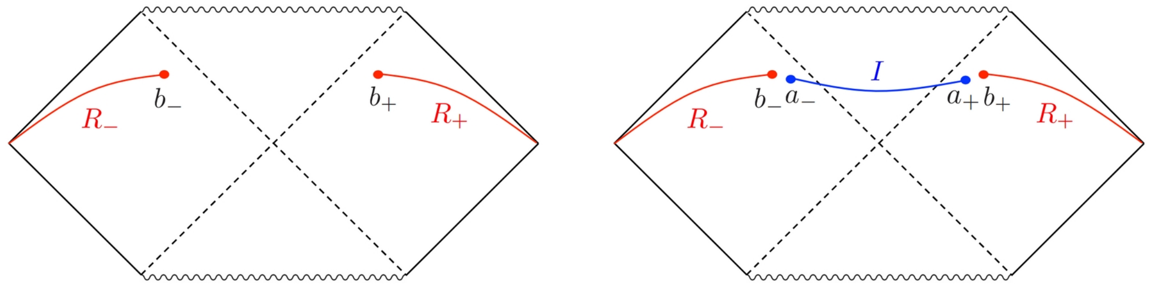
We assume $\rho_{\text{tot}}(t_0) = \rho^2_{\text{tot}}(t_0).$

If $\exists t_f > t_0 : \quad S_{\text{Rad}}(t_f) > S_{\text{therm}}(t_f)$

$\Rightarrow \rho_{\text{tot}}(t_f) \neq \rho^2_{\text{tot}}(t_f)$

The island formula reproduces the Page curve and suggests that the entropy of Hawking radiation is controlled by quantum extremal surfaces in a gravitational region [Almheir et al '2019]

$$S(R) = \min\left\{ \underset{\partial I}{\text{ext}} \left[\frac{\text{Area}(\partial I)}{4G} + S_{\text{matter}}(R \cup I) \right] \right\}$$



Total system “Hawking radiation (R) + black hole (BH)”

$$\mathcal{H}_{\rm tot} = \mathcal{H}_R \otimes \mathcal{H}_{\rm BH}$$

$$\rho_R = \underset{\mathcal{H}_{BH}}{\rm Tr}\, \rho_{\rm tot}, \quad \rho_{BH} = \underset{\mathcal{H}_R}{\rm Tr}\, \rho_{\rm tot}$$

The entanglement entropy of the subsystem is defined as $S(R) \equiv S_{\rm vN}(\rho_R) = -{\rm Tr}\rho_R \ln \rho_R$

The ”island formula” for entanglement entropy of quantum field theory in gravitational systems for

$$R \subset \Sigma, \quad \mathcal{H}_{\rm tot} = \mathcal{H}_R \otimes \mathcal{H}_{\bar{R}}, \quad \text{where } \bar{R} = \Sigma / R$$

Σ – constant time surface

$I \subset \bar{R}$ – ”island”, defined by extremization

∂I – its boundary

$S_{\rm matter}$ – entanglement entropy of the island plus the Hawking radiation, computed on a classical background

$$S(R) = \min \left\{ \underset{\partial I}{\rm ext} \left[\frac{{\rm Area}(\partial I)}{4G} + S_{\rm matter}(R \cup I) \right] \right\}$$

Introduction: fuzzballs

Different perspective from string theory: the classical horizon to be replaced in the microscopic theory by a horizonless cap or fuzzball state [Mathur '2005]

What becomes of the island prescription when the near-horizon region is replaced by a fuzzball-like cap?

Does a horizonless microstate geometry still admit a stable extremum of the generalized entropy, and if so, what geometric structure is responsible for it?

Main idea

$$S_{\rm gen}(R \cup I) = \frac{A(\partial I)}{4G} + S_{\rm bulk}(R \cup I)$$

We consider a two-dimensional approximation of a Schwarzschild black hole.

We replace the horizon by a stretched horizon* - a timelike reflecting wall placed just outside it.

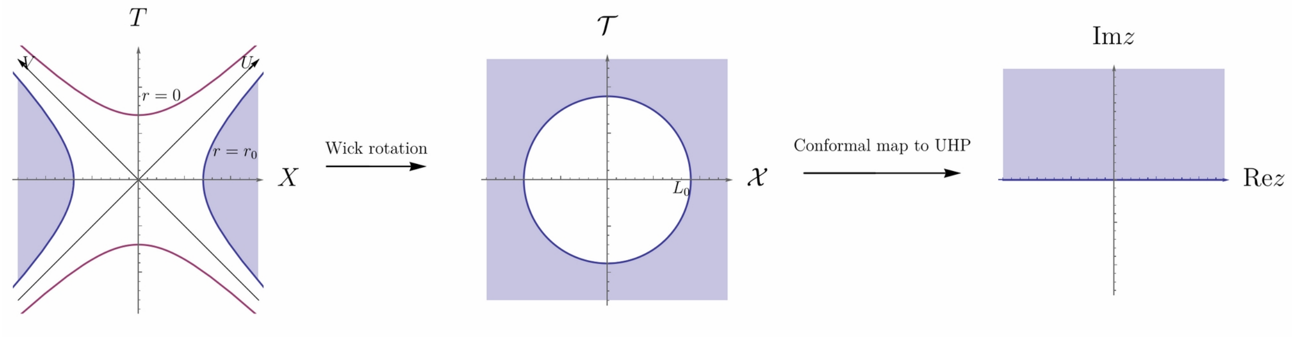
- No horizon
- Analytic island calculations
- Effect of the wall via stress tensor

We estimate the generalized entropy by combining the area term with the finite nonlocal part of the bulk entropy in a thin-slab approximation for the more realistic fuzzball models

- Different geometries, different area terms

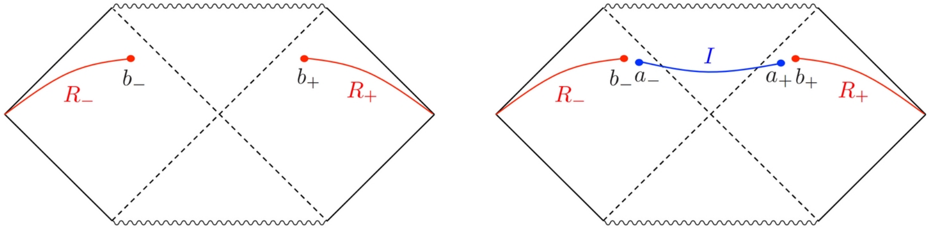
* [Price&Thorne'86, Susskind et al '93]

Island prescription



$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)}$$

$$r \geq r_0$$



$$S(R) = \min\left\{\underset{\partial I}{\text{ext}}\left[\frac{\text{Area}(\partial I)}{4G} + S_{\text{matter}}(R\cup I)\right]\right\}$$

$$\begin{cases} \partial_{r_a} S_{\text{gen}}[I,R](r_a,t_a) = 0, \\ \partial_{t_a} S_{\text{gen}}[I,R](r_a,t_a) = 0 \end{cases}$$

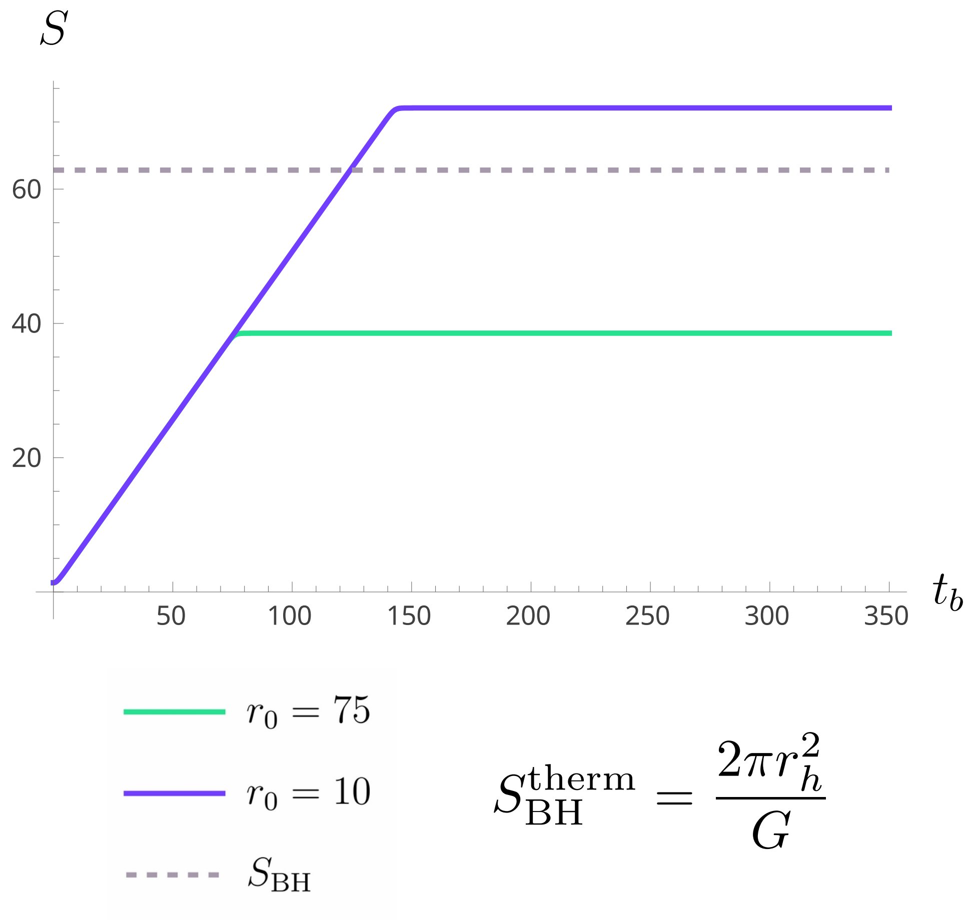
Island configuration extends between both exteriors $\quad I = [a_-, a_+], \quad a_\pm = \{r_a, \pm t_a\}$

Entanglement entropy without island

$$S(R) = \min\left\{ \underset{\partial I}{\text{ext}} \left[\frac{\text{Area}(\partial I)}{4G} + S_{\text{matter}}(R \cup I) \right] \right\}$$

$$I = \emptyset$$

As the wall is moved closer to the event horizon, the saturation value of the entropy increases and may exceed the thermodynamic entropy. Thus, in the no-island case, the information paradox is most acute when the stretched horizon lies very close to the event horizon.

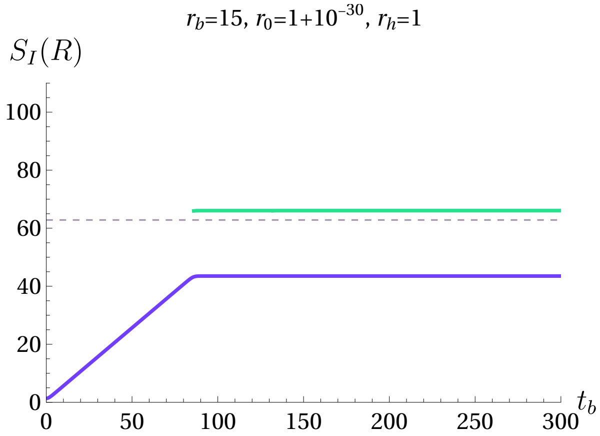


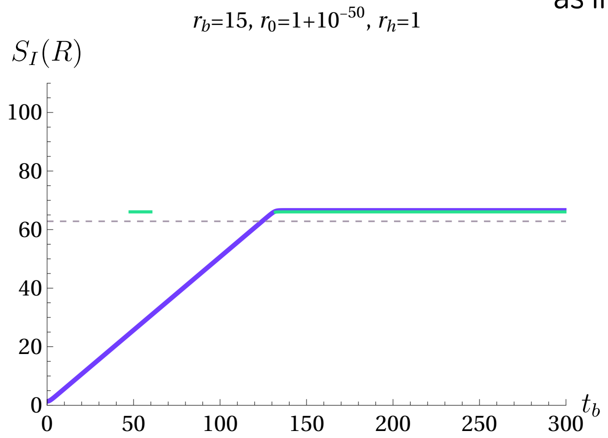
$$S_{\text{BH}}^{\text{therm}} = \frac{2\pi r_h^2}{G}$$

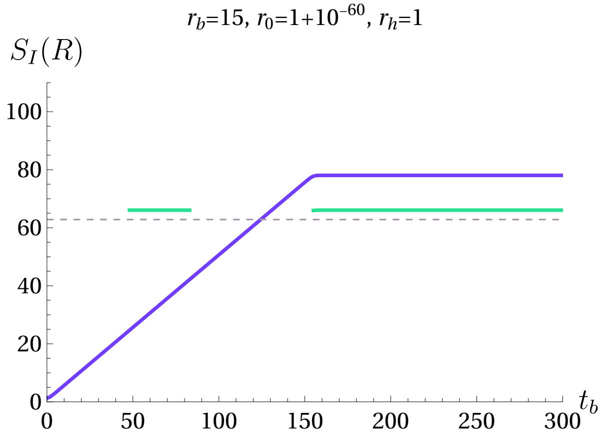
Entanglement entropy with island

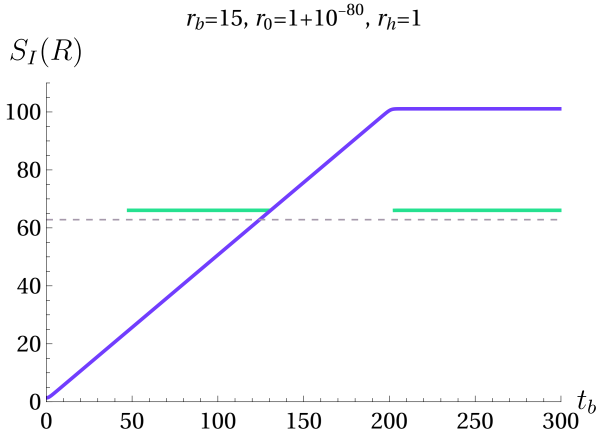
→ Blinking islands

as in [Ageev et al. '24]



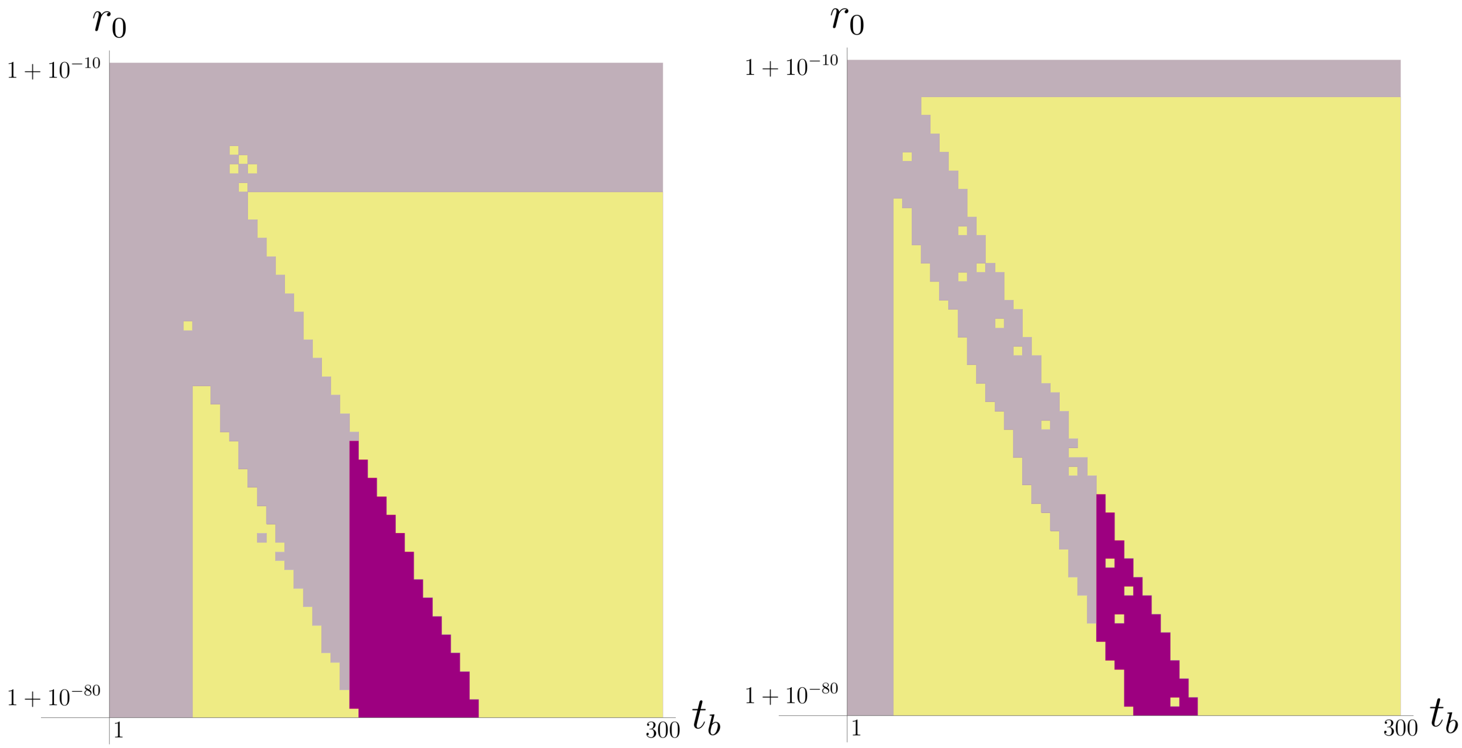






Entanglement entropy with island

Existence of an island and the appearance of the information paradox as functions of the wall position r_0 and the time t_b .



Gray - the absence of an island,
Yellow - the region where an island exists,
Pink - the region, where no island exists and the information paradox arises.

Stress tensor from the boundary anomaly

Two-dimensional CFT on a static background

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)}\,, \quad r \geq r_0$$

$$\langle T^\mu{}_\mu \rangle = \frac{c}{24\pi}\Big(R + 2K\,\delta(x_\perp)\Big)$$

$$\nabla_\mu \langle T^\mu{}_\nu \rangle = 0$$

A reflecting wall imposes vanishing energy flux through the boundary $\quad \Phi = 0$

$$\langle T_{tt} \rangle_{\text{bulk}} = \Omega + \frac{c}{24\pi} f f'' - \frac{c}{96\pi}\,(f')^2$$

$$\langle T_{rr} \rangle_{\text{bulk}} = \frac{1}{f^2}\left(\Omega - \frac{c}{96\pi}\,(f')^2\right)$$

$$\langle T_{tr} \rangle_{\text{bulk}} = \frac{\Phi}{f}$$

We require the state to be thermal at infinity

$$\Omega = \lim_{r\to\infty} \langle T_{tt} \rangle_{\text{bulk}} = \frac{\pi c}{6} T_\infty^2$$

The force at the wall

The boundary Ward identity

$$\nabla_\mu T^{\mu x} = \delta(x) D$$

The proper wall temperature

$$T_0 \equiv T_{\rm loc}(r_0) = \frac{T_\infty}{\sqrt{1 - r_h/r_0}}$$

Two-dimensional conformal field theory in the presence of the reflecting wall

$$ds^2 = -f(r)\, dt^2 + \frac{dr^2}{f(r)}$$

$$f(r) = 1 - \frac{r_h}{r}, \quad r \geq r_0$$

The displacement expectation value for the Schwarzschild metric

$$D = \frac{\pi c}{6} T_0^2 + \frac{c}{96\pi} \frac{r_h^2}{r_0^4 \left(1 - \frac{r_h}{r_0}\right)}$$

At fixed proper wall temperature T_0 , the geometric contribution to D diverges as $r_0 \to r_h$. This divergence reflects the large proper acceleration, directed toward increasing r , required to hold the wall static near the horizon.

Stretched horizon in higher dimensional black hole

$$S_{\text{gen}}(R \cup I) = \frac{A(\partial I)}{4G} + S_{\text{bulk}}(R \cup I)$$

We use thin slab approximation with transverse area $A_{\perp}{=}r_h^{d-2}$ for the bulk entropy

[Bousso&Penington'2024]

$$S_{\text{gen}}(\rho_I) = \frac{A(\rho_I)}{4G} - \kappa \frac{A_{\perp}}{(\rho_R - \rho_I)^{d-2}}$$

For a Schwarzschild black hole, the near-horizon area exhibits the standard quadratic dependence on proper distance,

$$A \sim A_h + \rho_I^2$$

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{d-2}^2$$

$$f(r) = 1 - \frac{r_h^{d-3}}{r^{d-3}} \qquad 0 < \rho_I < \rho_R$$

Proper distance near horizon

$$\rho = 2\sqrt{\frac{r_h(r - r_h)}{d-3}}$$

We restrict to a static setup and the boundary ∂I is a sphere of fixed radius on the $t{=}0$ slice

In the regime $\rho_I \ll \rho_R \qquad \rho_I \sim \dfrac{G\, r_h^2}{\rho_R^{d-1}}$

D1–D5 and superstrata

$$\frac{\partial S_{\text{gen}}(\rho_I)}{\partial \rho_I} = \frac{A'(\rho_I)}{4G} - \frac{\kappa(d-2)A_\perp}{(\rho_R - \rho_I)^{d-1}} = 0$$

$$0 < \rho_I < \rho_R$$

$$ds_6^2 = -\frac{2}{\sqrt{\mathcal{P}}}(dv+\beta)\left(du + \omega + \frac{\mathcal{F}}{2}(dv+\beta)\right) + \sqrt{\mathcal{P}}\, ds_4^2$$

Cap $r \to 0$	AdS_3	$A_{\text{cap}}(\rho) \approx 4\pi^3\ell^4\rho$	Local maximum $\rho_I = 0$
Throat Intermediate region	Extremal BTZ black hole	$\frac{dA_{\text{throat}}}{d\rho} \ll 1$ Deep in the throat	The extremization condition can be satisfied only if the radiation region is placed at parametrically large proper distance $\rho_R - \rho_I \to \infty$

Bubbling solution

$$\frac{\partial S_{\text{gen}}(\rho_I)}{\partial \rho_I} = \frac{A'(\rho_I)}{4G} - \frac{\kappa(d-2)A_\perp}{(\rho_R - \rho_I)^{d-1}} = 0$$

$$ds_5^2 = -\frac{1}{(Z_3\mathcal{P})^{2/3}}(dt+\omega)^2 + (Z_3\mathcal{P})^{1/3} ds_4^2$$

$$0 < \rho_I < \rho_R$$

Cap $r^2 \ll a^2$	$A_{\text{cap}}(\rho) \approx A_0 + \text{Const} \cdot \rho^2$	Local minimum in the region very close to the cap
Throat $a^2 \ll r^2 \ll Q_\alpha$	$A_{\text{throat}} = \text{Const}$ $A'_{\text{throat}} = 0$	The extremization equation has no solution supported purely inside the throat.

Conclusions

The main goal of this work was to explore the island prescription and how it resolves the information paradox in the context of fuzzball-like descriptions of black holes.

The setup with the stretched horizon captures a key feature of the fuzzball paradigm, namely, the absence of a traditional interior region, while remaining that of where the island prescription could be applied explicitly.

Once a boundary is introduced near the horizon, the island prescription is no longer guaranteed to provide a robust resolution of the paradox at all times.

More realistic fuzzball models do not necessarily have an island solution. We get that the island solution to the extremization equation of generalized entropy exists for the quadratic dependence on the island position of the area term.