Structure of $\mathcal{N} = 2$ superfield higher-spin Abelian cubic interactions

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Cubic (s_1, s_2, s_3) vertices in $4D$ flat space

Classification: [Metsaev, 2005; 2007], $\mathcal{N} = 1$, $\mathcal{N} = 4\mathbb{N}$ SUSY [Metsaev, 2019; 2019].

Vertices (with integer spins $s_1 \geq s_2 \geq s_3$) contain either $s_1 + s_2 + s_3$ derivatives or $s_1 + s_2 - s_3$ derivatives. Depending on the values of the spins and the number of derivatives, these vertices can be divided into several types:

I. **Trivially gauge invariant or Born-Infeld-type** ($s_1 \leq s_2 + s_3$; $s_1 + s_2 + s_3$ derivatives)
Such vertices can be constructed in terms of the abelian curvatures and do not deform the linearized gauge transformations [Damour, Deser 1987].

$$\mathcal{L}^{\mathrm{I}}_{int} \sim \epsilon^{\alpha_1\beta_1} \ldots \epsilon^{\alpha_{2s_1}\gamma_{2s_3}} \quad C_{\alpha(2s_1)} C_{\beta(2s_2)} C_{\gamma(2s_3)}.$$

II. **Abelian or Chern-Simons-type** ($s_1 \geq s_2 + s_3$; $s_1 + s_2 \pm s_3$ derivatives)
In this case, the gauge transformation receive non-trivial corrections that mix fields of different spins, although *the gauge algebra is abelian* [Berends, Burgers, van Dam 1985].

$$\mathcal{L}^{\mathrm{II}}_{int} \sim \Phi^{\alpha(s_1)\dot{\alpha}(s_1)} J_{\alpha(s_1)\dot{\alpha}(s_1)}, \qquad J \sim \partial C \partial \bar{C}.$$

III. **Non-abelian or Yang-Mills-type** ($s_1 < s_2 + s_3$; $s_1 + s_2 - s_3$ derivatives)
In this case, cubic vertices require deformation of gauge transformations which lead to *non-abelian gauge algebra*. The structure of such vertices is the most tricky, see e.g. examples [Berends 1984; Fradkin, Vasiliev 1987; Boulanger 2006; Zinoviev 2008].

A recent analysis [Tatarenko, Vasiliev 2024] confirms that, for $d = 4$, the vertices obtained from Metsaev classification match exactly the current deformation of the free higher-spin equations coming from the non-linear $4d$ higher-spin theory [Vasiliev 1990, 1992] .

Abelian higher-spin interactions

- $4D$ free massless higher-spin theories [Fronsdal 1978], [Fang, Fronsdal 1978] are described by real fields
$$\left\{\Phi^{\alpha(s)\dot\alpha(s)}, \Phi^{\alpha(s-2)\dot\alpha(s-2)}\right\}$$
with gauge transformations:
$$\delta_\xi \Phi^{\alpha(s)\dot\alpha(s)} = \partial^{(\alpha(\dot\alpha}\xi^{\alpha(s-1))\dot\alpha(s-1))}, \quad \delta_\xi \Phi^{\alpha(s-2)\dot\alpha(s-2)} = \partial_{\alpha\dot\alpha}\xi^{\alpha(s-1)\dot\alpha(s-1)}.$$
- Explicit construction of gauge-invariant conserved currents for massless fields of arbitrary Spin [Berends, Burgers, van Dam 1986], [Manvelyan, Mkrtchyan, Ruhl 2009]:
$$\boxed{s_1 \geq 2s_2} \quad J_{\alpha(s_1)\dot\alpha(s_1)} = c\,(i)^{s_1-2s_2} \sum_{p=0}^{s_1-2s_2} (-1)^p \frac{\binom{s_1}{2s_2+p}\binom{s_1}{p}}{\binom{s_1}{2s_2}}\,\partial^p C_{\alpha(2s_2)}\,\partial^{(s_1-2s_2-p)}\bar{C}_{\dot\alpha(2s_2)}.$$
Weyl-like higher-spin gauge invariant tensor is defined as:
$$C_{(\alpha_1\dots\alpha_{2s})} = \partial_{(\alpha_1}{}^{\dot\beta_1}\dots\partial_{\alpha_s}{}^{\dot\beta_s}\Phi_{\alpha_{s+1}\dots\alpha_{2s})(\dot\beta_1\dots\dot\beta_s)}, \qquad \partial_{\dot\alpha}{}^{\alpha_1} C_{(\alpha_1\dots\alpha_{2s})} \approx 0.$$
- Trilinear interaction between a massless spin-s_1 field and two spin- s_2 fields ($s_1 \geq 2s_2$):
$$\mathcal{L}_{int} \sim \phi^{\alpha(s_1)\dot\alpha(s_1)} \times J_{\alpha(s_1)\dot\alpha(s_1)}.$$
- The inverse Noether theorem determines the corresponding spin s_1 gauge transformations for spin s_2 fields.

Supersymmetric higher-spin theories

? $\mathcal{N}=2$ gauge-invariant higher-spin supercurrents and Abelian interactions

- The component approach to the description of $4D, \mathcal{N}=1$ supersymmetric free massless higher spin models was initiated in [Courtright 1979], [Vasiliev 1980].
- The complete off-shell Lagrangian formulation of $4D$ free higher spin $\mathcal{N}=1$ models (including those on the AdS background) has been given in terms of $\mathcal{N}=1$ superfields in a series of works by S. Kuzenko with collaborators [Kuzenko et al, 1993, 1994].
- $4D, \mathcal{N}=1$ unconstrained higher-spin prepotentials as irreducible representations of super Poincaré group [Gates, Koutrolikos 2013].
- Conserved $\mathcal{N}=1$ higher spin supercurrents and Abelian cubic interactions [Buchbinder, Gates, Koutrolikos 2018], [Gates, Koutrolikos 2019].
- An off-shell manifestly $\mathcal{N}=2$ supersymmetric unconstrained formulation of $4D, \mathcal{N}=2$ superextension of the Fronsdal theory for the integer spins in the harmonic superspace approach [Buchbinder, Ivanov, N.Z. 2021].
- Cubic interactions of $\mathcal{N}=2$ higher spins with hypermultiplet [Buchbinder, Ivanov, N.Z. 2022].
- Superconformal $\mathcal{N}=2$ higher spins and superconformal hypermultiplet higher-spin interactions [Buchbinder, Ivanov, N.Z. 2024].

Harmonic superspace

- Harmonic superspace [Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev 1984] is an efficient approach to deal with supersymmetric theories with 8 real SUSY generators in a manifestly covariant manner.
- In harmonic superspace there are auxiliary coordinates – $SU(2)$ harmonics $u_i^{\pm}$:

$$\mathbb{HR}^{4+2|8} := \frac{\{P_m, L_{mn}, Q^i_\alpha, \bar{Q}^i_{\dot{\alpha}}, su(2)\}}{\{L_{mn}, u(1)\}} = \mathbb{R}^{4|8} \times S^2 = \{x^m, \theta^+_\alpha, \bar{\theta}^+_{\dot{\alpha}}, \theta^-_\alpha, \bar{\theta}^-_{\dot{\alpha}}, u^{\pm i}\}$$

- Harmonic variables satisfy $SU(2)$ relations $u^-_i = (u^{+i})^*, u^{i+}u^-_i = 1$.
- Using harmonics, one can convert $SU(2)$ indices to $U(1)$:

$$Q^i_\alpha \quad \to \quad Q^+_\alpha = Q^i_\alpha u^+_i, \quad Q^-_\alpha = Q^i_\alpha u^-_i.$$

- In HSS we consider superfields with fixed $U(1)$ charge. This allows one to work effectively with functions on $S^2 \sim SU(2)/U(1)$.
- Superfields in HSS: $Q^{(n)} = Q^{(n)}(x^a, \theta^i_\alpha, u^{\pm i})$ have infinitely many components.
- Harmonic superspace has a new invariant subspace containing only half of the original Grassmann variables. Analytic superspace:

$$\mathbb{HA}^{4+2|4} := \frac{\{L_{mn}, P_m, Q^{\pm}_\alpha, \bar{Q}^{\pm}_{\dot{\alpha}}, su(2)\}}{\{L_{mn}, Q^+_\alpha, \bar{Q}^+_{\dot{\alpha}}, u(1)\}} = (x^m, \theta^+_\alpha, \bar{\theta}^+_{\dot{\alpha}}, u^{\pm i}) := \zeta.$$

- Analytic basis is defined by relations

$$x_A^m = x^m - 2i\theta^{(i}\sigma^m\bar{\theta}^{j)}u_i^+u_j^-, \qquad \theta^{\pm}_{A\,\alpha} = u_i^{\pm}\theta^i_{\alpha}, \qquad \bar{\theta}^{\pm}_{A\,\dot{\alpha}} = u_i^{\pm}\bar{\theta}^i_{\dot{\alpha}}.$$

- $\mathcal{N} = 2$ supersymmetry action on analytic coordinates

$$\delta_\epsilon x_A^m = -2i\left(\epsilon^i\sigma^m\bar{\theta}^+ + \theta^+\sigma^m\bar{\epsilon}^i\right)u_i^-, \qquad \delta_\epsilon\theta^{\pm}_{A\,\hat{\alpha}} = u_i^{\pm}\epsilon^i_{\hat{\alpha}}, \qquad \delta_\epsilon u_i^{\pm} = 0.$$

- Analytic superspace is real under *tilde-conjugation*:

$$\widetilde{x^{\alpha\dot{\alpha}}} = x^{\alpha\dot{\alpha}}, \quad \widetilde{\theta^{\pm}_{\alpha}} = \bar{\theta}^{\pm}_{\dot{\alpha}}, \quad \widetilde{\bar{\theta}^{\pm}_{\dot{\alpha}}} = -\theta^{\pm}_{\alpha}, \quad \widetilde{u^{\pm i}} = -u_i^{\pm}, \quad \widetilde{u_i^{\pm}} = u^{\pm i}.$$

- Covariant harmonic derivatives:

$$\mathcal{D}^{++} = \partial^{++} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \theta^{+\hat{\alpha}}\partial^+_{\hat{\alpha}} + i\theta^{+\hat{\alpha}}\theta^+_{\hat{\alpha}}\partial_5,$$

$$\mathcal{D}^{--} = \partial^{--} - 4i\theta^{-\alpha}\bar{\theta}^{-\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + \theta^{-\hat{\alpha}}\partial^-_{\hat{\alpha}} + i\theta^{-\hat{\alpha}}\theta^-_{\hat{\alpha}}\partial_5,$$

$$\mathcal{D}^0 = \partial^0 + \theta^{+\hat{\alpha}}\partial^-_{\hat{\alpha}} - \theta^{-\hat{\alpha}}\partial^+_{\hat{\alpha}}.$$

- Covariant spinor derivatives:

$$\begin{aligned}
&\mathcal{D}^+_{\hat{\alpha}} := \partial^+_{\hat{\alpha}},\\
&\mathcal{D}^-_{\alpha} := -\partial^-_{\alpha} + 4i\bar{\theta}^{-\dot{\alpha}}\partial_{\alpha\dot{\alpha}} - 2i\theta^-_{\alpha}\partial_5,\\
&\bar{\mathcal{D}}^-_{\dot{\alpha}} := -\partial^-_{\dot{\alpha}} - 4i\theta^{-\alpha}\partial_{\alpha\dot{\alpha}} - 2i\bar{\theta}^-_{\dot{\alpha}}\partial_5.
\end{aligned}$$

$\mathcal{N}=2$ vector multiplet in harmonic superspace

- $\mathcal{N}=2$ vector multiplet is described by the unconstrained analytic prepotential $\mathbf{V}^{++}$ ($\delta_\epsilon V^{++}=0$) with the gauge transformation:

$$\delta_\lambda V^{++} = \mathcal{D}^{++}\lambda.$$

- V^{++} contains **infinite set of gauge d.o.f.** One can fix the Wess-Zumino-type gauge:

$$\begin{aligned} V^{++}_{WZ} =& -4i\theta^+_\beta\bar\theta^+_{\dot\beta}A^{\beta\dot\beta} + i(\bar\theta^+)^2\phi - i(\theta^+)^2\bar\phi \\ &+ (\bar\theta^+)^2\theta^{+\beta}\psi^i_\beta u^-_i + (\theta^+)^2\bar\theta^{+\dot\beta}\bar\psi^i_{\dot\beta}u^-_i + (\theta^+)^2(\bar\theta^+)^2 D^{(ij)}u^-_i u^-_j\,. \end{aligned}$$

- Supersymmetric gauge invariant action is

$$S_{(s=1)} = \frac{1}{2}\int d^4x d^8\theta du\; V^{++}V^{--} = \int d^4x d^8\theta du_1 du_2 \frac{V^{++}(u_1)V^{++}(u_2)}{(u^+_1u^+_2)^2},$$

where V^{--} is defined as the solution of the *zero-curvature equation*:

$$\mathcal{D}^{++}V^{--} = \mathcal{D}^{--}V^{++}.$$

- Supersymmetric equations of motion are

$$(\mathcal{D}^+)^4 V^{--} \approx 0.$$

- Gauge invariant superfield strengths are

$$\mathcal{W} := (\bar{\mathcal{D}}^+)^2 V^{--}, \qquad \bar{\mathcal{W}} := (\mathcal{D}^+)^2 V^{--},$$

and satisfy

$$\mathcal{D}^{\pm\pm}\mathcal{W} = 0, \qquad \bar{\mathcal{D}}^\pm_{\dot\beta}\mathcal{W} = 0, \qquad (\mathcal{D}^+)^2\mathcal{W} \approx 0, \qquad \partial^{\alpha}_{\dot\alpha}\mathcal{D}^+_\alpha\mathcal{W} \approx 0.$$

$\mathcal{N} = 2$ SYM theory

- Non-abelian supergauge transformations:

$$\delta V^{++} = [\mathcal{D}^{++} + iV^{++}, \lambda].$$

- $\mathcal{N} = 2$ SYM action [Zupnik 1987]:

$$S_{\text{SYM}}^{\mathcal{N}=2} = \frac{1}{2} \sum_{n=2}^{\infty} \frac{(-i)^n}{n} \text{Tr} \int d^4x d^8\theta du_1 \dots du_n \, \frac{V^{++}(x, \theta, u_1) \dots V^{++}(x, \theta, u_n)}{(u_1^+ u_2^+) \dots (u_n^+ u_1^+)}.$$

- The harmonic distributions are defined by the relations [Galperin 1985]:

$$\partial_1^{++} \frac{1}{(u_1^+ u_2^+)} = \delta^{(1,-1)}(u_1, u_2),$$

$$\partial_1^{++} \frac{1}{(u_1^+ u_2^+)^2} = \partial_1^{--} \delta^{(2,-2)}(u_1, u_2).$$

$\mathcal{N} = 2$ higher spins in harmonic superspace

- Analytic prepotentials for $\mathcal{N} = 2$ higher-spin integer spin multiplets:

$$h^{++\alpha(s-1)\dot\alpha(s-1)}, \quad h^{++\alpha(s-1)\dot\alpha(s-2)}, \quad h^{++\alpha(s-2)\dot\alpha(s-1)}, \quad h^{++\alpha(s-2)\dot\alpha(s-2)}.$$

- Gauge transformations are given by:

$$\begin{aligned}
\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-1)} &= \mathcal{D}^{++}\lambda^{\alpha(s-1)\dot\alpha(s-1)} \\
&\qquad + 4i\big[\lambda^{+\alpha(s-1)(\dot\alpha(s-2)}\bar\theta^{+\dot\alpha_{s-1})} + \theta^{+(\alpha_{s-1}}\bar\lambda^{+\alpha(s-2))\dot\alpha(s-1)}\big], \\
\delta_\lambda h^{++\alpha(s-2)\dot\alpha(s-2)} &= \mathcal{D}^{++}\lambda^{\alpha(s-2)\dot\alpha(s-2)} \\
&\qquad - 2i\,\big[\lambda^{+(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)}\theta^{+}_{\alpha_{s-1}} + \bar\lambda^{+(\dot\alpha(s-2)\dot\alpha_{s-1})\alpha(s-2)}\bar\theta^{+}_{\dot\alpha_{s-1}}\big], \\
\delta_\lambda h^{++\alpha(s-1)\dot\alpha(s-2)+} &= \mathcal{D}^{++}\lambda^{+\alpha(s-1)\dot\alpha(s-2)}, \\
\delta_\lambda h^{++\dot\alpha(s-1)\alpha(s-2)+} &= \mathcal{D}^{++}\bar\lambda^{+\dot\alpha(s-1)\alpha(s-2)}.
\end{aligned}$$

- $\mathcal{N} = 2$ rigid supersymmetry transformations ($\epsilon^{\pm\alpha} := \epsilon^{\alpha i}u^{\pm}_i$, $\bar\epsilon^{\pm\dot\alpha} := \bar\epsilon^{\dot\alpha i}u^{\pm}_i$):

$$\begin{aligned}
\delta_\epsilon h^{++\alpha(s-1)\dot\alpha(s-1)} &= -4i\big[h^{++\alpha(s-1)(\dot\alpha(s-2)+}\bar\epsilon^{-\dot\alpha_{s-1})} - h^{++\dot\alpha(s-1)(\alpha(s-2)+}\,\epsilon^{-\alpha_{s-1})}\big], \\
\delta_\epsilon h^{++\alpha(s-2)\dot\alpha(s-2)} &= 2i\big[h^{++(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)+}\epsilon^{-}_{\alpha_{s-1}} + h^{++\alpha(s-2)(\dot\alpha(s-2)\dot\alpha_{s-1})+}\,\bar\epsilon^{-}_{\dot\alpha_{s-1}}\big], \\
\delta_\epsilon h^{++\alpha(s-1)\dot\alpha(s-2)+} &= 0, \\
\delta_\epsilon h^{++\dot\alpha(s-1)\alpha(s-2)+} &= 0.
\end{aligned}$$

- Using gauge freedom, one can impose the Wess-Zumino-type gauge:

$$
\begin{aligned}
h_{WZ}^{++\alpha(s-1)\dot\alpha(s-1)} &= -4i\theta^+_\beta\bar\theta^+_{\dot\beta}\Phi^{(\beta\alpha(s-1))(\dot\beta\dot\alpha(s-1))} - 4i\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}\Phi^{\alpha(s-2))\dot\alpha(s-2))} \\
&+ (\bar\theta^+)^2\theta^{+\beta}\psi_\beta^{\alpha(s-1)\dot\alpha(s-1)i}u_i^- + (\theta^+)^2\bar\theta^{+\dot\beta}\bar\psi_{\dot\beta}^{\alpha(s-1)\dot\alpha(s-1)i}u_i^- \\
&+ (\theta^+)^2(\bar\theta^+)^2 V^{\alpha(s-1)\dot\alpha(s-1)(ij)}u_i^-u_j^- \;, \\
h_{WZ}^{++\alpha(s-2)\dot\alpha(s-2)} &= -4i\theta^+_\beta\bar\theta^+_{\dot\beta}C^{(\beta\alpha(s-2))(\dot\beta\dot\alpha(s-2))} - 4i\theta^{+(\alpha}\bar\theta^{+(\dot\alpha}C^{\alpha(s-3))\dot\alpha(s-3))} \\
&+ (\bar\theta^+)^2\theta^+_\beta\rho^{(\beta\alpha(s-2))\dot\alpha(s-2)i}u_i^- + (\bar\theta^+)^2\theta^{+(\alpha}\bar\psi^{\alpha(s-3))\dot\alpha(s-2)i}u_i^- \\
&+ (\theta^+)^2\bar\theta^+_{\dot\beta}\bar\rho^{\alpha(s-2)(\dot\beta\dot\alpha(s-2))i}u_i^- + (\theta^+)^2\bar\theta^{+(\dot\alpha}\psi^{\alpha(s-2)\dot\alpha(s-3))i}u_i^- \\
&+ (\theta^+)^2(\bar\theta^+)^2 S^{\alpha(s-2)\dot\alpha(s-2)(ij)}u_i^-u_j^- \;, \\
h_{WZ}^{++\alpha(s-1)\dot\alpha(s-2)+} &= (\theta^+)^2\bar\theta^+_{\dot\beta}P^{\alpha(s-1)\dot\alpha(s-2)\dot\beta} + \left(\bar\theta^+\right)^2\theta^+_\beta T^{\dot\alpha(s-2)\alpha(s-1)\beta} \\
&+ (\theta^+)^2(\bar\theta^+)^2\chi^{\alpha(s-1)\dot\alpha(s-2)i}u_i^- \;, \\
h_{WZ}^{++\dot\alpha(s-1)\alpha(s-2)+} &= \left(\widetilde{h_{WZ}^{++\alpha(s-1)\dot\alpha(s-2)+}}\right).
\end{aligned}
$$

- In this gauge we obtain fields of the $\mathcal{N}=2$ spin s off-shell supermultiplet:

$$
\begin{aligned}
\textit{Physical fields}: \quad & \left(\Phi^{\alpha(s)\dot\alpha(s)}, \Phi^{\alpha(s-1)\dot\alpha(s-1)}\right), \left(\psi_\rho^{\alpha(s-1)\dot\alpha(s-1))i}, \psi^{\alpha(s-2)\dot\alpha(s-3)i}\right), \\
& \left(C^{\alpha(s-1)\dot\alpha(s-1)}, C^{\alpha(s-3)\dot\alpha(s-3)}\right), \\
\textit{Auxiliary fields}: \quad & V^{\alpha(s-1)\dot\alpha(s-1)(ij)}, \rho^{\alpha(s-1)\dot\alpha(s-2)\,i}, S^{\alpha(s-2)\dot\alpha(s-2)(ij)}, \\
& T^{\alpha(s-1)\beta\dot\alpha(s-2)}, P^{\alpha(s-1)\dot\alpha(s-2)\dot\beta}, \chi^{\alpha(s-1)\dot\alpha(s-2)i}.
\end{aligned}
$$

- Physical fields correspond to the massless Fronsdal spin s, doublet of spin $s-\frac{1}{2}$ Fang-Fronsdal fields and spin $s-1$ Fronsdal field.

- $\mathcal{N}=2$ supersymmetry invariant superfields ($\delta_\epsilon G^{++\cdots}=0$) are defined by relations:

$$G^{++\alpha(s-1)\dot\alpha(s-1)} = h^{++\alpha(s-1)\dot\alpha(s-1)} + 4i\big[h^{++\alpha(s-1)(\dot\alpha(s-2)+}\bar\theta^{-\dot\alpha_{s-1})} - h^{++\dot\alpha(s-1)(\alpha(s-2)+}\,\theta^{-\alpha_{s-1})}\big],$$

$$G^{++\alpha(s-2)\dot\alpha(s-2)} = h^{++\alpha(s-2)\dot\alpha(s-2)} - 2i\big[h^{++(\alpha(s-2)\alpha_{s-1})\dot\alpha(s-2)+}\theta^{-}_{\alpha_{s-1}} + h^{++\alpha(s-2)(\dot\alpha(s-2)\dot\alpha_{s-1})+}\,\bar\theta^{-}_{\dot\alpha_{s-1}}\big],$$

$$G^{++\alpha(s-1)\dot\alpha(s-2)+} = -h^{++\alpha(s-1)\dot\alpha(s-2)+},$$

$$G^{++\alpha(s-2)\dot\alpha(s-1)+} = -h^{++\alpha(s-2)\dot\alpha(s-1)+}.$$

- Using zero-curvature conditions,

$$\mathcal{D}^{++}G^{--\alpha(s-2)\dot\alpha(s-2)M} = \mathcal{D}^{--}G^{++\alpha(s-2)\dot\alpha(s-2)M},$$

$$\mathcal{D}^{++}G^{--\alpha(s-2)\dot\alpha(s-2)\hat\beta-} + G^{--\alpha(s-2)\dot\alpha(s-2)\hat\beta+} = 0.$$

  one can construct negatively charged potentials $G^{--\cdots}$, which play important role in the harmonic superfield formulation.

- We use notations for indices $M=\{\alpha\dot\alpha,\alpha,\dot\alpha,5\}$ and $\hat\alpha=\{\alpha,\dot\alpha\}$. We also omit the index 5, so, $h^{++\alpha(s-2)\dot\alpha(s-2)5}=h^{++\alpha(s-2)\dot\alpha(s-2)}$.

Superfield action and equations of motions

- $\mathcal{N} = 2$ supersymmetric action invariant under gauge transformations has a universal form for all spin **s** supermultiplets:

$$S_{(s)} = (-1)^{s+1} \int d^4x d^8\theta du \left\{ G^{++\alpha(s-1)\dot{\alpha}(s-1)} G^{--}_{\alpha(s-1)\dot{\alpha}(s-1)} \right.$$
$$\left. + \; 4G^{++\alpha(s-2)\dot{\alpha}(s-2)} G^{--}_{\alpha(s-2)\dot{\alpha}(s-2)} \right\}.$$

- Varying this action with respect to analytic prepotentials, one can derive superfield equations of motion:

$$(\bar{\mathcal{D}}^+)^2 \mathcal{D}^{+\alpha_{s-1}} G^{--}_{\alpha(s-1)\dot{\alpha}(s-1)} - 2(\mathcal{D}^+)^2 \bar{\mathcal{D}}^+_{(\dot{\alpha}} G^{--}_{\alpha(s-2)\dot{\alpha}(s-2))} \approx 0.$$

- In components, these equations lead to the free equations for physical gauge fields. Auxiliary fields are equal to zero on-shell (for $\mathbf{s} \geq 3$).

Abelian cubic vertex in terms of Mezincescu-type prepotential

- Supersymmetry invariant analytic differential operator:

$$\hat{\mathcal{H}}^{++}_{(s)} = h^{++\alpha(s-2)\dot{\alpha}(s-2)M}\partial_M\partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)},$$

$$\partial_M := (\partial_{\alpha\dot{\alpha}}, \partial^-_{\alpha}, \partial^-_{\dot{\alpha}}, \partial_5).$$

- Gauge transformation law can be presented as:

$$\delta_\lambda \hat{\mathcal{H}}^{++}_{(s)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(s)}],$$

$$\hat{\Lambda}_{(s)} := \lambda^{\alpha(s-2)\dot{\alpha}(s-2)M}\partial_M\partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)}.$$

- This operator is the natural covariantization of the harmonic derivative under spin **s** gauge transformations:

$$\mathcal{D}^{++} \quad \to \quad \mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)}$$

and determines the cubic interaction of $\mathcal{N} = 2$ higher spins with the hypermultiplet [Buchbinder, Ivanov, N. Z. 2022].

- Alternative representation of prepotentials is given by Mezincescu non-analytic prepotentials:

$$\hat{\mathcal{H}}^{++}_{(s)} = (\mathcal{D}^+)^4 \left[\Psi^{-(\alpha(s-2)\beta)\dot\alpha(s-2)}\mathcal{D}^-_\beta + \bar\Psi^{-\alpha(s-2)(\dot\alpha(s-2)\dot\beta)}\bar{\mathcal{D}}^-_{\dot\beta}\right]\partial^{s-2}_{\alpha(s-2)\dot\alpha(s-2)}.$$

- Mezincescu prepotentials are defined up to the gauge freedom and pre-gauge freedom:

$$\begin{aligned}\delta_{\lambda,b}\Psi^-_{\alpha(s-1)\dot\alpha(s-2)} = {} & \mathcal{D}^{++}K^{(-3)}_{\alpha(s-1)\dot\alpha(s-2)} + \mathcal{D}^+_{(\alpha}B^{--}_{\alpha(s-2))\dot\alpha(s-2)} \\ & + \mathcal{D}^{+\beta}B^{--}_{(\alpha(s-1)\beta)\dot\alpha(s-2)} + \bar{\mathcal{D}}^{+\dot\beta}B^{--}_{\alpha(s-1)(\dot\alpha(s-2)\dot\beta)} \\ & + \bar{\mathcal{D}}^+_{(\dot\alpha}B^{--}_{\alpha(s-1)\dot\alpha(s-3))},\end{aligned}$$

- This motivates us to consider cubic couplings in the general form:

$$S_{int} = \int d^4x d^8\theta du \left(\Psi^{-\alpha(s-1)\dot\alpha(s-2)}\mathcal{J}^+_{\alpha(s-1)\dot\alpha(s-2)} + c.c.\right) \quad \Rightarrow$$

$$\begin{cases}\mathcal{D}^{++}\mathcal{J}^+_{\alpha(s-1)\dot\alpha(s-2)} \approx 0, \\ \mathcal{D}^{+\alpha}\mathcal{J}^+_{\alpha(s-1)\dot\alpha(s-2)} + \bar{\mathcal{D}}^{+\dot\alpha}\bar{\mathcal{J}}^+_{\alpha(s-2)\dot\alpha(s-1)} \approx 0, \\ \mathcal{D}^+_{(\beta}\mathcal{J}^+_{\alpha(s-1))\dot\alpha(s-2)} \approx 0, \\ \bar{\mathcal{D}}^+_{(\dot\alpha}\mathcal{J}^+_{\alpha(s-1)\dot\alpha(s-2))} - \mathcal{D}^+_{(\alpha}\bar{\mathcal{J}}^+_{\alpha(s-2))\dot\alpha(s-1)} \approx 0, \\ \bar{\mathcal{D}}^{+\dot\alpha}\mathcal{J}^+_{\alpha(s-1)\dot\alpha(s-2)} \approx 0.\end{cases}$$

- If these superfield equations are satisfied on-shell, cubic coupling is consistent in leading order.

- The solution for supercurrents has a special form [N.Z. 2024]:

$$\begin{cases} \mathcal{J}^+_{(\alpha_1\ldots\alpha_{s-1})\dot\alpha(s-2)} = \mathcal{D}^+_{(\alpha_1}\mathcal{J}_{\alpha_2\ldots\alpha_{s-1})\dot\alpha(s-2)}, \\ \bar{\mathcal{J}}^+_{\alpha(s-2)(\dot\alpha_1\ldots\dot\alpha_{s-1})} = -\bar{\mathcal{D}}^+_{(\dot\alpha_1}\mathcal{J}_{\alpha(s-2)\dot\alpha_2\ldots\dot\alpha_{s-1})}. \end{cases}$$

- Here we introduce the real *principal* $\mathcal{N}=2$ *supercurrent* $\mathcal{J}_{\alpha(s-2)\dot\alpha(s-2)}$, which satisfies the on-shell equations:

$$\begin{cases} \widetilde{\mathcal{J}}_{\alpha(s-2)\dot\alpha(s-2)} = \mathcal{J}_{\alpha(s-2)\dot\alpha(s-2)}, \\ \mathcal{D}^{++}\mathcal{J}_{\alpha(s-2)\dot\alpha(s-2)} \approx 0, \\ \mathcal{D}^{+\beta}\mathcal{J}_{(\beta\alpha_2\alpha_{s-2})\dot\alpha(s-2)} \approx 0, \\ \bar{\mathcal{D}}^{+\dot\beta}\mathcal{J}_{\alpha(s-2)(\dot\beta\dot\alpha_2\ldots\alpha_{s-2})} \approx 0. \end{cases}$$

- In the case of $s=2$, one needs to replace the third and fourth equations by $(\mathcal{D}^+)^2\mathcal{J}\approx 0$ and $(\bar{\mathcal{D}}^+)^2\mathcal{J}\approx 0$. This case corresponds to $\mathcal{N}=2$ supergravity supercurrent.
- In terms of principal $\mathcal{N}=2$ supercurrent, cubic vertex has the form:

$$S_{int} = \int d^4x d^8\theta du \left(\Psi^{-\alpha(s-1)\dot\alpha(s-2)}\mathcal{D}^+_\alpha + \bar\Psi^{-\alpha(s-2)\dot\alpha(s-1)}\bar{\mathcal{D}}^+_{\dot\alpha}\right)\mathcal{J}_{\alpha(s-2)\dot\alpha(s-2)}.$$

- All components of the $\mathcal{N}=2$ higher-spin principle supercurrent are conserved currents:

$$\partial^{\beta\dot\beta}\mathcal{J}_{(\alpha(s-3)\beta)(\dot\alpha(s-3)\dot\beta)} \approx 0.$$

Structure of $\mathcal{N}=2$ Mezincescu-type higher spin prepotentials (comment)

- Mezincescu-type higher spin prepotentials are invariant under rigid $\mathcal{N}=2$ supersymmetry:

$$\delta_\epsilon \Psi^{-(\alpha(s-2)\beta)\dot{\alpha}(s-2)} = 0, \qquad \delta_\epsilon \bar{\Psi}^{-\alpha(s-2)(\dot{\alpha}(s-2)\dot{\beta})} = 0.$$

- Equation for analytic differential operator

$$\hat{\mathcal{H}}^{++}_{(s)} := h^{++\alpha(s-2)\dot{\alpha}(s-2)M}\partial_M \partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)} = (\mathcal{D}^+)^4 \left[\Psi^{-(\alpha(s-2)\beta)\dot{\alpha}(s-2)}\mathcal{D}^-_\beta + c.c.\right] \partial^{s-2}_{\alpha(s-2)\dot{\alpha}(s-2)}$$

determines the structure of non-analytical Mezincescu prepotentials in terms of analytical:

$$\begin{aligned}
\Psi^{-(\alpha(s-1))\dot{\alpha}(s-2)}(\zeta,\theta^-) \sim \quad & (\theta^-)^4 h^{++\alpha(s-1)\dot{\alpha}(s-2)+}(\zeta) \\
& + (\theta^-)^2 \bar{\theta}^-_{\dot{\alpha}} \left[h^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta) + i g^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta)\right] \\
& + (\bar{\theta}^-)^2 \theta^{-(\alpha} \left[h^{++\alpha(s-2))\dot{\alpha}(s-2)}(\zeta) + i g^{++\alpha(s-2))\dot{\alpha}(s-2)}(\zeta)\right] \\
& + \theta^-_\alpha \bar{\theta}^-_{\dot{\alpha}} g^{+\alpha(s)\dot{\alpha}(s-1)}(\zeta) + \dots
\end{aligned}$$

- Analytical superfields $g^{\cdots}(\zeta)$ are the "pre-gauge" d.o.f., so, can be gauged away using the pre-gauge parameters $B^{\cdots}$.

$\mathcal{N}=2$ higher spin Weyl-like tensor

- To construct gauge invariant $\mathcal{N}=2$ supercurrents, we need gauge invariant superfields.
- Gauge invariant $\mathcal{N}=2$ supertensor [Ivanov, N.Z. 2024]:

$$\mathcal{W}_{\alpha(2s-2)} := \left(\bar{\mathcal{D}}^{+}\right)^2 \partial_{(\alpha_1}^{\dot{\beta}_1} \dots \partial_{\alpha_{s-2}}^{\dot{\beta}_{s-2}} \Big\{ \mathcal{D}^{+}_{\alpha_{s-1}} G^{---}_{\alpha_s \dots \alpha_{2s-2})(\dot{\beta}_1 \dots \dot{\beta}_{s-2})} + \mathcal{D}^{-}_{\alpha_{s-1}} G^{--+}_{\alpha_s \dots \alpha_{2s-2})(\dot{\beta}_1 \dots \dot{\beta}_{s-2})} + \partial_{\alpha_{s-1}}^{\dot{\beta}_{s-1}} G^{--}_{\alpha_s \dots \alpha_{2s-2})(\dot{\beta}_1 \dots \dot{\beta}_{s-1})} \Big\}.$$

- Superstrength $\mathcal{W}_{\alpha(2s-2)}$ is the *chiral* $\mathcal{N}=2$ superfield, i.e. $\bar{\mathcal{D}}^{\pm}_{\dot{\beta}} \mathcal{W}_{\alpha(2s-2)} = 0$, tilde conjugated strength $\bar{\mathcal{W}}_{\dot{\alpha}(2s-2)}$ is the *anti-chiral* superfield, $\mathcal{D}^{\pm}_{\beta} \bar{\mathcal{W}}_{\dot{\alpha}(2s-2)} = 0$.
- $\mathcal{W}_{\alpha(2s-2)}$ and $\bar{\mathcal{W}}_{\dot{\alpha}(2s-2)}$ are *covariantly harmonic independent*, i.e., satisfy:

$$\mathcal{D}^{\pm\pm} \mathcal{W}_{\alpha(2s-2)} = 0, \qquad \mathcal{D}^{\pm\pm} \bar{\mathcal{W}}_{\dot{\alpha}(2s-2)} = 0.$$

- On-shell superfield strengths satisfy the equations:

$$\mathcal{D}^{\pm\alpha} \mathcal{W}_{\alpha(2s-2)} \approx 0, \qquad \bar{\mathcal{D}}^{\pm\dot{\alpha}} \bar{\mathcal{W}}_{\dot{\alpha}(2s-2)} \approx 0, \qquad \partial_{\dot{\beta}}^{\alpha} \mathcal{W}_{\alpha(2s-2)} \approx 0, \qquad \partial_{\beta}^{\dot{\alpha}} \bar{\mathcal{W}}_{\dot{\alpha}(2s-2)} \approx 0.$$

- In components, superfield strengths contain the Weyl-like higher spin tensor:

$$\mathcal{W}_{\alpha(2s-2)} \sim \theta^{+\beta} \theta^{-\gamma} C_{(\alpha(2s-2)\beta\gamma)} + \dots$$

Thus, one can consider this superfield strength as a higher spin generalization of the linearized $\mathcal{N}=2$ super Weyl tensor.

General gauge-invariant ansatz for $\mathcal{N}=2$ primary supercurrent

$$
\begin{aligned}
\mathcal{J}_{\alpha(s_1-2)\dot\alpha(s_1-2)} = & \sum_{p=0}^{s_1-2s_2} \alpha_p\, \partial^p \mathcal{W}_{\alpha(2s_2-2)} \partial^{s_1-2s_2-p} \bar{\mathcal{W}}_{\dot\alpha(2s_2-2)} \\
& + \sum_{p=0}^{s_1-2s_2-1} \beta_p\, \partial^p \mathcal{D}^-_\alpha \mathcal{W}_{\alpha(2s_2-2)} \partial^{s_1-2s_2-p-1} \bar{\mathcal{D}}^+_{\dot\alpha} \bar{\mathcal{W}}_{\dot\alpha(2s_2-2)} \\
& - \sum_{p=0}^{s_1-2s_2-1} \beta^*_{s_1-2s_2-p-1}\, \partial^p \mathcal{D}^+_\alpha \mathcal{W}_{\alpha(2s_2-2)} \partial^{s_1-2s_2-p-1} \bar{\mathcal{D}}^-_{\dot\alpha} \bar{\mathcal{W}}_{\dot\alpha(2s_2-2)} \\
& + \sum_{p=0}^{s_1-2s_2-2} \gamma_p\, \partial^p \mathcal{D}^+_\alpha \mathcal{D}^-_\alpha \mathcal{W}_{\alpha(2s_2-2)} \partial^{s_1-2s_2-p-2} \bar{\mathcal{D}}^+_{\dot\alpha} \bar{\mathcal{D}}^-_{\dot\alpha} \bar{\mathcal{W}}_{\dot\alpha(2s_2-2)}.
\end{aligned}
$$

$$
\alpha_p = c\,(i)^{s_1-2s_2}(-1)^p \frac{\binom{s_1-2}{2s_2-2+p}\binom{s_1-2}{p}}{\binom{s_1-2}{2s_2-2}}, \qquad p = 0,1,2\ldots, s_1-2s_2,
$$

$$
\beta_p = c\,(i)^{s_1-2s_2}(-1)^{p+1} \frac{1}{4i} \frac{\binom{s_1-2}{2s_2-1+p}\binom{s_1-2}{p}}{\binom{s_1-2}{2s_2-2}}, \qquad p = 0,1,2\ldots, s_1-2s_2-1,
$$

$$
\gamma_p = c\,(i)^{s_1-2s_2}(-1)^p \frac{1}{16} \frac{\binom{s_1-2}{2s_2+p}\binom{s_1-2}{p}}{\binom{s_1-2}{2s_2-2}}, \qquad p = 0,1,2\ldots, s_1-2s_2-2.
$$

Analytic structure of $\mathcal{N}=2$ principal supercurrent

$$
\begin{aligned}
\mathcal{J}_{\alpha(s)\dot\alpha(s)} &= \mathbb{J}_{\alpha(s)\dot\alpha(s)} + \theta^{-\beta}\mathbb{J}^{+}_{\beta\alpha(s)\dot\alpha(s)} + \bar\theta^{-\dot\beta}\bar{\mathbb{J}}^{+}_{\alpha(s)\dot\beta\dot\alpha(s)} \\
&\quad + i(\theta^-)^2\mathbb{J}^{++}_{\alpha(s)\dot\alpha(s)} - i(\bar\theta^-)^2\bar{\mathbb{J}}^{++}_{\alpha(s)\dot\alpha(s)} - 4i\theta^{-\beta}\bar\theta^{-\dot\beta}\mathbb{J}^{++}_{\beta\alpha(s)\dot\beta\dot\alpha(s)} \\
&\quad + (\bar\theta^-)^2\theta^{-\beta}\mathbb{J}^{(+3)}_{\beta\alpha(s)\dot\alpha(s)} + (\theta^-)^2\bar\theta^{-\dot\beta}\bar{\mathbb{J}}^{(+3)}_{\alpha(s)\dot\beta\dot\alpha(s)} + (\theta^-)^4\mathbb{J}^{(+4)}_{\alpha(s)\dot\alpha(s)}.
\end{aligned}
$$

On-shell condition $\mathcal{D}^{+\alpha}\mathcal{J}_{\alpha(s)\dot\alpha(s)} \approx 0$ lead to the constraints on the analytical superfields $\mathbb{J}$:

$$
\begin{aligned}
&\mathbb{J}^{+}_{\beta\alpha(s)\dot\alpha(s)} \approx \mathbb{J}^{+}_{(\beta\alpha(s))\dot\alpha(s)}, \quad \bar{\mathbb{J}}^{+}_{\alpha(s)\dot\beta\dot\alpha(s)} \approx \bar{\mathbb{J}}^{+}_{\alpha(s)(\dot\beta\dot\alpha(s))}, \quad \mathbb{J}^{++}_{\alpha(s)\dot\alpha(s)} \approx \bar{\mathbb{J}}^{++}_{\alpha(s)\dot\alpha(s)} \approx 0, \\
&\mathbb{J}^{++}_{\beta\alpha(s)\dot\beta\dot\alpha(s)} \approx \mathbb{J}^{++}_{(\beta\alpha(s))(\dot\beta\dot\alpha(s))}, \quad \mathbb{J}^{(+3)}_{\beta\alpha(s)\dot\alpha(s)} \approx 0, \quad \bar{\mathbb{J}}^{(+3)}_{\alpha(s)\dot\beta\dot\alpha(s)} \approx 0, \quad \mathbb{J}^{(+4)}_{\alpha(s)\dot\alpha(s)} \approx 0,
\end{aligned}
$$

These equations are *algebraic* and indicate that a number of terms in the expansion vanish on the equations of motion, and therefore lead to fake vertices.

$$
\boxed{\mathcal{J}_{\alpha(s)\dot\alpha(s)} \approx \mathbb{J}_{\alpha(s)\dot\alpha(s)} + \theta^{-\beta}\mathbb{J}^{+}_{(\beta\alpha(s))\dot\alpha(s)} + \bar\theta^{-\dot\beta}\bar{\mathbb{J}}^{+}_{\alpha(s)(\dot\beta\dot\alpha(s))} - 4i\theta^{-\beta}\bar\theta^{-\dot\beta}\mathbb{J}^{++}_{(\beta\alpha(s))(\dot\beta\dot\alpha(s))}.}
$$

Analytic supercurrents

$$\mathbb{J}^{++}_{\alpha(s-1)\dot\alpha(s-1)} = +i(\mathcal{D}^+)^4 \left\{\theta^-_{(\alpha}\bar\theta^-_{(\dot\alpha}\mathcal{J}_{\alpha(s-2))\dot\alpha(s-2))}\right\} = \frac{1}{4i}\mathcal{D}^+_{(\alpha}\bar{\mathcal{D}}^+_{(\dot\alpha}\mathcal{J}_{\alpha(s-2))\dot\alpha(s-2))}\Big|_{\theta^-=0},$$
$$\mathbb{J}^{+}_{\alpha(s-1)\dot\alpha(s-2)} = -2(\mathcal{D}^+)^4 \left\{\theta^-_{(\alpha}(\bar\theta^-)^2\mathcal{J}_{\alpha(s-2))\dot\alpha(s-2)}\right\} = \mathcal{D}^+_{(\alpha}\mathcal{J}_{\alpha(s-2))\dot\alpha(s-2)}\Big|_{\theta^-=0},$$
$$\bar{\mathbb{J}}^{+}_{\alpha(s-2)\dot\alpha(s-1)} = +2(\mathcal{D}^+)^4 \left\{\bar\theta^-_{(\dot\alpha}(\theta^-)^2\mathcal{J}_{\alpha(s-2)\dot\alpha(s-2))}\right\} = \bar{\mathcal{D}}^+_{(\dot\alpha}\mathcal{J}_{\alpha(s-2)\dot\alpha(s-2))}\Big|_{\theta^-=0}.$$

fully determine $\mathcal{N}=2$ Abelian cubic vertices!

$$S^A_{int} = \int d\zeta^{(-4)}\Big(h^{++\alpha(s-1)\dot\alpha(s-1)}J^{++}_{\alpha(s-1)\dot\alpha(s-1)} + h^{++\alpha(s-1)\dot\alpha(s-2)+}J^{+}_{\alpha(s-1)\dot\alpha(s-2)} + h^{++\alpha(s-2)\dot\alpha(s-1)+}\bar J^{+}_{\alpha(s-2)\dot\alpha(s-1)}\Big).$$

Harmonic independence condition $\mathcal{D}^{++}\mathcal{J}_{\alpha(s)\dot\alpha(s)} \approx 0$ imply additional relations on $\mathbb{J}$ supercurrents:

$$\begin{cases}\mathcal{D}^{++}\mathbb{J}^{++}_{(\beta\alpha(s))(\dot\beta\dot\alpha(s))} \approx 0,\\ \mathcal{D}^{++}\mathbb{J}^{+}_{(\beta\alpha(s))\dot\alpha(s)} - 4i\bar\theta^{+\dot\beta}\mathbb{J}^{++}_{(\beta\alpha(s))(\dot\beta\dot\alpha(s))} \approx 0,\\ \mathcal{D}^{++}\bar{\mathbb{J}}^{+}_{\alpha(s)(\dot\beta\dot\alpha(s))} + 4i\theta^{+\beta}\mathbb{J}^{++}_{(\beta\alpha(s))(\dot\beta\dot\alpha(s))} \approx 0,\\ \mathcal{D}^{++}\mathbb{J}_{\alpha(s)\dot\alpha(s)} + \theta^{+\beta}\mathbb{J}^{+}_{(\beta\alpha(s))\dot\alpha(s)} + \bar\theta^{+\dot\beta}\bar{\mathbb{J}}^{+}_{\alpha(s)(\dot\beta\dot\alpha(s))} \approx 0.\end{cases}$$

$\boxed{\text{Off-shell analyticity}}$ $\longleftrightarrow$ $\boxed{\text{Off-shell harmonic analyticity}}$

Analytic action — Full superspace action

Example: $(3,1,1)$ vertex

- Principle $\mathcal{N}=2$ supercurrent:

$$\mathcal{J}_{\alpha\dot\alpha} = i\left(\partial_{\alpha\dot\alpha}\mathcal{W}\bar{\mathcal{W}} - \mathcal{W}\partial_{\alpha\dot\alpha}\bar{\mathcal{W}}\right) - \frac{1}{4}\left(\mathcal{D}^-_\alpha\mathcal{W}\bar{\mathcal{D}}^+_{\dot\alpha}\bar{\mathcal{W}} - \mathcal{D}^+_\alpha\mathcal{W}\bar{\mathcal{D}}^-_{\dot\alpha}\bar{\mathcal{W}}\right).$$

- Cubic vertex:

$$S_{(3,1,1)} = \int d^4x d^8\theta du\ \left(\Psi^{-(\alpha\beta)\dot\alpha}\mathcal{D}^+_\beta + \bar\Psi^{-\alpha(\dot\alpha\dot\beta)}\bar{\mathcal{D}}^+_{\dot\beta}\right)\mathcal{J}_{\alpha\dot\alpha}.$$

- Spin 3 current:

$$J_{(\alpha\beta)\rho(\dot\alpha\dot\beta)\dot\rho} \sim i\left(C_{(\alpha\dot\rho)}\overleftrightarrow{\partial}_{\beta\dot\beta}\bar{C}_{(\dot\alpha\dot\rho)}\right) + \partial_{(\rho\dot\rho}\psi^i_{\alpha)}\overleftrightarrow{\partial}_{\beta\dot\beta}\bar\psi_{\dot\alpha i} - \psi^i_\alpha\overleftrightarrow{\partial}_{\beta\dot\beta}\partial_{\rho(\dot\rho}\bar\psi_{\dot\alpha)i} + i\partial_{\alpha\dot\rho}\phi\overleftrightarrow{\partial}_{\beta\dot\beta}\partial_{\rho\dot\alpha}\bar\phi.$$

- Vector part of spin 3 current corresponds to *zilch pseudotensor:*

$$z_{\mu\nu|\rho} = 2iF_{\mu\sigma}\overleftrightarrow{\partial}_\rho{}^\star F^\sigma{}_\nu = i\left(F_{\mu\sigma}\partial_\rho{}^\star F^\sigma{}_\nu - \partial_\rho F_{\mu\sigma}{}^\star F^\sigma{}_\nu\right).$$

- Gauge spin **3** transformations in $\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}$ sector:

$$\delta V^{++} = -32\kappa_3(\mathcal{D}^+)^4\Big\{\partial_{\alpha\dot\alpha}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}[\theta^+_\beta\bar\theta^-_{\dot\beta} - \theta^-_\beta\bar\theta^+_{\dot\beta}]V^{--} \\ + 2\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)}[\theta^+_\beta\bar\theta^-_{\dot\beta} - \theta^-_\beta\bar\theta^+_{\dot\beta}]\partial_{\alpha\dot\alpha}V^{--}\Big\}.$$

- In the rigid limit ($\partial_{\rho\dot\rho}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} = 0$, $\mathcal{D}^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} = 0$), using the Wess–Zumino type gauge for vector superfield

$$V^{--}_{WZ} \quad \to \quad 8(\theta^-)^2\bar\theta^{-(\dot\rho}\bar\theta^{+\dot\sigma)}\bar{\mathcal{F}}_{(\dot\rho\dot\sigma)} - 8(\bar\theta^-)^2\theta^{-(\rho}\theta^{-\sigma)}\mathcal{F}_{(\rho\sigma)}.$$

$$\delta A_{\beta\dot\beta} = -64i\lambda^{(\alpha_1\alpha_2)(\dot\alpha_1\dot\alpha_2)}\partial_{\alpha_1\dot\alpha_1}\left(\epsilon_{\alpha_2\beta}\bar{\mathcal{F}}_{(\dot\alpha_2\dot\beta)} - \epsilon_{\dot\alpha_2\dot\beta}\mathcal{F}_{(\alpha_2\beta)}\right).$$

- In vector indices:

$$\delta A_b \sim i\lambda^{(a_1a_2)}\partial_{a_1}{}^\star F_{[a_2b]}, \quad \text{where} \quad {}^\star F_{[ab]} := \frac{i}{2}\epsilon_{abcd}F^{[cd]} = i\left(\partial_a B_b - \partial_b B_a\right).$$

The action of these transformations on the electromagnetic field strength has the form:

$$\delta F_{[ab]} \sim i\lambda^{(cd)}\partial_c\partial_d{}^\star F_{[ab]}, \qquad \delta A_a = -\xi^{bc}\partial_b\partial_c B_a.$$

Arbitrary spin transformation for vector multiplet

$$\delta V^{++} = -8\kappa_s i^{s+1}(\mathcal{D}^+)^4 \left[\sum_{p=0}^{s-2} \binom{s-1}{p}\binom{s-1}{p+1}\partial^p\Big\{\lambda\cdot\left(\theta^+\bar\theta^- + (-1)^s\theta^-\bar\theta^+\right)\partial^{s-p-2}V^{--}\Big\}\right].$$

- In components, these transformations correspond to:

$$\delta_k^{(1)}A_\sigma = \xi^{\mu_1\cdots\mu_k}\partial_{\mu_1}\dots\partial_{\mu_k}A_\sigma \quad\Rightarrow\quad \delta_k^{(1)}F_{\rho\sigma} = \xi^{\mu_1\cdots\mu_k}\partial_{\mu_1}\dots\partial_{\mu_k}F_{\rho\sigma},$$
$$\delta_k^{(2)}A_\sigma = \xi^{\mu_1\cdots\mu_k}\partial_{\mu_1}\dots\partial_{\mu_k}B_\sigma \quad\Rightarrow\quad \delta_k^{(2)}F_{\rho\sigma} = -i\xi^{\mu_1\cdots\mu_k}\partial_{\mu_1}\dots\partial_{\mu_k}{}^\star F_{\rho\sigma}.$$

  Both types of transformations preserve the invariance of the Maxwell equations:

$$\partial_\mu F^{\mu\nu}\approx 0, \qquad \partial_\mu{}^\star F^{\mu\nu} = 0.$$

- However, one can verify that the invariance of the Maxwell action is preserved only for

$$\delta_k^{(1)}\ \text{ with }\ k=1,3,5,\dots \quad\text{ and for }\quad \delta_k^{(2)}\ \text{ with }\ k=0,2,4,\dots$$

  It is precisely these series of rigid higher-spin transformations, that correspond to the Abelian interaction of the vector field with even and odd spin higher-spin fields.

- At the same time, the higher-spin supersymmetry transformations have the same form for all spin value:

$$\delta A_{\beta\dot\beta} \sim \epsilon_\beta^{\alpha(s-1)\dot\alpha(s-1)}\partial^p_{\alpha(s-1)\dot\alpha(s-1)}\bar\psi_{\dot\beta}$$

Outlook

- *Alternative representations for* $\mathcal{N}=2$ *Abelian vertices?*

$$S_{NL}^{(s=1)} = \int d^4x d^8\theta du\, E\, V^{++}V^{--},$$

$$(\mathcal{D}^{++} + \kappa\hat{\mathcal{H}}^{++})V^{--} = (\mathcal{D}^{--} + \kappa\hat{\mathcal{H}}^{--})V^{++}.$$

- *General* $\mathcal{N}=2$ *Abelian interactions*
- *Non-Abelian interactions*
- *Supersymmetric Born-Infeld cubic vertices?*
- *Fradkin-Vasiliev mechanism and AdS harmonic superspace*

Thank you for your attention!