Black hole spectroscopy

(Particle quantum levels in the gravitational field of black hole)

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I elucidate the quantization of particle trajectories in the strong gravitational fields of black hole

Qualitative quantization of particle trajectories in the qualitative quantization of hydrogen atom by Niels Borh (old quantum mechanics) gravitational fields of black holes may be done by the relativistic generalization of the Niels Borh electron orbits in hydrogen atom.

For the strong calculations of quantum orbits of particles in the gravitational fields of black hole it must be used Clein--Gordon equation for scalar particles and Dirack equation for spinor-particles.

Schrödinger's equation is nonrelativistic and cannot be used for description of black hole quantization.

Niels Bohr model of hydrogen atom -- old quantum theory I

Quantization condition: $L=n\hbar$, L – impulse angular momentum, n=1, 2,...

Atom radiation frequency: $\hbar\omega_{nk}$

E_k, E_n – energies of stationary quantum states; k, n – numbers of quantum states, k>n

X-ray notation

Квантовые числа				Атомная номенклатура	Рентгеновская номенклатура
n e	l	s	j	Место для уравнения.	
1	0	±1/2	1/2	$1s_{(1/2)}$	K_1
2	1	-1/2	1/2	$2p_{1/2}$	L_2
2	1	+1/2	3/2	$2p_{3/2}$	L_3
2	0	±1/2	1/2	$2s_{(1/2)}$	L_1
3	2	-1/2	3/2	$3d_{3/2}$	M_4
3	2	+1/2	5/2	$3d_{5/2}$	M_5
3	1	-1/2	1/2	$3p_{1/2}$	M_2
3	1	+1/2	3/2	$3p_{3/2}$	M_3
3	0	±1/2	1/2	3s	M_1

Niels Bohr model of hydrogen atom -- old quantum theory II

Rotation of electron with mass m_0 on circular orbit:

$$\frac{m_0 \mathbf{v}^2}{r} = \frac{e^2}{r^2};\ m_0 \mathbf{v};\ m_0 \mathbf{vr} = \mathbf{n}\,\hbar$$

Radii of stationary quantum orbits of electron in hydrogen atom:

$$r_n = \frac{\hbar^2}{m_0 e^2}\, n^2 = 0.529 * 10^{10}\, n^2 \text{ meters , } n=1,\ 2, \ldots$$

Electron velocity:

$$v_n = \frac{e^2}{\hbar\, n} = \frac{1}{n}\, 2.29 * 10^6\, \frac{\text{meters}}{\text{sec}},\ \mathbf{Period} = \frac{1}{n}\, 1.5 * 10^{-16} \text{ sec}$$

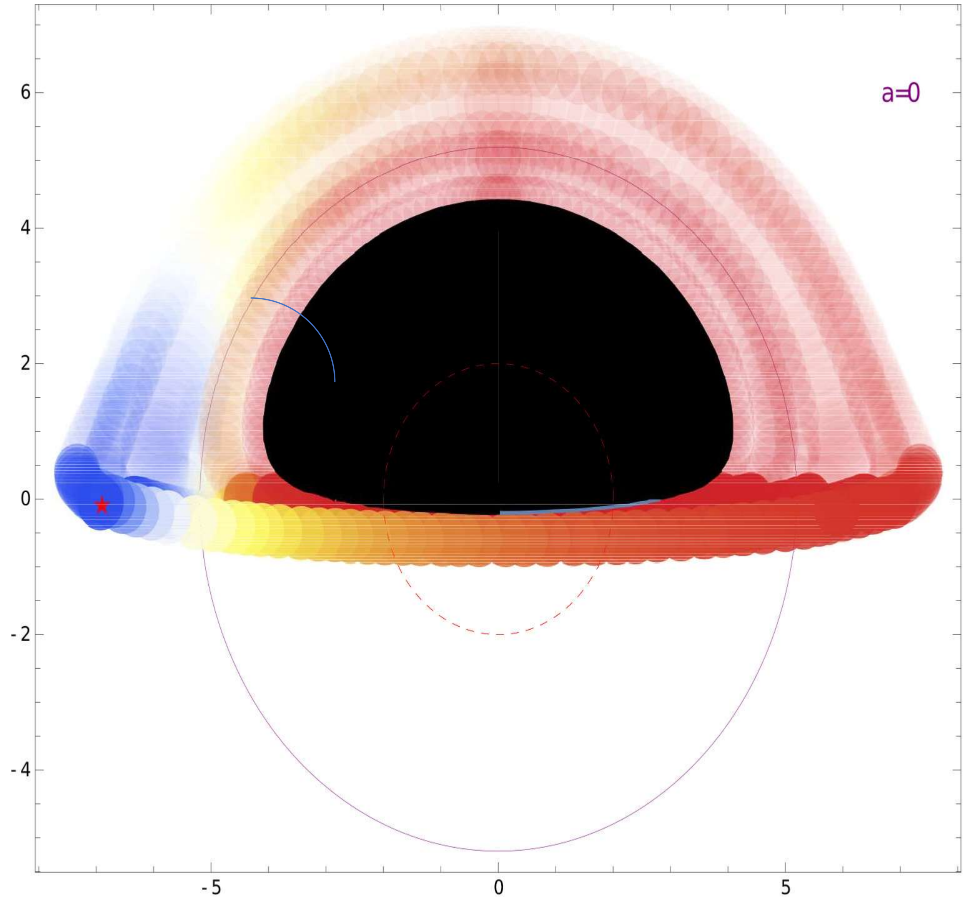
Luminosity of the accretion disc very near from the Schwarcshild black hole (spin a=0), viewed by a distant observer and multicolored closed curves of the particle quantum orbits

Red star marks the brightest point in the accretion disk

Black spot is the lensed North hemisphere of the black hole event horizon horizon globus.

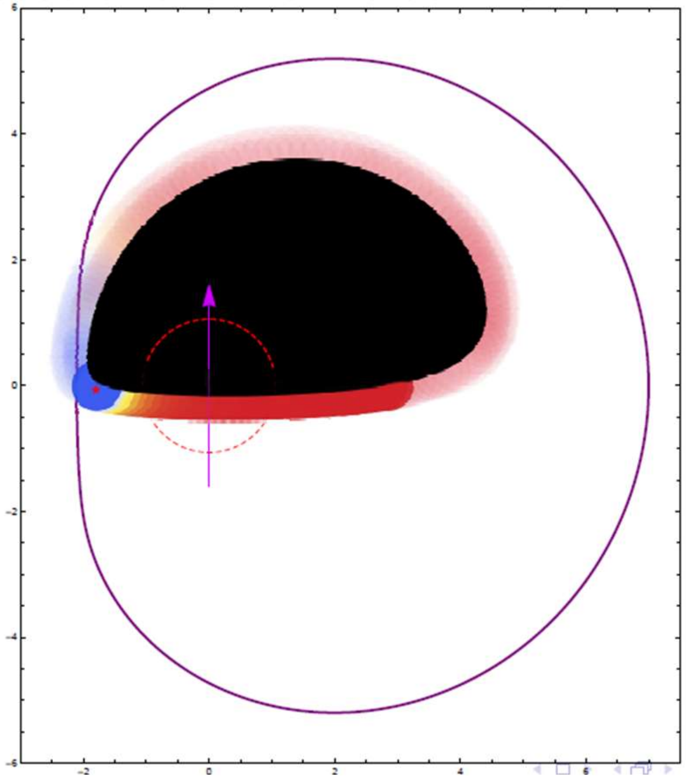
Multicolored closed curves are the quantum orbits of particles very near the black hole event horizon at $\mathrm{r}g = 2$MGc^2.

These orbits are shmoozed due to the quantum uncertainty (Heisenberg's indeterminacy principle)



Luminosity of the accretion disc very near from the fast rotating Kerr black hole viewed by a distant observer

Internal part of the lensed accretion disk at $r_{rh} \leq r \leq r_{ISCO}$ adjoining (примыкающая к) the event horizon in a case of the Kerr black hole with $a = 0.9982$

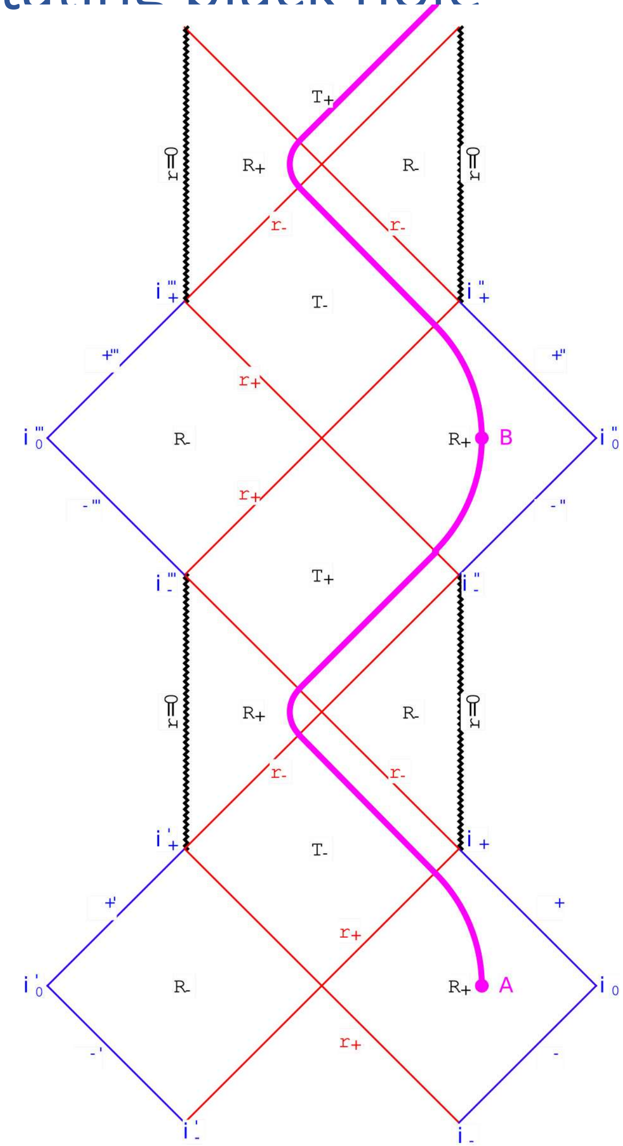


Spinor particles in the black hole gravitational field

1. The Quantum Model of Spinning Black Holes. V.P.Neznamov, S.Yu.Sedov, V.E.Shemarulin. Int.J. Mod. Phys. A (2024) 2450012.
2. Quantum probing of singularities at event horizons of black holes. V.P.Neznamov. International Journal of Modern Physics A 36, 2150147 (2021).
3. Quantum mechanics of stationary states of particles in a space-time of classical black holes. M.V.Gorbatenko, V.P.Neznamov. Theor Math Phys 205, 1492-1526 (2020).
4. Solution of the problem of uniqueness and hermiticity of hamiltonians for Dirac particles in gravitational fields. M.V. Gorbatenko, V.P. Neznamov. Phys.Rev.D82:104056 (2010).Stationary solutions of the Dirac equation in the gravitational field of a charged black hole.
5. V. I. Dokuchaev, Yu. N. Eroshenko, JETP, 117} 72–77 (2013).

Carter-Penrose conformal diagram for rotating black hole

Red curve is a trajectory of massive particle moving in the gravitational filed of rotating black hole,starting at point A outside the black hole event horizon at radius r>r+. There are turning points for radial motion in R+ regions. The long living quasi-stationary quantum states exist in R+ region as outside and inside black hole event horizon. Neznamov and coathors considered only solutions of Dirack equations only outside event horizon.



Scalar and spinor particles in the black hole gravitational field

[55] G. V. Kraniotis, "The Kerr-Gordon-Fock equation in the curved spacetime of the Kerr-newman (anti) de Sitter black hole", *Class. Quantum Grav.*, **33**, 225011 (2016), arXiv: 1602.04830v5 [gr-qc] (2016).

[56] G. V. Kraniotis, "The massive Dirac equation in the Kerr-Newman-de Sitter and Kerr-Newman black hole spacetime", *J. Phys. Commun.*, **3**, 035026 (2019), arXiv: 1801.03157v5 [gr-qc] (2018).

In fact, the Chandrasekhar ansatz [49]

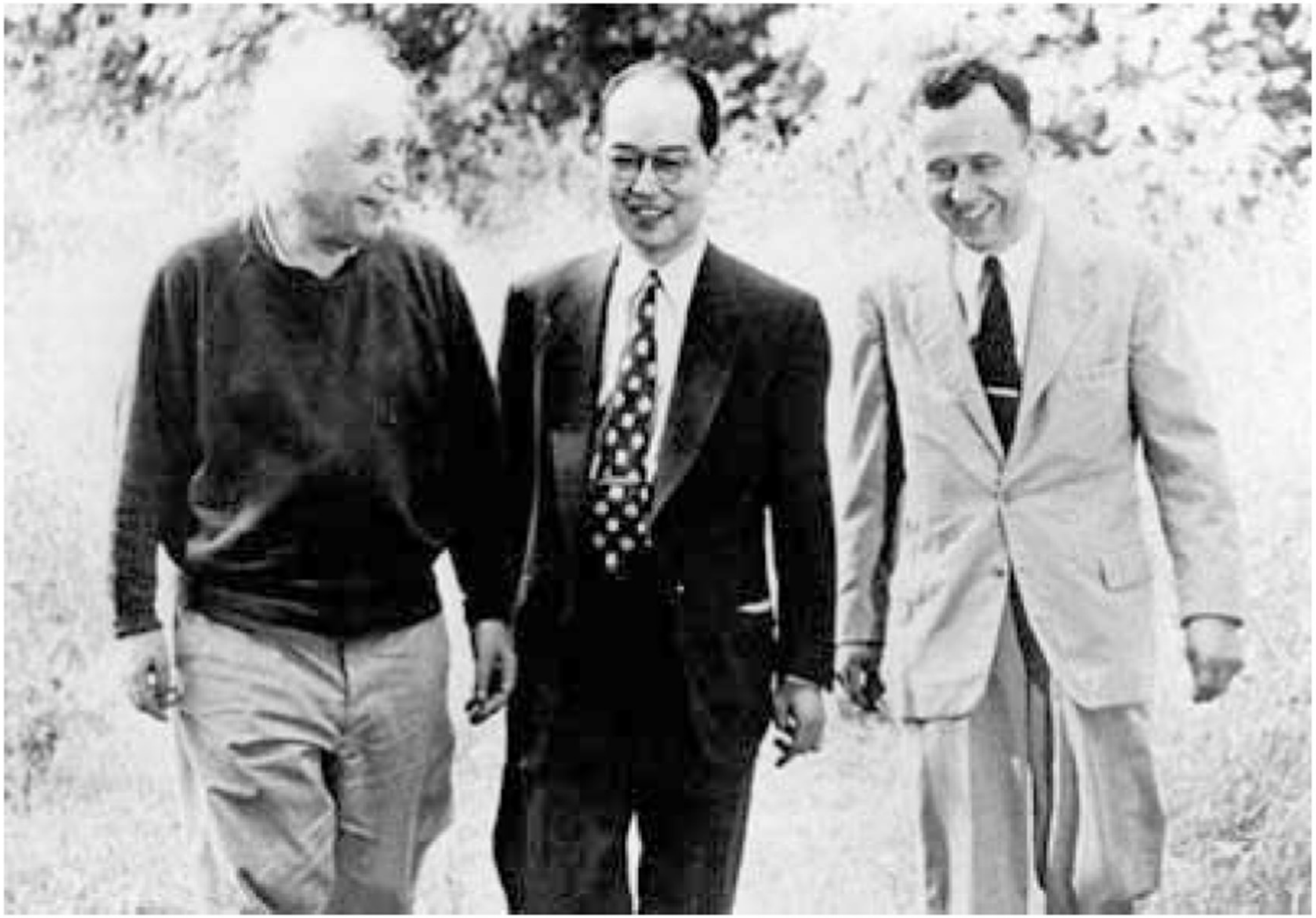
$$\psi(\mathbf{r},t) = \begin{pmatrix} \frac{1}{r - ia\cos\theta} R^{(-)}(r)\, S^{(-)}(\theta) \\ \frac{1}{\sqrt{\Delta_r^{KN}}} R^{(+)}(r)\, S^{(+)}(\theta) \\ \frac{1}{\sqrt{\Delta_r^{KN}}} R^{(+)}(r)\, S^{(-)}(\theta) \\ -\frac{1}{r + ia\cos\theta} R^{(-)}(r)\, S^{(+)}(\theta) \end{pmatrix} e^{-iEt + im_\varphi \varphi} \qquad (108)$$

was used to separate variables for the wave function ψ of the Dirac equation with the mass m and charge q in [56], where the system of radial equations was obtained

$$\sqrt{\Delta_r^{KN}} \frac{dR^{(-)}}{dr} - \left[\frac{i\Xi\left(E\left(r^2 + a^2\right) - m_\varphi a\right) - iqQr}{\sqrt{\Delta_r^{KN}}} \right] R^{(-)}(r) = (\lambda + imr)\, R^{(+)}(r)\,, \qquad (109)$$

$$\sqrt{\Delta_r^{KN}} \frac{dR^{(+)}}{dr} + \left[\frac{i\Xi\left(E\left(r^2 + a^2\right) - m_\varphi a\right) - iqQr}{\sqrt{\Delta_r^{KN}}} \right] R^{(+)}(r) = (\lambda - imr)\, R^{(-)}(r)\,. \qquad (110)$$

Discussion of Gravity — 1954



Albert Einstein Hideki Yukawa John Wheeler