Cosmology in Cuscuton Theory and Minimally Modified Gravity

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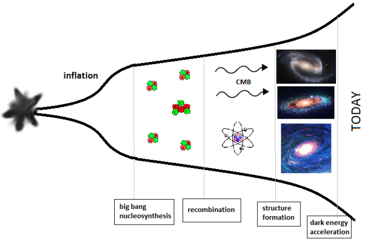
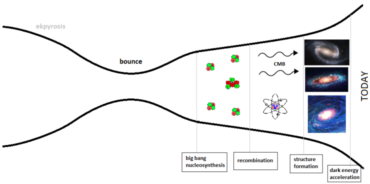
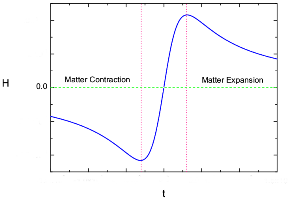
Motivation



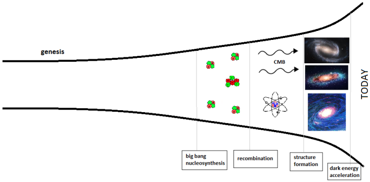
- Standard Big Bang cosmology has its characteristic set of problems at early times including an initial singularity problem.
- Inflation can successfully solve the majority of issues, but it **is geodesically incomplete in the past**.
- Alternative scenarios propose non-singular solutions:
  - Bouncing models (contraction → bounce → expansion)
  - Genesis models (expansion from Minkowski space)

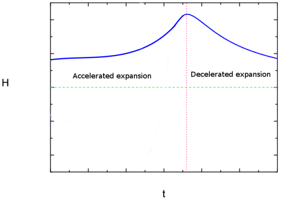
- Both scenarios require the Hubble parameter H to grow:
  - a Universe with a bounce ($\dot{H} > 0$ during the bouncing stage)





  - a Universe starting off with Genesis ($\dot{H} > 0$ at the onset of expansion)





Non-singular cosmologies and NEC violation

- Both scenarios require **violation of the Null Energy Condition** (NEC):

$$T_{\mu\nu}k^\mu k^\nu > 0 \quad (g_{\mu\nu}k^\mu k^\nu = 0)$$

NEC for a homogeneous stationary fluid: $\boxed{p+\rho > 0}$

- NEC ensures that the Hubble parameter never grows

$$\dot{H} = -4\pi G(p+\rho) + \cancel{\frac{\kappa}{a^2}} < 0$$

and the energy density in standard cosmologies always decreases

$$\frac{d\rho}{dt} = -3H(\rho + p) < 0.$$

- How to violate the NEC: *scalar-tensor theories like Horndeski theories*

Horndeski theory

Horndeski (1974), Nicolis, Rattazzi, Trincherini (2009), Deffayet, Gao, Steer, Zahariade (2011)

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$
$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$
$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right].$$

π is a scalar field, $X = \pi_{,\mu}\pi^{,\mu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{iX} = \partial G_i/\partial X$.

- Equations of motion are 2nd order (2+1 DOFs and no Ostrograsky ghost)
- Generality: any STT with 2nd order EOMs, belong to the Horndeski group.
- Special cases: Brans-Dicke theory, $f(R)$-gravity, k-essence, kinetic gravity braiding, Fab Four, any inflation on a scalar and many more...
- *Allow healthy NEC violation → suitable for non-singular cosmologies*

Stability at the perturbative level

- Quadratic action for tensor h_{ij}^T and scalar ζ DOFs (in unitary gauge $\delta\pi = 0$):

$$S^{(2)}_{h+\zeta} = \int \mathrm{d}t\mathrm{d}^3x\; a^3 \left[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}_{ij}^T\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_k h_{ij}^T\right)^2 + \mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \right]$$

Stability and (sub)luminality conditions

$$\mathcal{G}_\mathcal{T}, \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S}, \mathcal{F}_\mathcal{S} > \epsilon > 0, \quad \mathcal{F}_\mathcal{T} \le \mathcal{G}_\mathcal{T}, \quad \mathcal{F}_\mathcal{S} \le \mathcal{G}_\mathcal{S}$$

- ***Complete*** stability for $\forall t$? $\longrightarrow$ *No-go theorem*

Libanov, Mironov, Rubakov (2016)
Kobayshi (2016)

Ways to evade the no-go theorem:

- Go beyond Horndeski: generalization up to DHOST theories

  (for a review see e.g. 2409.16108)
- Reduce to specific parameter space in Horndeski theory (with a necessary control of strong coupling regime)

  Ageeva, Petrov, Rubakov (2021), *Choi, Petrov, Park (2025)*
- **Minimally modified gravity (MMG) theories** $\longrightarrow$ ***Cuscuton***

Cusсuton theory

- MMG theory = modified gravity theory with 2 DOFs
- Cuscuton was formulated as a subclass of k-essence:

$$S = S_{\rm EH} + \int d^4x \sqrt{-g}\, F(X,\varphi), \qquad \left(X \equiv -\tfrac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi\right)$$

$$\delta_\varphi S = 0 \quad \xrightarrow{\text{FLRW}+\varphi(t)} \quad (F_{,X} + 2XF_{,XX})\ddot{\varphi} + 3HF_{,X}\dot{\varphi} + F_{,X\varphi}\dot{\varphi}^2 - F_{,\varphi} = 0$$

Afshordi et al. (2007)

- By definition: cuscuton is a homogeneous scalar field φ that is
  (1) non-dynamical at the background level

$$F_{,X} + 2XF_{,XX} = 0 \qquad \longrightarrow \qquad \boxed{F(X,\varphi) = \pm\mu^2\sqrt{2X} - V(\varphi)}$$

  (2) has non-propagating scalar DOF at the perturbation level

$$S^{(2)}_\zeta = \int \mathrm{d}t\mathrm{d}^3x\, a^3\left[\mathcal{G}_\mathcal{S}\dot{\zeta}^2 - \mathcal{F}_\mathcal{S}\frac{(\nabla\zeta)^2}{a^2}\right]$$

$$\mathcal{G}_\mathcal{S} \equiv \frac{X}{H^2}(F_{,X} + 2XF_{,XX}) = 0, \qquad \mathcal{F}_\mathcal{S} = -\frac{\dot{H}}{H^2}M_{Pl}^2$$

Cuscuton theory and modification of gravity

- Cuscuton and Einstein equations:

$$\boxed{(1)\ \pm\,\mathrm{sign}(\dot\varphi)\cdot 3H\mu^2 = -V'(\varphi)} \quad (2)\ H^2 = \frac{8\pi}{3M_{Pl}^2}V(\varphi) \quad (3)\ \dot H = \mp\frac{4\pi}{M_{Pl}^2}(\mu^2\sqrt{2X})$$

- Elimination of the Cuscuton field in (2) with (1) yields a modified Friedmann equation (effectively renormalizes κ and M_{Pl} for $V \propto \varphi^2$):

$$H^2 + \frac{\kappa}{a^2} = \frac{8\pi}{3M_{Pl}^2}(\rho_m + V[V'^{-1}(3\mu^2 H)])$$

Approach 1

Q: *which V is allowed for non-trivial $\varphi(t)$?*
Combine (1) and (2) with $8\pi G = 1$:

$$V' = \mu^2\sqrt{3V} \ \rightarrow\ V = \tfrac{3}{4}\mu^4(\varphi - \varphi_0)^2.$$

Outcome: *V is uniquely quadratic.*
Then from (2): $H^2 = \frac{\mu^2}{4}[\varphi(t) - \varphi_0]^2$

Approach 2

Q: *which $\varphi(t)$ are allowed for generic V ?*

For $V' = \mu^2\sqrt{3V}$ and non-quadratic V the only solution is

$$\varphi = \varphi_* = \mathrm{const} \ \rightarrow\ H = \mathrm{const}\ \text{(de Sitter)}$$

Outcome: *generic V allows only frozen Cuscuton $\varphi = \mathrm{const}$*

Two faces of Cuscuton constraint: special V or frozen φ, rooted in non-propagating scalar DOF.

Genesis driven by the cuscuton field

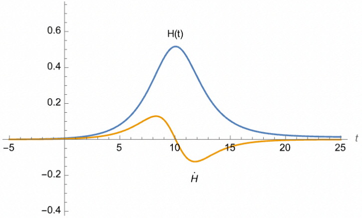
Available choice of the sign:

$$F(X,\varphi) = \pm\mu^2\sqrt{2X} - V(\varphi)$$

$$(1)\quad 3H\mu^2 = V'$$

$$(2)\quad H^2 = \frac{8\pi}{3M_{Pl}^2}V(\varphi)$$

$$(3)\quad \dot{H} = \frac{4\pi}{M_{Pl}^2}(\mu^2\sqrt{2X})$$



- For $V(\varphi) \propto \varphi^2$ one has arbitrary $\varphi(t)$, which "follows" $H(t)$ in (2)
- Hubble parameter for Genesis model (*Mironov, Rubakov, VV (2019)*):

$$H(t) = \left[\left(4\,\frac{\Lambda^3}{f^3}\cdot\frac{t^2\,(1-\tanh(t/\tau))}{2\,(1+\alpha/3)} + 3\cdot\frac{1+\tanh(t/\tau)}{2}\right)\sqrt{2\tau^2+t^2}\right]^{-1}$$

$$t\to-\infty:\quad H(t)\sim\frac{1}{(-t)^3},\qquad\qquad t\to+\infty:\quad H(t)\sim\frac{1}{3t}$$

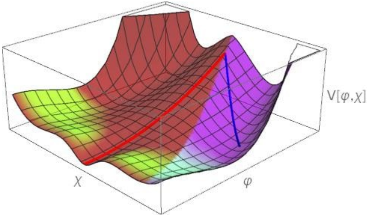
- The scenario is completely stable by design (no other matter is present)

Genesis scenario with Cuscuton + additional matter

- Additional homogeneous scalar field $\chi(t)$:

$$\mathcal{L}_{\varphi+\chi} = -[1-\theta(\varphi_s,\chi_s)]\,\mu^2\sqrt{\dot\varphi^2} + \frac{1}{2}\dot\chi^2 + \theta(\varphi_s,\chi_s)\cdot\frac{1}{2}\dot\varphi^2 - V(\varphi,\chi),$$

- Early times ($\varphi < \varphi_s$): Genesis $H \propto (-t)^{-3}$ with a pure cuscuton $\varphi \propto (-t)^{-3}$ and a frozen scalar χ



$$H^2 = \frac{8\pi}{3M_{Pl}^2}V(\varphi) \qquad\qquad H^2 = \frac{1}{2}(\dot\varphi^2+\dot\chi^2)+V(\varphi,\chi)$$

$$3H\mu^2 = V_\varphi \qquad \rightarrow \qquad \ddot\chi + 3H\dot\chi + V_\chi(\varphi,\chi) = 0$$

$$\chi = \dot\chi = 0 \qquad\qquad \ddot\varphi + 3H\dot\varphi + V_\varphi(\varphi,\chi) = 0$$

- Late times: two conventional scalar fields $\varphi(t)$ and $\chi(t)$ and inflationary-like stage
- Stability:
  - during Genesis no dynamical DOF from cuscuton, the additional scalar χ is free from ghosts and gradient instabilities
  - at late times the NEC is restored and both scalar fields are standard $\longrightarrow$ no issues with stability are expected

Summary

- Cuscuton theories and other Minimally Modified Gravity Theories are promising theoretical frameworks for non-singular scenarios like bouncing Universe or cosmological Genesis
- MMG theories resolve the problem of potential instability of the scalar sector at the linearized level of perturbations
- There exist examples of non-singular cosmologies in Cuscuton theories

Summary

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- There exist examples of non-singular cosmologies in Cuscuton theories

Thank you for your attention!