Inflationary models with the Gauss-Bonnet term and nonminimal coupling

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based on

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JCAP **09** (2024) 050, *arXiv:2403.06147*;
JCAP **05** (2025) 081, *arXiv:2502.13008*

Quarks–2026
Petrozavodsk, Russia, 18 May 2026

Let us consider a single-field inflationary model with

$$S=\frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}R-g^{\mu\nu}\partial_\mu\tilde{\varphi}\partial_\nu\tilde{\varphi}-2V_E(\tilde{\varphi})\right].$$

In the spatially flat Friedmann-Lemaître-Robertson-Walker metric with

$$ds^2=\,-\,dt_E^2+a_E^2(t_E)\,d\mathbf{x}^2\,,$$

the evolution equations are

$$3\,M_{PL}^2\,H_E^2=\frac{1}{2}\left(\frac{d\tilde{\varphi}}{dt_E}\right)^2+V_E\,, \tag{1}$$

$$\frac{dH_E}{dt_E}=\,-\,\frac{1}{2M_{PL}^2}\left(\frac{d\tilde{\varphi}}{dt_E}\right)^2\,, \tag{2}$$

$$\frac{d^2\tilde{\varphi}}{dt_E^2}+3H_E\frac{d\tilde{\varphi}}{dt_E}+V_{E,\tilde{\varphi}}=0\,, \tag{3}$$

where $V_{E,\tilde{\varphi}}=\frac{dV_E(\tilde{\varphi})}{d\tilde{\varphi}}$.

The slow-roll parameters in the Einstein frame are defined by

$$\varepsilon^{(E)} = -\frac{1}{H_E^2}\frac{dH_E}{dt_E} = \frac{3\left(\frac{d\tilde{\varphi}}{dt_E}\right)^2}{\left(\frac{d\tilde{\varphi}}{dt_E}\right)^2 + 2V_E}\,, \tag{4}$$

$$\eta^{(E)} = \varepsilon^{(E)} - \frac{1}{2\,\varepsilon^{(E)}\,H_E}\frac{d\varepsilon^{(E)}}{dt_E} = -\frac{1}{2\,H_E}\frac{\frac{d^2H_E}{dt_E^2}}{\frac{dH_E}{dt_E}}\,. \tag{5}$$

If $\varepsilon^{(E)} \ll 1$ and $|\eta^{(E)}| \ll 1$, then the slow-roll inflation takes place and the evolution equations can be simplified:

$$3\,M_{PL}^2\,H_E^2 \approx V_E\,, \tag{6}$$

$$\frac{dH_E}{dt_E} = -\frac{1}{2M_{PL}^2}\left(\frac{d\tilde{\varphi}}{dt_E}\right)^2, \tag{7}$$

and

$$3H_E\frac{d\phi}{dt_E} \approx -V_{E,\tilde{\varphi}}\,. \tag{8}$$

Note that differentiating Eq. (6) and using Eq. (7), one gets Eq. (8).

THE EINSTEIN AND JORDAN FRAMES

Let us generalize this analysis on the model with nonminimal coupling
Models with a nonminimally coupled scalar field can be obtained by the conformal transformation

$$g_{\mu\nu} = \Omega^2 g^E_{\mu\nu} \tag{9}$$

with $\Omega^2 = \frac{M_{\rm Pl}^2}{2U(\sigma)}$.
We express the Einstein frame Hubble parameter H_E through Jordan frame variables,

$$H_E = \frac{d\ln(a_E)}{dt_E} = \Omega^{-1}\left(H + \frac{d\ln\Omega}{dt}\right) = \frac{M_{\rm Pl}}{\sqrt{2U}}\left(H + \frac{U_{,\sigma}\dot{\sigma}}{2U}\right), \tag{10}$$

where H is the Hubble parameter in the Jordan frame.

SLOW-ROLL PARAMETERS

The slow-roll parameter

$$\varepsilon^{(E)} = -\frac{1}{H_E^2}\frac{dH_E}{dt_E} = \varepsilon_1 + \frac{\zeta_1\left(1-\varepsilon_1\right)}{2+\zeta_1} - \frac{2\zeta_1\zeta_2}{\left(2+\zeta_1\right)^2}$$
$$\approx \varepsilon_1 + \frac{1}{2}\zeta_1 - \frac{\zeta_1}{4}\left(\zeta_1+2\zeta_2+2\varepsilon_1\right), \qquad (11)$$

where

$$\varepsilon_1 = -\frac{\dot H}{H^2} = \frac{d\ln(H^{-1})}{dN} = -\frac{1}{2}\frac{d\ln(H^2)}{dN}, \qquad \varepsilon_n = \frac{\dot\varepsilon_{n-1}}{H\varepsilon_{n-1}} = \frac{d\ln(\varepsilon_{n-1})}{dN},$$

$$\zeta_1 = \frac{\dot F}{HF} = \frac{d\ln(F)}{dN}, \qquad \zeta_n = \frac{\dot\zeta_{n-1}}{H\zeta_{n-1}} = \frac{d\ln(\zeta_{n-1})}{dN}.$$

So, there are two sets of the slow-roll parameters.

MODELS WITH NONMINIMAL COUPLING

The action of a generic model with a nonminimally coupled scalar field ϕ,

$$S = \int d^4x\sqrt{-g}\left[U(\sigma)R - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\sigma)\right],$$

includes the coupling function $U(\sigma) > 0$ and the potential $V(\sigma)$.
In the spatially flat Friedmann-Lemaître-Robertson-Walker metric with

$$ds^2 = -dt^2 + a^2(t)\,d\mathbf{x}^2\,,$$

the evolution equations are

$$6UH^2 = \frac{1}{2}\dot{\sigma}^2 + V - 6U_{,\sigma}\dot{\sigma}H, \tag{12}$$

$$4U\dot{H} = -\dot{\sigma}^2 + 2U_{,\sigma}\dot{\sigma}H - 4U_{,\sigma\sigma}\dot{\sigma}^2 - 2_{,\sigma}\ddot{\sigma}, \tag{13}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + V_{,\sigma} - 6U_{,\sigma}\left(\dot{H} + 2H^2\right) = 0, \tag{14}$$

where dots denote the time derivatives and $A_{,\sigma} = \frac{dA(\sigma)}{d\sigma}$ for any A.

Equations (13) and (14) are not written in the dynamical system form!
There are 2 ways to transform Eqs. (13) and (14) into a dynamical system.
The first way gives the following dynamical system

$$\dot{\sigma} =\psi,$$
$$\dot{\psi} =\frac{1}{E}\left\{ -6U_{,\sigma}\left[2U_{,\sigma\sigma}+1\right]\psi^2+3\left[2U_{,\sigma}^2-4U\right]H\psi-4U\left[V_{,\sigma}-12U_{\sigma}H^2\right]\right\},$$
$$\dot{H} =\frac{1}{E}\left\{ -\left[2U_{,\sigma\sigma}+1\right]\psi^2+8U_{,\sigma}H\psi+2U_{,\sigma}\left[V_{,\sigma}-12U_{\sigma}H^2\right]\right\},$$

where

$$E=12U_{,\sigma}^2+2U.$$

Another way to get a dynamical system is to introduce the function

$$Y = \frac{M_{\rm Pl}}{\sqrt{F}}\left(H + \frac{U_{,\sigma}\dot\sigma}{2U}\right). \qquad (15)$$

and rewrite Eqs. (12)–(14) in the following form:

$$3M_{\rm Pl}^2 Y^2 = \frac{A}{2}\dot\sigma^2 + V_{\rm eff}, \qquad (16)$$

$$\dot Y = -\frac{A\sqrt{2U}}{2M_{\rm Pl}^3}\,\dot\sigma^2, \qquad (17)$$

$$\ddot\sigma = -3\sqrt{\frac{2U}{M_{\rm Pl}^2}}\,Y\dot\sigma - \frac{A_{,\sigma}}{2A}\dot\sigma^2 - \frac{V_{{\rm eff},\sigma}}{A}, \qquad (18)$$

where

$$V_{\rm eff}(\sigma) = \frac{M_{\rm Pl}^4 V}{4U^2}, \quad A(\sigma) = \frac{M_{\rm Pl}^4}{4U^2}\left(1 + \frac{3U_{,\sigma}^2}{U}\right), \qquad (19)$$

M.A. Skugoreva, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **90** (2014) 064044 [arXiv:1404.6226],
A.Yu. Kamenshchik, E.O. Pozdeeva, A. Tribolet, A. Tronconi, G. Venturi and S.Yu. Vernov, Phys. Rev. D **110** (2024) 104011 [arXiv:2406.19762].

NEW SLOW-ROLL APPROXIMATION

The proposed slow-roll approximation is based on system (27)–(18). Neglecting the proportional to $\dot{\sigma}^2$ term in Eq. (27), we get

$$Y^2 \approx \frac{V_{\mathrm{eff}}}{3M_{\mathrm{Pl}}^2} = \frac{M_{\mathrm{Pl}}^2 V}{3F^2}\,. \tag{20}$$

Differentiating this equation over time and using Eq. (17), we obtain and a linear algebraic equation for $\chi(\sigma)$:

$$\psi \approx -\frac{M_{\mathrm{Pl}} V_{\mathrm{eff},\sigma}}{3YA\sqrt{2U}} = -\frac{M_{\mathrm{Pl}}\left(V_{,\sigma}U - 2VU_{,\sigma}\right)}{3Y\sqrt{2U}\left(U + 3U_{,\sigma}^2\right)} = \frac{-\left(V_{,\sigma}U - 2VU_{,\sigma}\right)}{3H\left(1 + \frac{\zeta_1}{2}\right)\left(U + 3U_{,\sigma}^2\right)}\,.$$

We obtain the first-order differential equation that defines slow-roll dynamic of σ:

$$\chi = \frac{d\sigma}{dN} = \frac{U\left(4VU_{,\sigma} - 2V_{,\sigma}U\right)}{VU + V_{,\sigma}U_{,\sigma}U + VU_{,\sigma}^2}\,. \tag{21}$$

$$H^2 \approx \frac{V}{6U}\left(1 + \frac{1}{2}\zeta_1\right)^{-2} = \frac{\left(UV + UU_{,\sigma}V_{,\sigma} + U_{,\sigma}^2 V\right)^2}{6UV\left(U + 3U_{,\sigma}^2\right)^2}\,, \tag{22}$$

This approximation has been generalized to $F(R)$ models in S.V. Ketov, E.O. Pozdeeva and S.Yu. Vernov, *JCAP* **12** (2025) 040 [arXiv:2508.08927].

Let us consider a generic metric $F(R)$ gravity model with the action

$$S_F = \int d^4x\sqrt{-g}F(R), \tag{23}$$

where F is a differentiable function of the Ricci scalar R.

The following action

$$S_J = \int d^4x\sqrt{-g}\left[F_{,\sigma}(R-\sigma)+F\right], \tag{24}$$

with a scalar field σ is equivalent to action (23).

For $F(R)$ models, we get

$$Y = \frac{M_{\rm Pl}}{\sqrt{2F_{,R}}}\left(H + \frac{F_{,RR}\dot{R}}{2F_{,R}}\right), \tag{25}$$

$$A = \frac{3M_{\rm Pl}^4 F_{,RR}^2}{4F_{,R}^3}, \qquad V_{\rm eff} = \frac{M_{\rm Pl}^4}{4F_{,R}^2}\left(RF_{,R} - F\right). \tag{26}$$

These functions satisfy the equation

$$3M_{\rm Pl}^2 Y^2 = \frac{A}{2}\dot{R}^2 + V_{\rm eff}. \tag{27}$$

$Y(t)$ is a monotonically decreasing function, because

$$\dot{Y} = -\frac{A\sqrt{2F_{,R}}}{2M_{\rm Pl}^3}\dot{R}^2. \tag{28}$$

$Y(t)$ is the Hubble parameter in the Einstein frame H_E as a function of the cosmic time in the Jordan frame t:

$$H_E = \frac{d\ln(a_E)}{dt_E} = \frac{M_{\rm Pl}}{\sqrt{2F_{,R}}}\left(H + \frac{F_{,RR}\dot{R}}{2F_{,R}}\right) = Y. \tag{29}$$

The slow-roll and inflationary parameters in the Einstein frame

The Hubble flow parameters in the Einstein frame are defined by

$$\varepsilon^{(E)} = -\frac{1}{H_E^2}\frac{dH_E}{dt_E} \tag{30}$$

and

$$\eta^{(E)} = \varepsilon^{(E)} - \frac{1}{2\,\varepsilon^{(E)}\,H_E}\frac{d\varepsilon^{(E)}}{dt_E} = -\frac{1}{2\,H_E}\frac{\frac{d^2H_E}{dt_E^2}}{\frac{dH_E}{dt_E}}\,. \tag{31}$$

The CMB observables are given by the amplitude of scalar perturbations A_s, the scalar spectral index n_s, the tensor-to-scalar ratio r, and their running, α_s and α_t, respectively.

In the *leading* approximation with respect to the SR parameters, we get

$$\begin{gathered} n_s \approx 1 - 2\varepsilon^{(E)} + \frac{d\ln(\varepsilon^{(E)})}{dN_E} = 1 - 4\varepsilon^{(E)} + 2\eta^{(E)}\,, \quad \alpha_s \approx -\frac{d\,n_s}{dN_E}\,, \\ r \approx 16\varepsilon^{(E)}\,, \qquad A_s \approx \frac{2H_E^2}{\pi^2 M_{\mathrm{Pl}}^2 r^{(E)}}\,, \quad \alpha_t \approx \frac{1}{8}\frac{d\,r}{dN_E}\,, \end{gathered} \tag{32}$$

the e-folds number $N_E = -\ln(a_E/a_{E_e})$.

To describe cosmic evolution during inflation, we use the e-folds number $N = -\ln(a/a_e)$, where a_e is a constant.
Using the relation

$$\frac{d}{dt} = -H\frac{d}{dN},$$

we find that Eq. (12) is equivalent to

$$\begin{cases} \left(H^2\right)' = 4H^2 - \dfrac{R}{3}, \\ R' = -\dfrac{\left(R - 6H^2\right)F_{,R} - F}{6H^2 F_{,RR}}, \end{cases} \tag{33}$$

where the primes denote the derivatives with respect to N.

The slow-roll parameters in the Jordan frame are as follows,

$$\varepsilon_1 = -\frac{\dot{H}}{H^2} = \frac{1}{2}\frac{d\ln(H^2)}{dN}, \qquad \varepsilon_n = \frac{\dot{\varepsilon}_{n-1}}{H\varepsilon_{n-1}} = -\frac{d\ln(\varepsilon_{n-1})}{dN},$$

$$\zeta_1 = \frac{\dot{F}_{,R}}{HF_{,R}} = \frac{F_{,RR}\dot{R}}{HF_{,R}} = -\frac{d\ln(F_{,R})}{dN}, \quad \zeta_n = \frac{\dot{\zeta}_{n-1}}{H\zeta_{n-1}} = -\frac{d\ln(\zeta_{n-1})}{dN}.$$

Using system (33), we get the SR parameters as

$$\varepsilon_1 = 2 - \frac{R}{6H^2}, \tag{34}$$

$$\zeta_1 = \frac{(6H^2 - R)\,F_{,R} + F}{6H^2 F_{,RR}\,(12H^2 - R)} - \frac{R}{3H^2}. \tag{35}$$

The slow-roll parameters are connected:

$$\zeta_2 = 1 + 2\frac{\varepsilon_1}{\zeta_1} + \varepsilon_1 - \zeta_1\,. \tag{36}$$

We can connect SR parameters in the Einstein and Jordan frames:

$$\varepsilon^{(E)} \approx \varepsilon_1 + \frac{1}{2}\zeta_1 - \frac{\zeta_1}{4}\left(\zeta_1 + 2\zeta_2 + 2\varepsilon_1\right) = \frac{1}{4}\zeta_1\left(\zeta_1 - 4\varepsilon_1\right) \approx 3\varepsilon_1^2 + \mathcal{O}\left(\varepsilon_1^3\right). \tag{37}$$

Similarly, we get

$$\eta^{(E)} = \varepsilon^{(E)} + \frac{1}{2}\frac{d\ln(\varepsilon^{(E)})}{dN_E} \approx -\varepsilon_2\frac{dN}{dN_E} + 3\varepsilon_1^2 + \mathcal{O}\left(\varepsilon_1^3\right) . \tag{38}$$

The relation

$$\frac{dN}{dN_E} = \frac{2}{2+\zeta_1} \approx 1 + \varepsilon_1 \tag{39}$$

gives

$$\eta^{(E)} \approx -\varepsilon_2 - \varepsilon_1\varepsilon_2 + 3\varepsilon_1^2 + \mathcal{O}\left(\varepsilon_1^3\right) . \tag{40}$$

The inflationary parameters are related to the SR parameters in the Jordan frame as

$$r \approx 16\varepsilon^{(E)} \approx 4\left|\zeta_1\left(\zeta_1 - 4\varepsilon_1\right)\right| \approx 48\varepsilon_1^2\,, \qquad (41)$$

$$n_s \approx 1 - 4\varepsilon^{(E)} + 2\eta^{(E)} \approx 1 - 2\varepsilon_2 - 6\varepsilon_1^2 - 2\varepsilon_1\varepsilon_2 \approx 1 - 2\varepsilon_2\,, \qquad (42)$$

$$\alpha_s \approx -2\varepsilon_2\varepsilon_3, \qquad \alpha_t \approx 2\,\frac{d\,\varepsilon^{(E)}}{dN_E} \approx -12\varepsilon_1^2\varepsilon_2. \qquad (43)$$

where we have kept only the leading contributions.
The amplitude of scalar perturbations is given by

$$A_s = \frac{H_E^2}{8\pi^2 M_{\mathrm{Pl}}^2\varepsilon^{(E)}} \approx \frac{H^2}{48\pi^2 F_{,R}\varepsilon_1^2} \approx \frac{H^2}{\pi^2 r F_{,R}}\,. \qquad (44)$$

Equation (27) in the SR approximation is just the Friedmann equation,

$$3M_{\rm Pl}^2 Y^2 \approx V_{\rm eff}\,. \tag{45}$$

After differentiating this equation, we obtain

$$R' \approx -\frac{2F_{,R}\,(RF_{,R}-2F)}{F_{,RR}\,(2RF_{,R}-F)}\,. \tag{46}$$

In addition, we get

$$\zeta_1(R) \approx \frac{2\,(RF_{,R}-2F)}{2RF_{,R}-F} \tag{47}$$

and

$$H^2(R) \approx \frac{(2RF_{,R}-F)^2}{54F_R\,(RF_R-F)}\,. \tag{48}$$

Equation (34) also implies

$$\varepsilon_1 = 2-\frac{R}{6H^2} \approx 2-\frac{9F_R R\,(RF_R-F)}{(2RF_{,R}-F)^2}\,. \tag{49}$$

Using Eqs. (47) and (49), we find in the SR approximation that

$$\varepsilon_1(R) = -\frac{1}{2}\zeta_1(R) + \frac{1}{4}\zeta_1^2(R). \tag{50}$$

Then, by using Eqs. (46), (47) and (49), we get

$$\varepsilon_2(R) = -\frac{1}{\varepsilon_1}\frac{d\varepsilon_1}{dN} \approx \frac{18FF_{,R}\left(FF_{,RR}R - F_{,R}^2R + F_{,R}F\right)}{F_{,RR}\left(4F_{,R}^3R^3 - 3F^2F_{,R}R + F^3\right)}\,, \tag{51}$$

$$\zeta_2(R) \approx \frac{6F_{,R}\left(\left[FF_{,RR} - F_{,R}^2\right]R + F_{,R}F\right)}{F_{,RR}\left(2F_{,R}R - F\right)^2}\,, \tag{52}$$

We also have

$$\epsilon_2 \approx \zeta_2(1 - \zeta_1) \approx \zeta_2 \tag{53}$$

and

$$\epsilon_3 \approx \zeta_3 - \frac{\zeta_1\zeta_2}{1 - \zeta_1} \approx \zeta_3. \tag{54}$$

Recent CMB observations due to Atacama Cosmology Telescope
T. Louis *et al.* [ACT], The Atacama Cosmology Telescope: DR6 Power Spectra, Likelihoods and ΛCDM Parameters," [arXiv:2503.14452].
combined with the Dark Energy Spectroscopic Instrument (DESI) data
A. G. Adame *et al.* [DESI], DESI 2024 VI: cosmological constraints from the measurements of baryon acoustic oscillations, JCAP **02** (2025), 021 [arXiv:2404.03002]
gives values of inflationary parameters

$$n_s = 0.9743 \pm 0.0034 \,, \quad \alpha_s = 0.0062 \pm 0.0052 \,. \tag{55}$$

These values are different from the Planck/BICEP data

$$n_s = 0.9651 \pm 0.0044 \,, \quad \alpha_s = -0.0069 \pm 0.0069 \,, \tag{56}$$

The ACT/DESI data does not significantly change the upper bound on r and the value of A_s,

$$A_s = (2.10 \pm 0.03) \times 10^{-9} \qquad \text{and} \qquad r < 0.028 \,. \tag{57}$$

Thus, the ACT/DESI data favours a *higher* scalar spectral index n_s with small *positive* running α_s.

Let us consider a new model, with

$$F_5(R) = \frac{M_{\mathrm{Pl}}^2}{2}\left(R + \frac{1}{6m^2}R^2 + \frac{c_3}{m^4}R^3 + \frac{c_4}{m^6}R^4 + \frac{c_5}{m^8}R^5\right) , \qquad (58)$$

where c_3, c_4 and c_5 are the dimensionless coupling constants.
The values of n_s, r, and α_s are independent upon the value of m that is fixed by the observed value of A_s.

Table: The values of R_{in} (in units of m^2), N_* and r for $n_s = 0.974$ and $\alpha_s = 0.0062$.

c_3	c_4	c_5	R_{in}	N_*	r
3.874×10^{-4}	-2.583×10^{-6}	6.084×10^{-9}	130	64.2	0.0030
3.412×10^{-4}	-2.282×10^{-6}	5.355×10^{-9}	130	56.1	0.0040
3.073×10^{-4}	-2.068×10^{-6}	4.845×10^{-9}	130	51.1	0.0050
2.805×10^{-4}	-1.903×10^{-6}	4.459×10^{-9}	130	47.6	0.0060
2.377×10^{-4}	-1.529×10^{-6}	3.361×10^{-9}	140	49.0	0.0060

Using the SR approximation formulae, the values of c_i can be chosen to meet the ACT favored values of n_s and α_s.

We consider models with the Gauss–Bonnet term, described by the following action:

$$S=\int d^4x\sqrt{-g}\left[U_0R-\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-V(\sigma)-\frac{1}{2}\xi(\sigma)\mathcal{G}\right], \qquad (59)$$

where $U_0=\frac{M_{\mathrm{Pl}}^2}{2}=\frac{1}{16\pi G}$,
the functions $V(\sigma)$ and $\xi(\sigma)$ are differentiable ones,
R is the Ricci scalar and

$$\mathcal{G}=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$$

is the Gauss–Bonnet term.

The perturbation theory for such types of models has been developed in
C. Cartier, J.c. Hwang, E.J. Copeland, Phys. Rev. D **64** (2001) 103504
J. c. Hwang and H. Noh, Phys. Rev. D **71** (2005) 063536
The standard approach to analyze an inflationary model with a nonminimally coupled scalar field involves the construction of the corresponding model in the Einstein frame. This method cannot be applied to models with the Gauss–Bonnet term.

EQUATIONS IN THE FLRW METRIC

In the spatially flat FLRW metric, one gets the evolution equations

$$12H^2\left(U_0-2\xi_{,\sigma}\psi H\right) = \psi^2+2V, \qquad (60)$$

$$4\dot{H}\left(U_0-2\xi_{,\sigma}\psi H\right) = 4H^2\left(\xi_{,\sigma\sigma}\psi^2+\xi_{,\sigma}\dot{\psi}-H\xi_{,\sigma}\psi\right)-\psi^2, \qquad (61)$$

$$\dot{\psi}+3H\psi = -V_{,\sigma}-12H^2\xi_{,\sigma}\left(\dot{H}+H^2\right). \qquad (62)$$

Using the relation $\frac{d}{dt}=H\frac{d}{dN}$ and introducing $\chi=\frac{\psi}{H}$, we get

$$\frac{d\sigma}{dN}=\chi,$$

$$\frac{d\chi}{dN}=\frac{1}{H^2\left(B-2\xi_{,\sigma}H^2\chi\right)}\left\{3\left[3-4\xi_{,\sigma\sigma}H^2\right]\xi_{,\sigma}H^4\chi^2 + \left[3B+2\xi_{,\sigma}V_{,\sigma}-6U_0\right]H^2\chi-\frac{V^2}{U_0}X\right\}-\frac{\chi}{2H^2}\frac{dH^2}{dN},$$

$$\frac{dH^2}{dN}=\frac{H^2}{2\left(B-2\xi_{,\sigma}H^2\chi\right)}\left[\left(4\xi_{,\sigma\sigma}H^2-1\right)\chi^2-16\xi_{,\sigma}H^2\chi-4\frac{V^2}{U_0^2}\xi_{,\sigma}X\right],$$

where $B=12\xi_{,\sigma}^2H^4+U_0$ and $X=\frac{U_0^2}{V^2}\left(12\xi_{,\sigma}H^4+V_{,\sigma}\right)$.

We consider the slow-roll parameters:

$$\varepsilon_1 = -\frac{\dot{H}}{H^2} = -\frac{d\ln(H)}{dN}, \qquad \varepsilon_{i+1} = \frac{d\ln|\varepsilon_i|}{dN}, \quad i \geqslant 1, \tag{63}$$

$$\delta_1 = \frac{2}{U_0}\xi_{,\sigma}H\psi = \frac{2}{U_0}\xi_{,\sigma}H^2\chi, \qquad \delta_{i+1} = \frac{d\ln|\delta_i|}{dN}, \quad i \geqslant 1. \tag{64}$$

The spectral index n_s and the tensor-to-scalar ratio r are connected with the slow-roll parameters as follows[1],

$$n_s = 1-2\varepsilon_1-\frac{2\varepsilon_1\varepsilon_2-\delta_1\delta_2}{2\varepsilon_1-\delta_1} = 1-2\varepsilon_1-\frac{d\ln(r)}{dN} = 1+\frac{d}{dN}\ln\left(\frac{H^2}{U_0 r}\right), \tag{65}$$

$$r = 8|2\varepsilon_1-\delta_1|. \tag{66}$$

The scalar perturbations amplitude

$$A_s = \frac{H^2}{\pi^2 U_0\, r}\,. \tag{67}$$

[1] Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897]

THE STANDARD APPROXIMATION

The standard approximate equations have been proposed in
Z.K. Guo, D.J. Schwarz, Phys. Rev. D **81** (2010), 123520
This way assumes that all inflationary parameters are negligibly small and can be removed from equations. In this slow-roll approximation, the leading order equations have the following form:

$$
\begin{aligned}
H^2 &\simeq \frac{V}{6U_0}\,,\\
\dot{H} &\simeq -\frac{\dot{\sigma}^2}{4U_0}-\frac{\xi_{,\sigma}H^3\dot{\sigma}}{U_0}\,,\\
\dot{\sigma} &\simeq -\frac{V_{,\sigma}+12\xi_{,\sigma}H^4}{3H}.
\end{aligned}
\tag{68}
$$

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\end{aligned}
\qquad (68)
$$

It has been shown by numerical calculations in
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) 063519
that the model with the potential $V=V_0\sigma^4$ and $\xi=\xi_0/V$, where V_0 and ξ_0 are positive constants, has no exit from inflation, whereas the standard slow-roll approximation shows that this exit does exist.
So, it is important to improve this slow-roll approximation.

We propose models with the potential

$$V = V_0\phi^n, \tag{69}$$

where $n = 2$ or $n = 4$, and

$$\xi = \frac{CU_0^2}{V + \Lambda}, \tag{70}$$

where C and Λ are positive constants.
Such a modification eliminate a singular behavior at $\phi = 0$ and allowing the Universe to exit the inflationary epoch when ϕ becomes sufficiently small.
The initial value of the scalar field ϕ_0 is positive and ϕ tends to zero during inflation.

To analyze the stability of de Sitter solutions in model (59) the effective potential has been proposed[2]:

$$V_{eff}(\sigma) = -\frac{U_0^2}{V(\sigma)} + \frac{1}{3}\xi(\sigma). \quad (71)$$

Calculating the derivative of the effective potential (71),

$$V_{eff\,,\sigma} = \frac{U_0^2 n\left(V_0^2\,(3-C)\,\sigma^{2\,n} + 6\,\Lambda\,V_0\sigma^n + 3\,\Lambda^2\right)}{3V_0\sigma^{n+1}\,(V_0\,\sigma^n + \Lambda)^2}\,, \quad (72)$$

we find that $V_{eff\,,\sigma} > 0$ for any $\sigma > 0$ at $C < 3$.
It is a sufficient condition that a de Sitter solution does not exist at any $\sigma > 0$.
This condition allows us to get an inflationary model without any fine-tuning of the initial data.

[2] E.O. Pozdeeva, M. Sami, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **100** (2019) 083527.

QUADRATIC EQUATION IN H^2

Equation (60) is equivalent to

$$12\,U_0\,(1-\delta_1)\,H^4 - 2VH^2 - \frac{\delta_1^2 U_0^2}{4\,\xi_{,\sigma}^2} = 0\,.$$

We consider the positive H^2 at $\delta_1 < 1$:

$$H^2 = \frac{V}{12\,U_0\,(1-\delta_1)} + \frac{\sqrt{V^2\xi_{,\sigma}^2 + 3\,U_0^3\delta_1^2\,(1-\delta_1)}}{12\,U_0\,(1-\delta_1)\,|\xi_{,\sigma}|}\,. \tag{73}$$

NEW APPROXIMATION I We expand this expression for H^2:

$$H^2 \approx \frac{V}{6\,U_0} + \frac{V}{6\,U_0}\delta_1 + \mathcal{O}(\delta_1^2)\,. \tag{74}$$

NEW APPROXIMATION II

The second way to get $\delta_1(\sigma)$ is the following.
We neglect the term proportional to δ_1^2 and get a nonzero solution:

$$H^2 = \frac{V}{6U_0(1-\delta_1)}\,. \tag{75}$$

FOURTH-ORDER POTENTIAL

We propose the model with the fourth-order potential $V = V_0\phi^4$.
For parameters

$$V_0 = 3.4 \times 10^{-11}, \quad C = 2.856, \quad \Lambda = 5.95 \times 10^{-13} M_{\rm Pl}^4 .$$

numeric calculations show that the inflation scenario does not contradict the current observation data.
The inflationary parameters are constrained by the combined analysis of Planck, BICEP/Keck and other observations as follows[3]:

$$A_s = (2.10 \pm 0.03) \times 10^{-9}, \qquad n_s = 0.9654 \pm 0.0040 , \qquad r < 0.028 .$$

[3] G. Galloni, N. Bartolo, S. Matarrese, M. Migliaccio, A. Ricciardone and N. Vittorio, JCAP **04** (2023) 062 [arXiv:2208.00188].

We fix the number of e-folding to be equal $N = 60.6$ and get unappropriated results for the standard approximations.
New approximations work essentially better (see Table).

Table: **Numerical and approximate values of parameters, characterizing the inflationary dynamic in the model with the quartic potential.**

Parameter	**Numeric result**	**Standard Approx**	**Approx I**	**Approx II**
$\phi_0/M_{\rm Pl}$	1.4019	4.9705	1.4898	1.3974
$10^9 A_s(\phi_0)$	2.096	117.2	2.599	2.017
$n_s(\phi_0)$	0.965	0.953	0.965	0.965
$r(\phi_0)$	0.0044	0.0120	0.0045	0.0045
$\phi_{end}/M_{\rm Pl}$	0.2000	0.8899	0.3048	0.3037
$\delta_1(\phi_{end})$	0.885	1.80	4.23	0.577
$N(\phi_{end})$	60.6	60.6	60.6	60.6

$$\varepsilon_1(\phi_{end}) = 1. \tag{76}$$

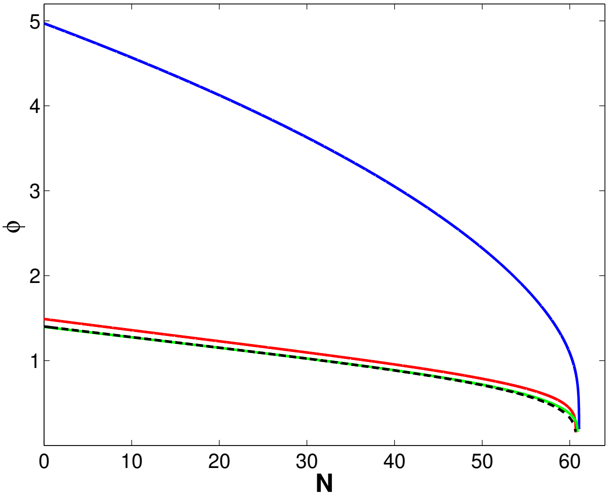

Figure: *4.* The inflationary model with $V(\phi) = V_0\phi^4$. Values of the function $\phi(N)$ in units of M_{Pl}. The black line is the result of the numerical integration. The blue curve is obtained in the standard approximation, red — in the approximation I , green — in the approximation II. The initial values $\phi(0) = \phi_0$ are given in Table 2.

CONCLUSIONS

- We propose new slow-roll approximations for modified gravity inflationary models, namely, for models with nonminimal coupling, $F(R)$ models, and models with the Gauss–Bonnet term. We find more accurate . expressions for the standard slow-roll parameters as functions of the scalar field. New slow-roll approximations are based on the use of the function $H(\sigma, \delta_1)$ rather than $H(\sigma)$.
- We proposed and studied a new inflation model that agrees with the ACT constraints to the CMB observables for some values of the model parameters.
- We construct new inflationary models with quadratic and quartic monomial potentials, $V = V_0\sigma^n$, and the function $\xi = \frac{CU_0^2}{V+\Lambda}$. Numerical analysis of these models indicates that the proposed inflationary scenarios are consistent with observation data. The standard approximation is not accurate enough to get correct values of inflationary parameters and correct number of e-folding during inflation. On the contrary, the inflationary parameters calculated using the proposed approximations are within the allowed ranges.

The research was conducted for the scientific program of the National Center for Physics and Mathematics, Section 5, Particle Physics and Cosmology.