

Higher-Spin Gauge Theory: Fundamentals & Recent Results

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Symmetries

Usual lower-spin symmetries:

- **Relativistic theories: Poincaré and (A)dS symmetry:** P_a and M_{ab}
- **SUSY:** $P_a, M_{ab} \longrightarrow P_a, M_{ab}, Q_\alpha$, $\alpha = 1, 2, 3, 4$
- **Inner symmetries:** $[T_i, (P_a, M_{ab})] = 0$
- **Conformal (super)symmetries:** $P_a, M_{ab} \longrightarrow P_a, M_{ab}, D, K^a$
- **Is it possible to go to higher HS symmetries?**

$$P_a, M_{ab} \longrightarrow P_a, M_{ab}, P_{a_1 \dots a_k} \dots$$

HS gauge theory: theory of maximal symmetries

Fronsdal Fields

All $m = 0$ HS fields are gauge fields

C. Fronsdal 1978

$\varphi_{a_1 \dots a_s}$ is a rank s symmetric tensor obeying $\varphi^c{}_c{}^b{}_{ba_3 \dots a_s} = 0$

Gauge transformation:

$$\delta \varphi_{a_1 \dots a_s} = \partial_{(a_1} \varepsilon_{a_2 \dots a_s)}, \quad \varepsilon^b{}_{ba_3 \dots a_{s-1}} = 0$$

Field equations: $G_{a_1 \dots a_s}(x) = 0$ $G_{a_1 \dots a_s}(x)$: Ricci-like tensor

$$G_{a_1 \dots a_s}(x) = \square \varphi_{a_1 \dots a_s}(x) - s \partial_{(a_1} \partial^b \varphi_{a_2 \dots a_s b)}(x) + \frac{s(s-1)}{2} \partial_{(a_1} \partial_{a_2} \varphi^b{}_{a_3 \dots a_s b)}(x)$$

Action

$$S = \int_{M^d} \left(\frac{1}{2} \varphi^{a_1 \dots a_s} G_{a_1 \dots a_s}(\varphi) - \frac{1}{8} s(s-1) \varphi_b{}^{ba_3 \dots a_s} G^c{}_{ca_3 \dots a_s}(\varphi) \right)$$

No-go and the Role of $(A)dS$

In 60th it was argued (Weinberg, Coleman-Mandula) that HS symmetries cannot be realized in a nontrivial local field theory in Minkowski space

In 70th it was shown by Aragone and Deser that HS gauge symmetries are incompatible with GR if expanding around Minkowski space

Green light: AdS background with $\Lambda \neq 0$ Fradkin, MV, 1987

In agreement with no-go statements the limit $\Lambda \rightarrow 0$ is singular

NonAbelian HS Algebra

3d Conformal HS symmetry = AdS_4 HS symmetry

HS gauge fields: $\omega(Y|x)$ 1986

$Y_A = (y_\alpha, \bar{y}_{\dot{\alpha}})$, $\alpha, \dot{\alpha} = 1, 2$ two-component spinor indices

$$\omega(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} \omega_{\alpha_1 \dots \alpha_n, \dot{\alpha}_1 \dots \dot{\alpha}_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar{y}^{\dot{\alpha}_1} \dots \bar{y}^{\dot{\alpha}_m}, \quad \omega = dx^\nu \omega_\nu$$

HS curvature and gauge transformation

$$R(Y|x) = d\omega(Y|x) + \omega(Y|x) * \wedge \omega(Y|x), \quad R = dx^\nu \wedge dx^\mu R_{\nu\mu}$$

$$\delta\omega(Y|x) = D\epsilon(Y|x) = d\epsilon(Y|x) + [\omega(Y|x), \epsilon(Y|x)]_*$$

$$[y_\alpha, y_\beta]_* = 2i\varepsilon_{\alpha\beta}, \quad [\bar{y}_{\dot{\alpha}}, \bar{y}_{\dot{\beta}}]_* = 2i\varepsilon_{\dot{\alpha}\dot{\beta}}$$

Star product is nonlocal in Y^A !

$$(f * g)(Y) = f(Y) \exp [i \overleftarrow{\partial}_A \overrightarrow{\partial}_B C^{AB}] g(Y)$$

Global symmetry of bosonic HS theory

Fradkin, MV 1986, MV 1988

Properties of HS Algebras

Let T_s be a homogeneous polynomial of degree $2(s - 1)$

$$[T_{s_1}, T_{s_2}] = T_{s_1+s_2-2} + T_{s_1+s_2-4} + \dots + T_{|s_1-s_2|+2}.$$

Once spin $s > 2$ appears, the HS algebra contains an infinite tower of higher spins: $[T_s, T_s]$ gives rise to T_{2s-2} as well as T_2 of $o(3, 2) \sim sp(4)$.

Usual symmetries: $\text{spin-}s \leq 2$ $u(1) \oplus o(3, 2)$: maximal finite-dimensional subalgebra of $hu(1, 0|4)$. $u(1)$ is associated with the unit element.

HS symmetries do not commute with space-time symmetries

$$[T^a, T^{HS}] = T^{HS}, \quad [T^{ab}, T^{HS}] = T^{HS}$$

HS transformations map gravitational fields (metric) to HS fields:

Riemann geometry is not appropriate for HS theory:

concept of local event may become illusive!

HS Gauge Theory and Quantum Gravity

HS symmetry is in a certain sense **maximal** relativistic symmetry. Hence, it cannot result from spontaneous breakdown of a larger symmetry:

HS symmetries are manifest at ultrahigh energies above any scale including Planck scale

- HS gauge theory should capture effects of Quantum Gravity:
restrictive HS symmetry versus unavailable experimental tests
- Lower-spin theories as low-energy limits of HS theory:
lower-spin symmetries: subalgebras of HS symmetry
- **String Theory as spontaneously broken HS theory?! ($s > 2, m > 0$)**

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983), Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

Infinite towers of spins \Rightarrow infinite towers of derivatives.

How (non)local is HS gauge theory?

HS Multiplets

Infinite set of spins $s = 0, 1/2, 1, 3/2, 2, \dots$

$\omega_{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_m}$ **and** $C_{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_m}$ **with all** $n \geq 0$ **and** $m \geq 0$ ($\alpha = 1, 2, \dot{\alpha} = 1, 2$)

Generating functions $\omega(Y|x)$ **and** $C(Y|x)$: **unrestricted functions of commuting spinor variables** $Y = (y_\alpha, \bar{y}_{\dot{\alpha}})$

$$A(Y|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} A_{\alpha_1 \dots \alpha_n, \dot{\alpha}_1 \dots \dot{\alpha}_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar{y}^{\dot{\alpha}_1} \dots \bar{y}^{\dot{\alpha}_m}$$

Gauge one-forms $\omega_{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_m}, \quad n + m = 2(s - 1)$

$$s = 1 : \quad \omega(x) = dx^\nu \omega_\nu(x)$$

$$s = 2 : \quad \omega_{\alpha\dot{\beta}}(x), \quad \omega_{\alpha\beta}(x), \quad \bar{\omega}_{\dot{\alpha}\dot{\beta}}(x)$$

$$s = 3/2 : \quad \omega_\alpha(x), \quad \bar{\omega}_{\dot{\alpha}}(x)$$

Frame-like fields: $|n - m| = 0$ **(bosons)** or $|n - m| = 1$ **fermions**

Auxiliary Lorentz-like fields: $|n - m| = 2$ **(bosons)**

Extra fields: $|n - m| > 2$

Zero-forms $C(Y|x)$: **matter fields and higher derivatives of massless fields**

Free Unfolded Massless Equations

The full unfolded system for free massless bosonic fields is

1989

$$\star \quad R_1(y, \bar{y} | x) = \frac{i}{4} \left(\eta \bar{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}} \partial \bar{y}^{\dot{\beta}}} C(0, \bar{y} | x) + \bar{\eta} H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0 | x) \right)$$

$$\star\star \quad \tilde{D}_0 C(y, \bar{y} | x) = 0$$

$$R_1(y, \bar{y} | x) := D_0^{ad} \omega(y, \bar{y} | x) \quad D_0^{ad} := D^L - e^{\alpha\dot{\beta}} \left(y_\alpha \frac{\partial}{\partial \bar{y}^{\dot{\beta}}} + \frac{\partial}{\partial y^\alpha} \bar{y}_{\dot{\beta}} \right)$$

$$\tilde{D}_0 = D^L + e^{\alpha\dot{\beta}} \left(y_\alpha \bar{y}_{\dot{\beta}} + \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\dot{\beta}}} \right) \quad D^L := d_x - \left(\omega^{\alpha\beta} y_\alpha \frac{\partial}{\partial y^\beta} + \bar{\omega}^{\dot{\alpha}\dot{\beta}} \bar{y}_{\dot{\alpha}} \frac{\partial}{\partial \bar{y}^{\dot{\beta}}} \right)$$

$$H^{\alpha\beta} := e^\alpha_{\dot{\alpha}} e^{\beta\dot{\alpha}}, \quad \bar{H}^{\dot{\alpha}\dot{\beta}} := e_{\alpha\dot{\alpha}} e^{\alpha\dot{\beta}}$$

$\star\star$ implies that higher-order terms in y and \bar{y} describe higher-derivative descendants of the primary HS fields

Zero-Form Sector

Equations on the gauge invariant zero-forms C

$$C(Y; K|x) = \sum_{n,m=0}^{\infty} \frac{1}{2n!m!} C_{\alpha_1 \dots \alpha_n, \dot{\alpha}_1 \dots \dot{\alpha}_m}(x) y^{\alpha_1} \dots y^{\alpha_n} \bar{y}^{\dot{\alpha}_1} \dots \bar{y}^{\dot{\alpha}_m}$$

decompose into independent subsystems associated with different spins

Spin- s zero-forms are $C_{\alpha_1 \dots \alpha_n, \dot{\alpha}_1 \dots \dot{\alpha}_m}(x)$ with

$$n - m = \pm 2s$$

Perturbative unfolded equations

$$d_x C = \sigma_- C + \text{lower-derivative and nonlinear terms}$$

$$\sigma_- := e^{\alpha\dot{\beta}} \frac{\partial^2}{\partial y^\alpha \partial \bar{y}^{\dot{\beta}}}, \quad \sigma_-^2 = 0$$

$C_{\alpha_1 \dots \alpha_n, \dot{\alpha}_1 \dots \dot{\alpha}_m}(x)$ contain $\frac{n+m}{2} - \{s\}$ space-time derivatives of the spin- s dynamical fields. Presence of zero-forms C in the HS vertices may induce infinite towers of derivatives and, hence, non-locality.

HS Vertices in the Unfolded Formalism

Diffeomorphisms without Riemannian geometry: Cartan formalism of differential forms with field equations in the **unfolded form**

$$d_x \omega = -\omega * \omega + \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \dots,$$

$$d_x C = -[\omega, C]_* + \Upsilon(\omega, C, C) + \dots$$

Unfolding: multidimensional cousin of the first-order form of differential equations

$$\frac{\partial}{\partial t} \rightarrow d, \quad q^i(t) \rightarrow W^\Omega(x) = dx^{n_1} \wedge \dots \wedge dx^{n_p} W_{n_1 \dots n_p}^\Omega(x)$$

$$dW^\Omega(x) = G^\Omega(W(x)), \quad d = dx^n \partial_n$$

$G^\Omega(W)$: **function of “supercoordinates”** W^Ω

$$G^\Omega(W) = \sum_{n=1}^{\infty} f^\Omega_{\Lambda_1 \dots \Lambda_n} W^{\Lambda_1} \wedge \dots \wedge W^{\Lambda_n}$$

$d > 1$: **Compatibility conditions**

$$G^\Lambda(W) \wedge \frac{\partial G^\Omega(W)}{\partial W^\Lambda} \equiv 0$$

Properties

- General applicability
- Manifest (HS) gauge invariance under the gauge transformation

$$\delta W^\Omega = d\varepsilon^\Omega + \varepsilon^\wedge \frac{\partial G^\Omega(W)}{\partial W^\wedge},$$

gauge parameter $\varepsilon^\Omega(x)$ is a $(p_\Omega - 1)$ -form.

- Invariance under diffeomorphisms

Exterior algebra formalism

- Interactions: nonlinear deformation of $G^\Omega(W)$
- Degrees of freedom are in 0-forms $C^i(x_0)$ at any $x = x_0$ (as $q(t_0)$)
infinite-dimensional module dual to the space of single-particle states
realized as a space of functions of auxiliary variables like $C(y, \bar{y})$ instead
of phase space coordinates in the Hamiltonian approach

Space-Time Metamorphoses and Holography

Independence of ambient space-time: geometry is encoded by $G^\Omega(W)$

Fundamental fact: unfolded equation makes sense in any space-time

$$dW^\Omega(x) = G^\Omega(W(x)), \quad x \rightarrow X = (x, y), \quad d_x \rightarrow d_X = d_x + d_y, \quad d_y = dy^u \frac{\partial}{\partial y^u}$$

X -dependence is reconstructed in terms of fields $W^\Omega(X_0) = W^\Omega(x_0, y_0)$

at any X_0 To take $W^\Omega(x_0, y_0)$ in space M_X with coordinates X_0 is the

same as to take $W^\Omega(x_0)$ in the space $M_x \in M_X$ with coordinates x

AdS_4 HS theory was conjectured to be holographically dual to $3d$ vectorial conformal models

Klebanov-Polyakov (2002), Sezgin-Sundell (2002); Giombi and Yin (2009)

Equations of motion in perturbatively local field theory $E_{A_0, s_0}(\partial, \phi) = 0$

$$E_{A_0, s_0}(\partial, \phi) = \sum_{k=0, l=1}^{\infty} a_{A_0 A_1 \dots A_l}^{n_1 \dots n_k}(s_0, s_1, \dots, s_l) \partial_{n_1} \dots \partial_{n_k} \phi_{s_1}^{A_1} \dots \phi_{s_l}^{A_l}$$

have a finite # of non-zero coefficients $a_{A_0 \dots A_l}^{n_1 \dots n_k}$ at any order l .

s_0 is the spin of the field on which the linearized equation is imposed

HS theory involves infinite towers of Fronsdal fields of all spins.

$a_{A_0 \dots A_l}^{n_1 \dots n_k}$ may take an infinite # of values.

It makes sense to distinguish between

Gelfond, MV 2018

local: finite number of derivatives at any order

$$a_{A_0 \dots A_l}^{n_1 \dots n_k}(s_0, s_1, \dots, s_l) = 0 \quad \text{at } k > k_{max}(l)$$

spin-local: finite number of derivatives for any finite subset of fields

$$a_{A_0 \dots A_l}^{n_1 \dots n_k}(s_0, s_1, s_2, \dots, s_l) = 0 \quad \text{at } k > k_{max}(s_0, s_1, s_2, \dots, s_l)$$

non-local: infinite number of derivatives for a finite subset of fields at some order.

Doubling of Spinors

How to find nonlinear corrections to HS equations?

The efficient trick 1992 reduces the problem to De Rham cohomology with respect to additional spinor variables $Z^A = (z^\alpha, \bar{z}^{\dot{\alpha}})$ in presence of Klein operators K

$$\omega(Y; K|x) \longrightarrow W(Z; Y; K|x), \quad C(Y; K|x) \longrightarrow B(Z; Y; K|x), \quad Y^A = (y^\alpha, \bar{y}^{\dot{\alpha}})$$

$$\Phi = (\mathcal{W}, B)$$

$$\begin{array}{ccc} d_z \Phi(z, x) & \longrightarrow & V_{z,x}(\Phi)(z, x), \quad d_z V(\Phi) = 0 \\ \uparrow & & \downarrow \\ d_x \Phi(x) & \longrightarrow & V_x(\Phi)(x), \quad d_x V(\Phi) = 0 \end{array}$$

Nonlinear HS Equations

Nonlinear HS equations

$$\left\{ \begin{array}{l} d_x W + W \star W = 0 \\ d_x B + W \star B - B \star W = 0 \\ d_x S + W \star S + S \star W = 0 \\ \mathbf{S \star B - B \star S = 0} \\ \mathbf{S \star S = i(\theta^A \theta_A + \eta \theta^\alpha \theta_\alpha B \star k \star \kappa + \bar{\eta} \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} B \star k \star \bar{\kappa})} \end{array} \right. \quad \mathbf{1992}$$

Some determine Z_A -dependence in terms of “initial data” $\omega(Y; K|x)$ and $C(Y; K|x)$

$S(Z; Y; K|x) = \theta^A S_A(Z; Y; K|x)$ is a connection along Z^A ($\theta^A \equiv dZ^A$)

HS star product

$$(f \star g)(Z, Y) = \int dS dT f(Z + S, Y + S) g(Z - T, Y + T) \exp -i S_A T^A$$

Klein operators $K = (k, \bar{k})$ generate chirality automorphisms

$$kf(A) = f(\tilde{A})k, \quad A_A = (a_\alpha, \bar{a}_{\dot{\alpha}}) : \quad \tilde{A}_A = (-a_\alpha, \bar{a}_{\dot{\alpha}})$$

Inner Klein operators: $\kappa = \exp iz_\alpha y^\alpha, \bar{\kappa} = \exp iz_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}, \kappa \star f = \tilde{f} \star \kappa, \quad \kappa \star \kappa = 1$

Dynamical content is in the d_x -independent twistor sector

Perturbative Analysis

Vacuum solution

$$B_0 = 0, \quad S_0 = \theta^A Z_A, \quad W_0 = \frac{1}{2} w^{AB}(x) Y_A Y_B$$

$$d_x W_0 + W_0 \star W_0 = 0, \quad w^{AB} : AdS_4$$

$$[S_0, f]_\star = -2 \text{id}_Z f, \quad d_Z = \theta^A \frac{\partial}{\partial Z^A}$$

First-order fluctuations

$$B_1 = C(Y), \quad S = S_0 + S_1, \quad W = W_0(Y) + W_1(Y) + W_0(Y)C(Y)$$

Order- n equations containing S have the form

$$d_Z U_n(Z; Y | dZ) = V[U_{<n}](Z; Y | \theta) \quad d_Z V[U_{<n}](Z; Y | \theta) = 0$$

can be solved by shifted homotopy with shift parameters Q

$$U_n(Z; Y | \theta) = d_Z^* V[U_{<n}](Z; Y | \theta) + \mathbf{h}(\mathbf{Y}) + d_Z \epsilon(Z; Y | \theta)$$

$$d_Z^* V(Z; Y | \theta) = (Z^A - Q^A) \frac{\partial}{\partial \theta^A} \int_0^1 \frac{dt}{t} V(tZ + (1-t)Q; Y | t\theta)$$

Interpretation

The contracting homotopy freedom encodes:

All possible gauge choices in d_z -exact forms $d_z \epsilon(Z; Y|dZ)$

All possible choices of field variables in d_z cohomology $h(Y)$

Any unfolded HS system is associated with one or another solution to the nonlinear HS system.

How to single out the proper (e.g., minimally nonlocal) frames?

Spin-local limit: $\beta \rightarrow -\infty$ with $Q_A = \beta \frac{\partial}{\partial Y^A}$

Didenko, Gelfond, Korybut, MV 1909.04876

Differential Homotopy

Homotopy and shift parameters t^a are treated as coordinates of some manifold \mathcal{M} with the total differential (2023)

$$d := d_Z + d_t, \quad d_Z := \theta^A \frac{\partial}{\partial Z^A}, \quad d_t := dt^a \frac{\partial}{\partial t^a},$$

Equations to be solved at every perturbation order still have the form

$$df(Z, t, \theta, dt) = g(Z, t, \theta, dt), \quad dg(Z, t, \theta, dt) = 0.$$

Fundamental Ansatz

Lower-order computation yields expressions of the remarkable form

$$f_\mu = \int_{p_i^2 r_i^2 u^2 v^2 \tau \sigma \beta \rho} \mu(\tau, \sigma, \beta, \rho, u, v, p, r) d\Omega^2 \mathcal{E}(\Omega) G(g(r)),$$

where $\mu(\tau, \sigma, \beta, \rho, \dots)$ is demanded to have compact support in $\tau, \sigma, \beta, \rho$

$$d\Omega^2 := d\Omega^\alpha d\Omega_\alpha$$

$$\Omega_\alpha(\tau, t) := \tau z_\alpha - (1 - \tau)(p_\alpha(\sigma) - \beta v_\alpha + \rho(y_\alpha + p_{+\alpha} + u_\alpha))$$

$$p_{+\alpha} := \sum_{i=1}^l p_{i\alpha}, \quad p_\alpha(\sigma) = \sum_{i=1}^k p_{i\alpha} \sigma_i, \quad p_{i\alpha} := -i \frac{\partial}{\partial r_i^\alpha}$$

$$\mathcal{E}(\Omega) := \exp i \left(\Omega_\beta (y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^l p_{i\alpha} r_i^\alpha - \sum_{k \geq j > i \geq 1} p_{i\beta} p_j^\beta \right),$$

$$G_l(g) := g_1(r_1) \dots g_l(r_l) k,$$

$g_i(y)$ are some functions of y_α (e.g., $C(y)$ or $\omega(y)$).

Projectively-Compact Spin-Local Vertex

Final result

$$\mathcal{DC} = J_{pc}^\eta + J_{pc}^{\bar{\eta}}$$

$$J_{pc}^\eta = \frac{i\eta}{4} \int_{u^2 v^2 \tau \rho \beta \sigma_1 \sigma_2} d^2 u d^2 v \mathbb{D}(\tau) \mathbb{D}(\sigma_1 + \rho) \mathbb{D}(\sigma_2 - (1 - \beta - \rho)) \vartheta(-\sigma_1) \vartheta(\sigma_2) d\Omega^2 \tilde{\mu}(\beta) \mathcal{E}(\Omega) \\ \left[P(\beta - 1, \sigma_\omega, \sigma_1, \sigma_2, 1 - \beta) \omega(r_\omega, \bar{y}; K) \bar{*} C(r_1, \bar{y}; K) \bar{*} C(r_2, \bar{y}; K) \right. \\ \left. + P(\beta - 1, \sigma_1, \sigma_\omega, \sigma_2, 1 - \beta) C(r_1, \bar{y}; K) \bar{*} \omega(r_\omega, \bar{y}; K) \bar{*} C(r_2, \bar{y}; K) \right. \\ \left. + P(\beta - 1, \sigma_1, \sigma_2, \sigma_\omega, 1 - \beta) C(r_1, \bar{y}; K) \bar{*} C(r_2, \bar{y}; K) \bar{*} \omega(r_\omega, \bar{y}; K) \right] \Big|_{r_\omega, C_i=0}^k,$$

$$P_k(\sigma_l, \sigma_{l+1}, \dots, \sigma_{l+k}) := \vartheta(\sigma_{l+k} - \sigma_{l+k-1}) \dots \vartheta(\sigma_{l+1} - \sigma_l), \quad \mathbb{D}(a) := da \delta(a)$$

J_{pc}^η is both spin-local and projectively-compact

containing an overall factor of $y^\alpha p_{\omega\alpha}$. To reach projective compactness at higher orders the preexponent factors have to be of the form

$$\prod_i \mathbb{D}(\sigma_{C_i} + \rho + \dots)$$

Coxeter HS Theory

To extend HS theory towards String theory more copies of oscillators are needed. A naive generalization is inconsistent.

In the Coxeter HS theory all fields W, S, B depend on p sets of variables like Y_A^n, Z_A^n, dZ_n^A ($A = 1, \dots, 4, n = 1, \dots, p$), idempotents I_n and Klein operators \hat{K}_v associated with all root vectors of \mathcal{C}

$$K_v q_\alpha^n = R_v^n{}_m q_\alpha^m K_v, \quad R_{v_a} x^i = x^i - 2v_a^i \frac{(v_a, x)}{(v_a, v_a)}, \quad R_{v_a}^2 = I.$$

Coxeter HS equations are

2018

$$S * S = -i \left(dZ^{An} dZ_{An} + \sum_i \sum_{v \in \mathcal{R}_i} F_{i*}(B) \frac{dZ_n^\alpha v^n dZ_{\alpha m} v^m}{(v, v)} * \hat{K}_v \right)$$

Further extension is via multiparticle HS algebras $U(\mathfrak{hs}(\dots))$ rich enough to generate String spectrum 2013

Conclusion

HS gauge theories contain gravity along with infinite towers of other fields with various spins including ordinary matter fields: singlet scalar!

Infinite-dimensional HS symmetry

Inner symmetries and SUSY are included

HS theories exist in various dimensions.

Unbroken HS symmetries demand $(A)dS$ background

HS vertices contain higher derivatives: Concepts of Riemann geometry are not applicable: study of exact solutions is very instructive:

BH-like solutions Didenko, MV 2009, Iazeola, Sundell 2010

Central problem: spontaneous breakdown of HS symmetries

Coxeter HS theory as a step to String Theory Tarusov, Ushakov, MV 2025, 2026

The new geometric differential homotopy approach is designed to figure out the actual level of non-locality of the HS theory. MV 2023, Kirakosyants,

Valerev, MV 2025

Recent progress in SDHS theory as a sector of HS theory

To do: higher orders and exact solutions in HS theories of various kind

Other directions

Massive HS fields: Zinoviev

Supersymmetric HS theory Buchbinder, Ivanov, Zaigraev; Misuna

Light-cone approach Metsaev, Ponomarev...

BRST approach Buchbinder, Fedoruk, Reshetnyak, ...

Self-Dual HS Theory

The dynamically nontrivial equation

$$S \star S = i(\theta^A \theta_A + \eta \theta^\alpha \theta_\alpha B \star k \star \kappa + \bar{\eta} \bar{\theta}^{\dot{\alpha}} \bar{\theta}_{\dot{\alpha}} B \star k \star \bar{\kappa})$$

contains two coupling constants that are complex conjugated in Minkowski signature but real in Euclidean and maximally split signatures

In 1992 the model with $(\eta = 0) \bar{\eta} = 0$ was identified with (anti)self-dual HS theory.

Lower-order spin-local vertices of the SDHS theory were found

Gelfond, Didenko, Koribut, MV 1909.04876, 2009.02811

An interesting new scheme leading to spin-local SD vertices in all orders was proposed by Didenko 2209.01966

and partly independently by Sharapov and Skvortsov 2209.01796, 2209.1544

Local Symmetries

Useful viewpoint: any global symmetry is the remnant of a local symmetry with parameters like $\varepsilon^a(x), \varepsilon^{ab}(x), \varepsilon^\alpha(x), \varepsilon^i(x)$ being arbitrary functions of space-time coordinates

Local symmetries: gauge fields A_a^n

$$\delta A_a^n = \partial_a \varepsilon^n + \dots$$

$$S \longrightarrow S + \Delta S + \dots, \quad \Delta S = \int_{M^d} J_n^a(\varphi) A_a^n(x)$$

ΔS : Noether current interaction.

Subtlety

If $\varphi(x)$ were gauge fields with gauge parameters ε' , $J_n^a(\varphi)$ may not be invariant under the ε' symmetry

Noether current interaction for several gauge fields may be obstructed

Holographic Higher Spins

Sezgin-Sundell-Klebanov-Polyakov conjecture: HS theory in AdS_4 is holographically dual to $3d$ vector model of scalar fields ϕ^i ($i = 1 \dots N$).

Sleight and Taronna argued 2017 that a HS theory resulting from holographic analysis based on the is essentially non-local

Since HS holography is a weak-weak duality, it should be possible to test it.

No locality analysis of the full HS theory in AdS_4 has been done except for that of the Lebedev group Didenko, Gelfond, Korybut, MV 2017-2022

What has been shown so far indicates that HS theory is spin-local?!

Suggests gauged version of the SSKP conjecture with conformal HS boundary theory MV 2012

Projectively-Compact Spin-Local Vertices in $d_x C$

The $d_x C$ vertex is 2017

$$\Upsilon = \Upsilon_\eta(e, C) + \Upsilon_{\bar{\eta}}(e, C)$$

$$\Upsilon_\eta(e, C) = \frac{1}{2} \eta \exp(i\bar{P}^{1,2}) \int_0^1 d\tau e(y, (1-\tau)\bar{p}_1 - \tau\bar{p}_2) C(\tau y, \bar{y}; K) C(-(1-\tau)y, \bar{y}; K),$$

$$\Upsilon_{\bar{\eta}}(e, C) = \frac{1}{2} \bar{\eta} \exp i(P^{1,2}) \int_0^1 d\tau e((1-\tau)p_1 - \tau p_2, \bar{y}) C(y, \tau\bar{y}; K) C(y, -(1-\tau)\bar{y}; K),$$

where $e(a, \bar{a}) := e^{\alpha\dot{\alpha}} a_\alpha \bar{a}_{\dot{\alpha}}$.

Being non-polynomial either in $P^{1,2}$ or in $\bar{P}^{1,2}$, Υ is spin-local

Since Υ contains either $e^{\alpha\dot{\alpha}} y_\alpha$ or $e^{\alpha\dot{\alpha}} \bar{y}_{\dot{\alpha}}$,

$$\Pi^{des} \Upsilon = 0 \quad \Rightarrow \quad \Upsilon \quad \text{is projectively-compact spin-local}$$

PCSL vertices contain the minimal possible number of derivatives.

Field Redefinitions

A local theory remains local under perturbatively local field redefinitions

$$\phi_{s_0}^B \rightarrow \phi_{s_0}^B + \delta\phi_{s_0}^B, \quad \delta\phi_{s_0}^B = \sum_{k=0, l=1}^{\infty} b_{A_1 \dots A_l}^{B n_1 \dots n_k}(s_0, s_1, \dots, s_l) \partial_{n_1} \dots \partial_{n_k} \phi_{s_1}^{A_1} \dots \phi_{s_l}^{A_l}$$

with a finite # of non-zero coefficients at any order.

Which field redefinitions leave vertices spin-local?

General spin-local field redefinitions do not work since contributions of all spin s_p redefined fields may develop non-locality

$$\delta E_{A_0, s_0}(\partial, \phi) = \sum_{s_p=0}^{\infty} \sum_{p, k, k'=0, l, l'=1}^{\infty} a_{A_0 A_1 \dots A_l}^{n_1 \dots n_k}(s_0, s_1, s_2, \dots, s_p, \dots, s_l) \partial_{n_1} \dots \partial_{n_k} \phi_{s_1}^{A_1} \dots \phi_{s_{p-1}}^{A_{p-1}} \phi_{s_{p+1}}^{A_{p+1}} \dots \phi_{s_l}^{A_l} b_{B_1 \dots B_{l'}}^{A_p m_1 \dots m_{k'}}(s_p, t_1, \dots, t_{l'}) \partial_{m_1} \dots \partial_{m_k} \phi_{t_1}^{B_1} \dots \phi_{t_{l'}}^{B_{l'}}$$

Spin-local-compact field redefinitions in spin-local theories:

proper substitute since summation over s_p is finite.

One of the central problems in HS theory is to find a field frame making it (spin-)local. Given non-locally looking field theory, the essential question is whether or not it is spin-local in some other variables.

Spinor Spin-Locality

Polynomiality of $F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$ in either P^{ij} or $\bar{P}^{ij} \forall i, j$ associated with C

Restriction to the fixed spin relates the degrees in P^{ij} and \bar{P}^{kl} since

$$n - m = \pm 2s$$

Non-linear corrections can affect the relation between spinor and space-time spin-locality making obscure the space-time interpretation of the locality analysis in the spinor space.

This does not happen for projectively-compact spin-local vertices with

$$F(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) = Q_\omega G(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl}) + \bar{Q}_\omega \bar{G}(Q^i, P^{ij}, \bar{Q}^j, \bar{P}^{kl})$$

Q_ω and \bar{Q}_ω being associated with the one-forms ω among Φ .

arXiv: 2208.02004

Projectively-Compact Spin-Local Vertices

Using background frame $e^{\alpha\dot{\beta}}$ HS equations can be represented as

$$D^L C(y, \bar{y}) = e^{\alpha\dot{\alpha}} \left(\partial_\alpha \bar{\partial}_{\dot{\alpha}} F^{++}(y, \bar{y}) + y_\alpha \bar{\partial}_{\dot{\alpha}} F^{-+}(y, \bar{y}) + \bar{y}_{\dot{\alpha}} \partial_\alpha F^{+-}(y, \bar{y}) + y_\alpha \bar{y}_{\dot{\alpha}} F^{--}(y, \bar{y}) \right)$$

Generally, nonlinear corrections can contribute to any of F^{ab} .

The contribution to F^{++} can be singled out by the projector

$$\Pi^{des} := N_y^{-1} \bar{N}_{\bar{y}}^{-1} y^\alpha \bar{y}^{\dot{\alpha}} \frac{\partial}{\partial e^{\alpha\dot{\alpha}}}, \quad N_y := y^\alpha \partial_\alpha, \quad N_{\bar{y}} := \bar{y}^{\dot{\alpha}} \bar{\partial}_{\dot{\alpha}}$$

A spin-local vertex Υ is called **projectively compact** if $\Pi^{des} \Upsilon$ is spin-local-compact. In particular, if $\Pi^{des} \Upsilon = 0$.

The contribution of the projectively-compact spin-local vertices can affect the expressions of the descendants in terms of derivatives of the ground fields only by spin-local-compact terms that preserve space-time locality of the vertex associated with the spin-local spinor vertex.

Vacuum

Equation of AdS_4 : $R_0 = 0$ **for** $\omega_0 \in sp(4) \sim o(3, 2)$

$$\omega_0(Y|X) = \frac{1}{4i}(w^{\alpha\beta}(X)y_\alpha y_\beta + \bar{w}^{\dot{\alpha}\dot{\beta}}(X)\bar{y}_{\dot{\alpha}}\bar{y}_{\dot{\beta}} + 2\lambda h^{\alpha\dot{\beta}}(X)y_\alpha\bar{y}_{\dot{\beta}})$$

Fluctuations

$$\omega = \omega_0 + \omega_1, \quad R_1 = D_0\omega_1 = d\omega_1 + [\omega_0, \omega_1]_*$$

Examples

$$s = 0 : \quad C_{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_n}^0 \sim C_{a_1 \dots a_n}, \quad C^b{}_{ba_3 \dots a_n} = 0$$

$$s = 2$$

Gauge fields: Lorentz connection $\omega_{\alpha\beta}, \bar{\omega}_{\dot{\alpha}\dot{\beta}}$ **and vierbein** $\omega_{\alpha, \dot{\beta}}$

Zero-forms $C_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}(X)$ **and** $\bar{C}_{\dot{\alpha}_1 \dot{\alpha}_2 \dot{\alpha}_3 \dot{\alpha}_4}(X)$: **Weyl tensor in terms of two-component spinors.**

Higher components $C_{\alpha_1 \dots \alpha_n, \dot{\beta}_1 \dots \dot{\beta}_m}^s$: $|n - m| = 4$: **all its derivatives**

$R_{\alpha, \dot{\beta}} = 0$: **expresses Lorentz connection via vierbein**

$R_{\alpha\beta} = H^{\gamma\delta} C_{\alpha\beta\gamma\delta}$, $R_{\dot{\alpha}\dot{\beta}} = \bar{H}^{\dot{\gamma}\dot{\delta}} \bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}$: **nonzero part of the Riemann tensor belongs to Weyl tensor**

Spin two in tensorial language

$$R_{\nu b}{}^a = 0, \quad R_{\nu b}{}^{ab} = e_{\nu}{}^c e_b{}^d C_{cd,}{}^{ab}, \quad C_{ab,}{}^b{}_c = 0$$

$C_{cd,}{}^{ab}$ coincides with Weyl tensor

$R_{\nu b} = R_{\nu c}{}^c{}_b = 0$: Einstein equations

HS counterparts impose Fronsdal equations $G_{\nu_1 \dots \nu_s} = 0$ and express generalized Weyl tensors in terms of Fronsdal fields

From COST to nonlinear theory

$$R(y, \bar{y} | X) = \bar{H}^{\dot{\alpha}\dot{\beta}} \frac{\partial^2}{\partial \bar{y}^{\dot{\alpha}} \partial \bar{y}^{\dot{\beta}}} C(0, \bar{y} | X) + H^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} C(y, 0 | X) + \dots$$

$$\tilde{D}C(y, \bar{y} | X) + \dots, \quad \dots = O(C, \omega_1)$$

$$R(y, \bar{y} | X) = d\omega(y, \bar{y} | X) + \omega(y, \bar{y} | X) * \omega(y, \bar{y} | X)$$

$$\tilde{D}C(y, \bar{y} | X) = dC(y, \bar{y} | X) + \omega(y, \bar{y} | X) * C(y, \bar{y} | X) - C(y, \bar{y} | X) * \omega(y, -\bar{y} | X)$$

Such field equations are **unfolded**: exterior differential of any field is expressed via the fields themselves

Problem: find gauge invariant nonlinear corrections

Idea of Nonlinear Construction

Being possible in a few first orders, straightforward construction of nonlinear deformation quickly gets complicated

- Trick: to find a larger algebra g' such that a substitution

$$\star \quad \omega \rightarrow W = \omega + \omega C + \omega C^2 + \dots$$

into g' reconstructs nonlinear equations via a zero-curvature condition

$$dW + W \wedge W = 0$$

To find restrictions on W that reconstruct \star in all orders

Doubling of spinors

$$\omega(Y|X) \longrightarrow W(Z; Y|X), \quad C(Y|X) \longrightarrow B(Z; Y|X)$$

to be accompanied by equations that determine dependence on the additional variables Z^A in terms of “initial data”

$$\omega(Y|X) = W(0; Y|X), \quad C(Y|X) = B(0; Y|X)$$

$S(Z, Y|X) = dZ^A S_A$ is connection along Z^A

HS star product

$$(f \star g)(Z, Y) = \int dS dT f(Z + S, Y + S) g(Z - T, Y + T) \exp -i S_A T^A$$

$$[Y_A, Y_B]_\star = -[Z_A, Z_B]_\star = 2i C_{AB}, \quad Z - Y : Z + Y \text{ normal ordering}$$

Inner Klein operators:

$$\kappa = \exp iz_\alpha y^\alpha, \quad \bar{\kappa} = \exp iz_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}, \quad \kappa \star f(y, \bar{y}) = f(-y, \bar{y}) \star \kappa, \quad \kappa \star \kappa = 1$$

Nonlinear HS Equations

$$\left\{ \begin{array}{l} dW + W \star W = 0 \\ dB + W \star B - B \star \tilde{W} = 0 \\ dS + W \star S + S \star W = 0 \\ S \star B - B \star S = 0 \\ S \star S = i(dZ^A dZ_A + dz^\alpha dz_\alpha F(B) \star \kappa + d\bar{z}^{\dot{\alpha}} d\bar{z}_{\dot{\alpha}} \bar{F}(B) \star \bar{\kappa}) \end{array} \right.$$

Manifest gauge invariance

$$\delta\mathcal{W} = [\varepsilon, \mathcal{W}]_\star, \quad \delta B = \varepsilon \star B - B \star \tilde{\varepsilon}, \quad \varepsilon = \varepsilon(Z; Y; K|x)$$

Nontrivial equations are free of space-time differential d

The equations are formally consistent and regular: no divergences despite non-polynomial Klein operators: $\kappa = \exp iz_\alpha y^\alpha$ and $\bar{\kappa} = \exp i\bar{z}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}}$

Perturbative analysis

Vacuum solution $B_0 = 0$, $S_0 = dZ^A Z_A$, $W_0 = \frac{1}{2}\omega_0^{b\nu}(x)Y_b Y_\nu$

$dW_0 + W_0 \star W_0 = 0 \longrightarrow \omega_0^{b\nu}(x)$ describes AdS_d

First-order analysis reproduces Central On-Shell Theorem

A particular form of star product plays crucial role

Multiparticle algebra as a symmetry of a multiparticle theory

$l(U(\mathfrak{h}))$ (2012)

- contains \mathfrak{h} as a subalgebra
- admits quotients containing up to k^{th} tensor products of \mathfrak{h} :
 k Regge trajectories?!
- Acts on all multiparticle states of HS theory

Promising candidate for a HS symmetry algebra of HS theory with mixed symmetry fields like String Theory

String Theory as a theory of bound states of HS theory

Chang, Minwalla, Sharma and Yin (2012)

4d HS systems

Three series of 4d HS algebras: $hu(n, m|4)$, $ho(n, m|4)$, $husp(2n, 2m|4)$

Spin-one YM sector:

$$g = u(n) \oplus u(m), o(n) \oplus o(m) \text{ or } usp(2n) \oplus usp(2m)$$

fermions: bifundamental.

Odd spins: adjoint representation of g .

Even spins: the opposite symmetry second rank representation of g ,

Particle spectrum always contains a singlet for

colorless graviton and colorless scalar

$ho(1, 0|4)$ is minimal HS algebra: even spins $s = 0, 2, 4, 6, \dots$

Colorless scalar is the prediction of HS symmetry!

Some Reviews

MV, hep-th/0401177; 9910096; 9611024

X. Bekaert, S. Cnockaert, C. Iazeolla and MV, hep-th/0503128

D. Sorokin, arXiv:hep-th/0405069

A. Fotopoulos and M. Tsulaia, 0805.1346

X. Bekaert, N. Boulanger and P. Sundell, 1007.0435

M. R. Gaberdiel and R. Gopakumar, 1207.6697

S. Giombi and X. Yin, 1208.4036

HS Holography

AdS_4 HS theory is dual to $3d$ vectorial conformal models

Klebanov-Polyakov (2002), Sezgin-Sundell (2002); Giombi and Yin (2009)

HS symmetry in AdS_{d+1} : maximal symmetry of a d -dimensional free conformal field(s)=singletons, usually, scalar and/or spinor.

Symmetries of KG equation in Minkowski space

Shaynkman, MV 2001 $3d$; Shapovalov, Shirokov 1992, Eastwood 2002 $\forall d$

Construction simplifies at $d = 3$ within spinor formalism

$3d$ Lorentz algebra: $o(2, 1) \sim sp(2, R) \sim sl_2(R)$.

Unfolded massless equations of the form

$$\left(\frac{\partial}{\partial x^{\alpha\beta}} + \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \right) C(y|x) = 0, \quad C(y|x) = \sum_{n=0}^{\infty} C^{\alpha_1 \dots \alpha_{2n}}(x) y_{\alpha_1} \dots y_{\alpha_{2n}}$$

are invariant under $\delta C(y|x) = \epsilon(y, \frac{\partial}{\partial y}|x) C(y|x)$

$$\epsilon(y, \frac{\partial}{\partial y}|x) = \exp \left[-x^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \right] \epsilon_{gl}(y, \frac{\partial}{\partial y}) \exp \left[x^{\alpha\beta} \frac{\partial^2}{\partial y^\alpha \partial y^\beta} \right]$$

$\epsilon_{gl}(y, \frac{\partial}{\partial y})$ describes global HS transformations

Space-Time and Spin

Space-time M is where symmetry $G = O(d - 1, 2)$ acts

Spin s : different G -modules V_s where fields $\phi^A(x)$ are valued.

V_s contain ground (primary) fields $\phi^A(x)$ along with their derivatives

$\partial_{n_1} \dots \partial_{n_k} \phi^A(x)$ (descendants)

HS vertices contain higher derivatives Bengtsson, Bengtsson, Brink (1983),

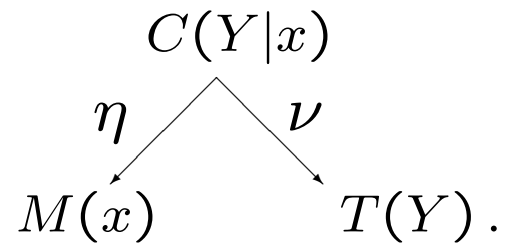
Berends, Burgers and H. Van Dam (1984), (1985), Fradkin, MV; Metsaev,...

HS symmetries Fradkin, MV 1986 are infinite dimensional extensions of G

Infinite towers of spins \Rightarrow infinite towers of derivatives.

How (non)local is HS gauge theory?

Unfolding as a Covariant Twistor Transform



MV 1404.1948
N.Misuna 2404.13212

$W^\Omega(Y|x)$ are functions on the “correspondence space” C .

Space-time M : coordinates x . Twistor space T : coordinates Y .

Unfolded equations: Penrose transform mapping functions on T to solutions of field equations in M .

Holographic duality: different space-times M for the same twistor space

Homology Map

Fundamental Ansatz has the remarkable property that

$$d\left(d\Omega^2 \exp i\left(\Omega_\beta(y^\beta + p_+^\beta + u^\beta) + u_\alpha v^\alpha - \sum_{i=1}^l p_{i\alpha} r_i^\alpha - \sum_{k \geq j > i \geq 1} p_{i\beta} p_j^\beta\right)\right) = 0$$

$(d\Omega)^3 = 0$ **since the one-forms** $d\Omega^\alpha$ **are anticommuting and** $\alpha = 1, 2$.

As a result d **effectively acts only on** μ

$$df_\mu = f_{d\mu},$$

mapping homological problem in terms of spinors Z^A **to that on** $\mu(t_i)$.

Great advantage of this formalism is that there is no more need to use the Schouten identity: the only formula where it manifests itself is the homology map.

The problem takes purely geometric form on \mathcal{M}

$$d\mu_f \cong \mu_g$$

That $d\mu_g \cong 0$ **implies** $\mu_g \cong dh_g$ **allowing to set**

$$\mu_f = h_g$$

HS Vertices

Diffeomorphisms without Riemannian geometry: Cartan formalism of differential forms with field equations in the **unfolded** form

$$d_x \omega = -\omega * \omega + \Upsilon(\omega, \omega, C) + \Upsilon(\omega, \omega, C, C) + \dots,$$

$$d_x C = -[\omega, C]_* + \Upsilon(\omega, C, C) + \dots$$

The problem: consistent non-linear corrections **1988** in the local frame

The vertices can be put into the form

$$\Upsilon(\Phi, \Phi, \dots) = F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl}) \Phi(Y_1) \dots \Phi(Y_n)|_{Y_i=0}$$

with $\Phi = \omega, C$ and some non-polynomial functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ of the Lorentz-covariant combinations

$$Q^i := y^\alpha \frac{\partial}{\partial y_i^\alpha}, \quad P^{ij} := \frac{\partial}{\partial y_i^\alpha} \frac{\partial}{\partial y_j^\alpha}, \quad \bar{Q}^i := \bar{y}^{\dot{\alpha}} \frac{\partial}{\partial \bar{y}_i^{\dot{\alpha}}}, \quad \bar{P}^{ij} := \frac{\partial}{\partial \bar{y}_i^{\dot{\alpha}}} \frac{\partial}{\partial \bar{y}_j^{\dot{\alpha}}}$$

The fundamental problem: find a proper class of functions $F(Q^i, P^{nm}; \bar{Q}^j, \bar{P}^{kl})$ guaranteeing spin-locality (minimal non-locality) of the HS theory

Compact Spin-Locality

The simplest option: replacement of the class of local field theories with the finite # of fields by spin-local models with infinite # of fields.

Spin-local-compact vertices in addition obey

$$a_{A_0 A_1 \dots A_l}^{n_1 \dots n_k}(s_0, s_1, \dots, s_k + t_k, \dots, s_l) = 0 \quad t_k > t_k^0 \quad \forall k$$

non-compact otherwise.

Compactness is in the space of spins, not in space-time

Both types of vertices in HS theory:

Cubic HS vertices $\omega * \omega$ built from HS gauge potentials are spin-local-compact: spins s_0, s_1, s_2 obey the triangle inequalities $s_0 \leq s_1 + s_2$ etc.

Vertices associated with the conserved currents built from gauge invariant field strength are spin-local non-compact. These include conserved currents of any integer s_0 built from two spin-zero fields ($s_1 = s_2 = 0$).