Scalar–Vector–Tensor Theories and Disformal Transformations

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Outline

1. Why Scalar–Tensor modifications of gravity?
   Why Scalar–Vector–Tensor theories?

2. Basis of higher derivative SVT theories

3. Conformal/ Disformal transformations

   Horndeski $\leftrightarrow$ Beyond Horndeski theories

   SVT $\leftrightarrow$ Beyond SVT theories

1. Introduction:

Why Scalar–Tensor modifications of GR?

Our reference theory, Einstein's gravity (GR)

- generally suffers singularities, if the Null Energy Condition (NEC) holds (Penrose, Hawking theorems),
- ΛCDM (Cosmological Constant Problem, Weinberg (1989)),
- challenges ahead?
  (DESI BAO data, *e.g.* A.G. Adame *et al.*, DESI 2024 VI. JCAP 02 (2025))

⋮

1. A paradigm: Horndeski theory

Scalar modification of GR that

- keeps second order equations (No Ghosts),
- does not satisfy the NEC in general,

Unique answer: **Horndeski theory** (1974)

Extensively used for

- early and late time cosmology
- compact objects and other modified gravity solutions

1. Horndeski theory

- Take a real scalar field ϕ besides the metric field,
- On top of GR take 4 general Scalar Potentials $G_i(\phi, X)$, $i = 2, 3, 4, 5$, where $X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$,
- Most general combinations of R and $(\nabla^2\phi)^p$ with $p \leq 3$ and G_i coefficients, with **second order equations of motion**.

$$\begin{aligned}
\mathcal{L}_H &= \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5, \qquad (1)\\
\mathcal{L}_2 &= G_2(\phi, X),\\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi,\\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right],\\
\mathcal{L}_5 &= G_5(\phi, X)G^{\mu\nu}\phi_{;\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\rho}\phi_{;\rho}^{\ \ \nu}\right],
\end{aligned}$$

Horndeski, 1974

1. Beyond Horndeski theory

Can Horndeski theory be generalized while avoiding Ostrogradsky's ghost? Yes! in a potentially key way (Gleyzes-Langlois-Piazza-Vernizzi, 2014)

$$\mathcal{L}_{BH} = \mathcal{L}_H + \mathcal{L}_{F4} + \mathcal{L}_{F5} \,, \tag{2}$$

$$\mathcal{L}_{F4} = F_4(\phi, X)\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon^{\mu'\nu'\rho'}{}_{\sigma}\, \phi_{;\nu\nu'}\, \phi_{;\rho\rho'}\, \phi_{;\mu}\, \phi_{;\mu'} \tag{3}$$

$$\mathcal{L}_{F5} = F_5(\phi, X)\, \epsilon^{\mu\nu\rho\sigma}\, \epsilon^{\mu'\nu'\rho'\sigma'}\, \phi_{;\nu\nu'}\, \phi_{;\rho\rho'}\, \phi_{;\sigma\sigma'}\, \phi_{;\mu}\, \phi_{;\mu'} \tag{4}$$

F_4 and F_5 are related as:

$$F_4\, G_{5,X}\, X = -3\, F_5\, (G_4 - 2\, X\, G_{4,X} + G_{5,\phi}\, X) \,, \tag{5}$$

What do we gain? with the extra function F_4

- Can avoid the No–Go theorems for stable, nonsingular cosmological solutions in Horndeski theory
- Can satisfy the constraints on the speed of GW with nonminimal couplings
- Relation Horndeski $\leftrightarrow$ Beyond Horndeski theory? Yes...

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1. Why Scalar–Vector–Tensor theories?

- Early universe: Primordial magnetic fields

  ⋮

- Late universe: to satisfy the constraint on the speeds

$$\left|\frac{c_g}{c} - 1\right| \leq 10^{-16}\,, \tag{6}$$

while preserving extra free functions.

For instance, adding an SVT to Horndeski theory

$$\mathcal{L} = \text{Horndeski Theory} + \mathcal{L}_{4A} + \mathcal{L}_{5A} \tag{7}$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,

$$\mathcal{L}_{4A} = -\frac{1}{4} G_4(\phi, X)\, F^2 - \frac{1}{2} G_{4,X}\, F_\mu{}^\sigma F_{\nu\sigma}\, \phi^{;\mu}\phi^{;\nu} \tag{8}$$

$$\mathcal{L}_{5A} \;=\; G_5(\phi)\left(\frac{1}{8} F^{\mu\nu} F_\mu{}^\rho(-4\,\phi_{;\nu\rho} + g_{\nu\rho}\Box\phi) + \frac{1}{2} F_{\mu\nu}\nabla_\sigma F^{\nu\sigma}\,\phi^{;\mu}\right)$$

S. Mironov, A. Shtennikova and **M. V-V** (2024), Phys.Lett.B 858, 139058 [arXiv:2405.02281]

1. The Speed of Perturbations in SVT theories

- Take the Perturbed metric

$$\mathrm{d}s^2 = (\eta_{\mu\nu} + \delta g_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu$$

with FLRW background

$$\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2\,\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j$$

and metric perturbation,

$$\begin{aligned} \delta g &= a^2(\eta)\left(-2\,\alpha\,\mathrm{d}\eta^2 + 2\,(\partial_i B + S_i)\,\mathrm{d}\eta\,\mathrm{d}x^i\right. \\ &+ \left.(-2\,\psi\,\delta_{ij} + 2\,\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + 2\,h_{ij})\,\mathrm{d}x^i\,\mathrm{d}x^j\right) \quad (9) \end{aligned}$$

- $A_i(x)$ is the perturbation for the vector
- $\phi(x) = \pi(t) + \delta\phi(x)$

1. The Speed of Perturbations in SVT theories

- Quadratic action for tensor modes,

$$\mathcal{S}_{Tensor} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\,\left(\mathcal{G}_\tau\,\dot{h}_{ij}^2 - \frac{\mathcal{F}_\tau}{a^2}\,(\partial_k h_{ij})^2\right)\,, \tag{10}$$

- Quadratic action for the vector modes,

$$\mathcal{S}_{Vector} = \frac{1}{4}\int \mathrm{d}t\,\mathrm{d}^3x\,a\,\left(\mathcal{G}_A\,\dot{A}_i^2 - \frac{\mathcal{F}_A}{a^2}\,(\partial_k A_i)^2\right)\,, \tag{11}$$

The speed of the tensor and vector modes through the cosmological background is,

$$c_g^2 = \frac{\mathcal{F}_\tau}{\mathcal{G}_\tau} = \frac{2G_4 + XG_{5,\pi}}{2G_4 - 4XG_{4,X} - XG_{5,\pi}} = \frac{\mathcal{F}_A}{\mathcal{G}_A} = c^2 \neq 1\,. \tag{12}$$

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2. Higher derivative SVT theories

- To date, the largest SVT theory (U(1) gauge invariant vector), of order $\mathcal{O}\left((\nabla\nabla\phi)^2\right)$, with second order equations of motion can be written as

$$L_{\rm SVT} = L_0 + L_1 + L_2 \; , \tag{13}$$

with $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, g_i and w_i functions of ϕ and X

$$L_0 = -\frac{1}{4}g_0 \; F_{\mu\nu}F^{\mu\nu} + g_2 \; F^{\mu}{}_{\rho}F^{\nu\rho}\phi_{;\mu}\phi_{;\nu} \tag{14}$$

$$L_1 = w_1 \; \tilde{F}^{\mu}{}_{\rho}\tilde{F}^{\nu\rho} \; \phi_{;\mu\nu} + w_2 \; \tilde{F}^{\mu\rho}\tilde{F}^{\nu\sigma} \; \phi_{;\rho} \; \phi_{;\sigma} \; \phi_{;\mu\nu} \tag{15}$$

$$L_2 = \left(w_3 \; R_{\mu\rho\nu\sigma} + w_{3,X} \; \phi_{;\mu\rho}\phi_{;\nu\sigma}\right)\tilde{F}^{\mu\nu}\tilde{F}^{\rho\sigma} \; . \tag{16}$$

L. Heisenberg (2018)

Ali Gorji, Mukohyama, Petrov, Yamaguchi (2025)

S. Mironov, A. Shtennikova and M. V-V (2025)

2. Higher derivative SVT theories

- The pure SVT part of Kaluza-Klein reductions of 5D Horndeski theory can be written in the latter basis.
- Larger sets of SVT theories also fit in the latter basis of SVT's (S. Mironov, A. Shtennikova and M. V-V (2025)), at least in 4D (Using Dimensional dependent identities).
- However, it is unknown if L_2 is the most general $\mathcal{O}\left((\nabla\nabla\phi)^2\right)$ with low order EOMs in 4D.
- Is L_{SVT} everything that there can be in U(1) gauge invariant SVT's? No!

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3. Conformal/ Disformal transformations of the metric

Horndeski (H) and Beyond Horndeski (BH) theory without extra matter can be seen in many cases (not all) as the same gravity theory, just two different geometries:

Conformal/ Disformal relation between two metrics, Jacob Bekenstein, 1993

$$\bar{g}_{\mu\nu} = C(\phi)\ g_{\mu\nu} + D(\phi,\ X)\ \phi_{;\mu}\phi_{;\nu}\ . \tag{17}$$

- $C(\phi)$ alters the measurement of lengths between the two geometries, but preserves angles and light cones.
- It is standard to have two conformally related metrics in a single gravity theory. Is this the most general relation? No:
- $D(\phi, X)\ \phi_{;\mu}\phi_{;\nu}$ alters angles and light cones Jacob Bekenstein, 1993

3. Horndeski ↔ Beyond Horndeski theory? One gravity theory?

- H and BH related by a transformation of the metric Eqn. (17) (e.g. with $C = 1$),

$$F_4 = \frac{1}{2}\frac{-D_{,X}}{1+2\ D_{,X}\ X^2}\left(G_4 - 2\ X\ G_{4,X} + G_{5,\phi}\ X\right) \tag{18}$$

$$F_5 = \frac{D_{,X}\ G_{5,X}\ X}{3\ (1+D_{,X}\ X^2)} \tag{19}$$

where the degeneracy condition is automatically satisfied

$$F_4\ G_{5,X}\ X = -3\ F_5\ \left(G_4 - 2\ X\ G_{4,X} + G_{5,\phi}\ X\right). \tag{20}$$

3. Conformal/ Disformal transformations in ST theories

- Need some assumptions: $\bar{g}$ non-degenerate $(C - 2X\ D) \neq 0$ ...

$$\det(\bar{g}) = C^3\ (C - 2X\ D)\ \det(g)\ , \tag{21}$$

Equivalence H↔BH breaks if:

- The relation between $\bar{g}$ and g is not invertible (No-Go theorem)
  Mironov, Rubakov, Volkova (2018)

- If the same matter Lagrangian is coupled to $\bar{g}$ and g.

All in all, ST theories behave as follows under these transformations:

- BH is closed under $C(\phi)/\ D(\phi, X)$ transformations

- DHOST is closed under $C(\phi, X)/\ D(\phi, X)$ transformations

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3. Conformal/ Disformal transformations in SVT theories

As a tool, C/D transformations are useful to uncover new theories.
For instance, H $\rightarrow$ BH.

What happens in the L_{SVT} theory? the theory is not closed under C/D transformations.

$$L_{\rm SVT} \xrightarrow{C/D} L_{\rm bSVT} = L_{\rm SVT} + L_{d\ 1} + L_{d\ 2} \tag{22}$$

where

$$L_{d\ 1} = w_0(\phi, X)\ \tilde{F}^{\alpha\zeta}\tilde{F}_{\zeta\beta}\phi_{;\alpha}\phi^{;\gamma}(2\ X\phi^{;\beta}_{;\gamma} + \frac{1}{2}\phi^{;\beta}\phi_{;\delta\gamma}\phi^{;\delta}) \tag{23}$$

$$\begin{aligned} L_{d\ 2} = w4(\phi, X)(&\tilde{F}^{\alpha\delta}\tilde{F}^{\gamma\zeta}{}_{;\delta}\ \phi_{;\alpha}\phi^{;\beta}\phi_{;\gamma}\phi_{;\zeta\beta} \\ &+ \tilde{F}^{\alpha\beta}\tilde{F}^{\gamma\delta}\phi_{;\delta\beta}(X\phi_{;\gamma\alpha} + \phi_{;\alpha}\phi_{;\zeta\gamma}\phi^{;\zeta})) \\ + w5(\phi, X)\ &\tilde{F}^{\alpha\beta}\tilde{F}^{\gamma\delta}\ \Big(2X^2\phi_{;\gamma\alpha}\phi_{;\delta\beta} \\ &+ \phi_{;\alpha}\phi_{;\zeta}(4X\phi^{;\zeta}_{;\gamma}\phi_{;\delta\beta} + \phi_{;\gamma}(-\phi^{;\zeta}_{;\beta}\phi_{;\nu\delta} + \phi_{;\delta\beta}\phi_{;\nu\zeta})\phi^{;\nu})\Big)\ . \end{aligned} \tag{24}$$

Where w_0, w_4 and w_5 are related as follows

$$w_0 = -\frac{w_1\ w_4}{w_3}, \qquad w_5 = \frac{w_4\ (X\ w_4 + w_{3,X})}{w_3 - 2X^2\ w_4} \tag{25}$$

Alternatively, in terms of the C/D factors,

$$w_4 = -\frac{w_3\ D_{,X}}{C}\ , \qquad w_5 = \frac{D_{,X}\ \left(\frac{X\ w_3\ D_{,X}}{C} - w_{3,X}\right)}{C + 2\ X^2\ D_{,X}}\ . \tag{26}$$

- $L_{d\,1} + L_{d\,2}$ lead to higher order EOMs, but they are free of ghosts.
- The equivalence $L_{SVT} \leftrightarrow L_{bSVT}$ is broken if the same matter Lagrangian for χ is coupled to $\bar{g}$ and g. The degeneracy is not spoiled, as long as they do not alter the off–diagonal terms in the kinetic matrix,

$$\frac{\partial^2 L}{\partial \dot{g}\ \partial \dot{\chi}}\ , \qquad \frac{\partial^2 L}{\partial \dot{\phi}\ \partial \dot{\chi}}\ , \qquad \frac{\partial^2 L}{\partial \dot{A}\ \partial \dot{\chi}}\ , \tag{27}$$

3. Conformal/ Disformal transformations in SVT theories

- The speed of the vector modes about the cosmological background acquires new terms after the C/D transformation

$$
\begin{aligned}
c_A^2 &= \frac{F_A}{G_A} && (28) \\
F_A &= g_0 + 4\,H^2\,w_3 + 4\,\dot{H}\,w_3 - 2\,\ddot{\pi}\,(w_1 + 2\,X^2\,w_0) \\
&\quad + H\,(4\,\dot{\pi}\,\ddot{\pi}\,w_{3,X} - 2\,\dot{\pi}\,(w_1 - 2\,w_2\,X)) && (29) \\
G_A &= g_0 + 4\,g_2\,X - 4\,H\,\dot{\pi}\,w_1 \\
&\quad + 4\,H^2\,(w_3 + 2\,X\,(w_{3,X} + X\,(w_4 + 2\,w_5\,X))) && (30)
\end{aligned}
$$

- Recall that the light cones are modified by the Disformal transformation.

- Nevertheless $c_g^2 = c_A^2$ remains impossible if $G_{5,X} \neq 0$.

Conclusions

- We showed that the most general higher derivative SVT theory is not closed under conformal/ disformal transformations of the metric
- We obtained a new, degenerate (Beyond) SVT theory, that is related to the initial SVT in a similar way as Beyond Horndeski relates to Horndeski theory.
- We showed that $c_g^2 = c_A^2$ remains impossible if $G_{5,X} \neq 0$.

Thanks for your attention!

Support slides:

Basic Blocks for C/D transformations:

$$\det(\bar{g}) = C^3\ (C - 2X\ D)\ \det(g)\ , \tag{31}$$

$$\bar{g}^{\mu\nu} = \frac{g^{\mu\nu}}{C} - \frac{D}{C\ (C - 2\ D\ X)} \phi^{;\mu}\phi^{;\nu}\ . \tag{32}$$

$$\bar{\phi}_{;\nu\mu} = \phi_{;\nu\mu} - \Delta^{\rho}{}_{\mu\nu}\ \phi_{;\rho} \tag{33}$$

$$\bar{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \Delta^{\rho}{}_{\mu\nu}\ , \tag{34}$$

$$\bar{R}^{\rho}{}_{\sigma\mu\nu} = R^{\rho}{}_{\sigma\mu\nu} + \nabla_{\mu}\Delta^{\rho}{}_{\nu\sigma} - \nabla_{\nu}\Delta^{\rho}{}_{\mu\sigma} + \Delta^{\rho}{}_{\mu\lambda}\Delta^{\lambda}{}_{\nu\sigma} - \Delta^{\rho}{}_{\nu\lambda}\Delta^{\lambda}{}_{\mu\sigma}\ , \tag{35}$$

Basic Blocks for C/D transformations:

$$\Delta^{\alpha}{}_{\beta\gamma} = \frac{1}{2\,C\,(C-2\,D\,X)}\Bigg(D\,\Big(-\phi_{;\beta}\phi_{;\gamma}(2X\phi^{;\alpha}{}_{;\delta}\phi^{;\delta} + \phi^{;\alpha}\phi^{;\zeta}\phi_{;\zeta;\eta}\phi^{;\eta})D_{,X} - 2(\delta^{\alpha}{}_{\gamma}X\phi_{;\beta} + (\delta^{\alpha}{}_{\beta}X + \phi^{;\alpha}\phi_{;\beta})\phi_{;\gamma})C_{,\phi}\Big) + C\,\Big(2D\,\phi^{;\alpha}\phi_{;\beta;\gamma} + (-\phi^{;\alpha}\phi_{;\gamma}\phi_{;\beta;\delta} + \phi_{;\beta}(\phi_{;\gamma}\phi^{;\alpha}{}_{;\delta} - \phi^{;\alpha}\phi_{;\gamma;\delta}))\phi^{;\delta}D_{,X} + (-g_{\beta\gamma}\phi^{;\alpha} + \delta^{\alpha}{}_{\gamma}\phi_{;\beta} + \delta^{\alpha}{}_{\beta}\phi_{;\gamma})C_{,\phi} + \phi^{;\alpha}\phi_{;\beta}\phi_{;\gamma}D_{,\phi}\Big)\Bigg)\,. \tag{36}$$

$$\bar{F}_{\mu\nu} = F_{\mu\nu}\,, \qquad \bar{F}^{\mu\nu} = \bar{g}^{\mu\alpha}\bar{g}^{\nu\beta}\,F_{\mu\nu}F_{\alpha\beta}\,, \qquad \bar{\tilde{F}}^{\mu\nu} = \frac{\tilde{F}^{\mu\nu}}{C^{\frac{3}{2}}\sqrt{C-2\,D\,X}}\,. \tag{37}$$

Basic Blocks for C/D transformations:

With $\bar{X} = \frac{X}{j}$ and $j = C - 2\,D\,X$,

$$g_0(\phi, X) = \frac{1}{C^{5/2}\ \ j^{\ 3/2}}\left(C^2\,\bar{g}_0\ \ j^{\ 2} + 4C\,\bar{w}_2\,X^2 C_{,\phi} + 4C\,\bar{w}_1\,X^2(D\,C_{,\phi} - C\,D_{,\phi}) + 4\bar{w}_3\,X\left(C_{,\phi}\Big((-C + D\,X)C_{,\phi} + C\,X D_{,\phi}\Big) + C\ \ j\ C_{,\phi\phi}\right)\right), \tag{38}$$

$$g_2(\phi, X) = \frac{1}{2\ C^{5/2}\ \ j^{\ 5/2}}\left(2C^3\,\bar{g}_2\ \ j\ + C^2\,D\,\bar{g}_0\ \ j^{\ 2} - 2C\,\bar{w}_2\,X\ j\ C_{,\phi} + 2C\,X\bar{w}_{3,\bar{X}}\,(C_{,\phi})^2 + 2C\,\bar{w}_1\ \ j\ \Big(-(C + D\,X)C_{,\phi} + C\,X D_{,\phi}\Big) + \bar{w}_3\ \ j\ \left(C_{,\phi}\Big((3C - 2D\,X)C_{,\phi} - 2C\,X D_{,\phi}\Big) - 2C\ \ j\ C_{,\phi\phi}\right)\right), \tag{39}$$

Basic Blocks for C/D transformations:

$$w_1(\phi, X) = \frac{1}{C^{1/2} \;\; j^{\,5/2}} \left(C\,\bar{w}_1 \;\, j \; - \bar{w}_3 \;\, j\; C_{,\phi} - 2X\bar{w}_{3,\bar{X}} C_{,\phi} \right), \quad (40)$$

$$w_2(\phi, X) = \frac{1}{C^{3/2} \;\; j^{\,5/2}} \Bigg(C\,D\,\bar{w}_1 \;\, j \; + C\,\bar{w}_2 \;\, j \; + \bar{w}_3 \;\, j\; (-2D\,C_{,\phi} + C\,D_{,\phi}) + \bar{w}_{3,\bar{X}} \Big((-2C \; + D\,X) C_{,\phi} + 2X(C \; - D\,X) D_{,\phi} \Big) \Bigg), \quad (41)$$

$$w_3(\phi, X) = \frac{\bar{w}_3}{C^{1/2} \;\; j^{\,1/2}} \; . \quad (42)$$