Thermalization of Inelastic Dark Matter in the Sun

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Introduction

- Dark matter has many possible explanations. One of them is new particles beyond the Standard Model, including WIMP-type particles (Weakly Interacting Massive Particles).
- Such particles can be produced in thermodynamic equilibrium in the early Universe via the freeze-out mechanism, which requires a certain annihilation cross section:
$$\langle \sigma_{ann} v \rangle \sim 10^{-26} \mathrm{cm}^3 \mathrm{s}^{-1}$$
- Search methods: direct (nuclear recoil from WIMP collisions), indirect (measurement of annihilation products and effects), collider searches. The absence of signals constrains model parameters.

Dark matter interaction with matter

- DM particle velocities are small ($v \sim 10^{-3}c$), so the interaction with the nucleon reduces to a finite set of local non-relativistic operators [1]:

$$V(\vec{r}_\chi - \vec{r}_N) = \sum_i c_i \hat{O}_i(\vec{r}_\chi - \vec{r}_N)$$

- Here are several examples of operators in momentum space:

$$\hat{O}_1 = 1 \quad \hat{O}_4 = \vec{S}_\chi \cdot \vec{S}_n \quad \hat{O}_6 = \vec{S}_\chi \cdot \frac{\vec{q}}{m_N} \vec{S}_n \cdot \frac{\vec{q}}{m_N}$$

  (where $\vec{q}$ is the momentum transfer, $\vec{S}_\chi$ and $\vec{S}_n$ are the DM and nucleon spins, m_N is the nucleon mass)

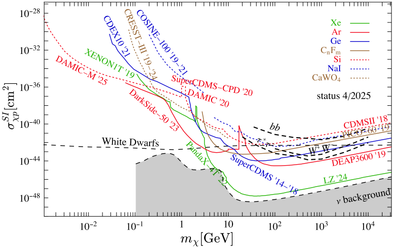
- We considered O_1, O_4 and magnetic dark matter (see 3).

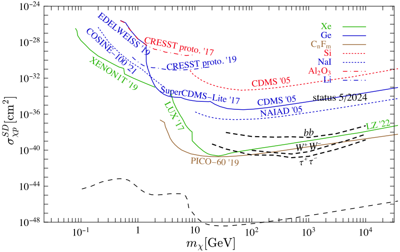
Dark matter interaction with matter

- In experiments, one needs to know the DM scattering cross section on the nucleus $\sigma_{\chi T}$. It is expressed through the nucleon scattering cross section $\sigma_{\chi p}$ and the nuclear form factor:
$$\frac{d\sigma_{\chi N}}{d\Omega} = \frac{\sigma_{\chi p}}{4\pi} \frac{(m_\chi + m_N)^2}{(m_\chi + m_T)^2} f(q^2)$$
- In the case of spin-independent scattering (operator $\hat{O}_1$), one can use a form factor of the form:
$$f(q^2) = A^4 \left(\frac{3J_0(qR)}{qR} e^{-s^2 q^2/2} \right)^2$$
The cross section grows as $\sigma_{\chi T} \propto A^4$.
- In the case of spin-dependent scattering (operator $\hat{O}_4$), the cross section grows as $\sigma_{\chi T} \propto A^2$. $f(q^2)$ is found in [2].

Constraints on WIMP

- The strongest constraints on the nucleon scattering cross section $\sigma_{\chi p}$ come from direct searches, ruling out the simplest models.





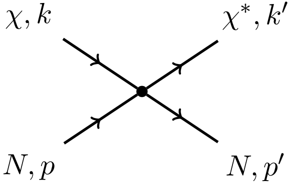
Constraints on spin-independent $\sigma_{\chi p}^{SI}$ and spin-dependent $\sigma_{\chi p}^{SD}$

[3]

Inelastic dark matter

- One way to weaken the constraints on $\sigma_{\chi p}$ is to make the interactions between DM and matter inelastic.
- For this purpose, we consider inelastic DM consisting of 2 components: χ with mass m_χ and χ^* with mass $m_\chi + \delta$
- Scattering $\chi N \to \chi^* N$ is a threshold reaction.

$N + \chi(m)$



$N + \chi^*(m + \delta)$

Inelastic dark matter

- Inelastic dark matter can naturally arise in various theories.
- The simplest example is a Dirac fermion with a small Majorana mass and a vector interaction

$$\mathcal{L}_{kin} \supset \overline{\chi}(i\gamma^\mu \partial_\mu - m)\chi + \frac{\delta}{4}\overline{\chi}\chi^C + \frac{\delta}{4}\overline{\chi^C}\chi$$

$$\mathcal{L}_{int} \supset g\bar{\chi}\gamma^\mu \chi \bar{q}\gamma_\mu q = i\frac{g}{2}\left[\bar{\chi_2}\gamma^\mu \chi_1 - \bar{\chi_1}\gamma^\mu \chi_2\right]\bar{q}\gamma^\mu q$$

(arises in the higgsino sector in SUSY extensions, in models with a dark photon [4, 5], and with complex scalar fields in the sneutrino sector [6])

- Magnetic Majorana dark matter has the following interaction [7]:

$$\mathcal{L}_{int} = \frac{\mu_{12}}{2}\bar{\Psi}_1 \Sigma_{\mu\nu}\Psi_2 F^{\mu\nu} + \frac{\mu_{21}}{2}\bar{\Psi}_2 \Sigma_{\mu\nu}\Psi_1 F^{\mu\nu}$$

Capture and annihilation of dark matter in the Sun

- Dark matter can be captured and annihilate in the Sun. These processes are described by the balance equation

$$\frac{dN(t)}{dt} = C - C_A N^2$$

(where $C(\sigma_{\chi p})$ is the capture rate, proportional to $\sigma_{\chi p}$, C_A is the annihilation coefficient)

- The solution to this equation has the form:

$$N = \sqrt{\frac{C}{C_A}}\,\mathrm{th}\,[\sqrt{C_A t^2 C}] \quad \Gamma(t) = \frac{1}{2} C\,\mathrm{th}^2\,[\sqrt{C_A t^2 C}] \qquad (1)$$

Capture and annihilation of dark matter in the Sun

- The annihilation rate at the final time $T_\odot$ (Sun's lifetime) has the form:

$$\Gamma = \frac{1}{2C_A T_\odot^2} F(C_A T_\odot^2 C) \quad F(x) = x\,\mathrm{th}^2\sqrt{x}$$

- From this, constraints on $\sigma_{\chi p}$ can be derived from constraints on Γ (which, in turn, can be obtained from neutrino telescope data):

$$\sigma_{\chi p} < \sigma_{\chi p,0} \frac{1}{C_A T_\odot^2 \cdot C(\sigma_{\chi p,0})} F^{-1}\left(2\Gamma_{max} \cdot C_A T_\odot^2\right) \qquad (2)$$

- For sufficiently large C_A ($C_A T_\odot^2 C >> 1$), equilibrium holds: $\Gamma = \frac{C}{2}$. In this case, the formula simplifies:

$$\sigma_{\chi p} < \sigma_{\chi p,0} \frac{2\Gamma_{max}}{C(\sigma_{\chi p,0})}$$

Capture and annihilation of dark matter in the Sun

- If dark matter is elastic, it is in thermodynamic equilibrium, and C_A has the form:

$$C_A T_\odot^2 \approx 9 \cdot 10^{-23}\mathrm{s} \left(\frac{\langle \sigma_a v \rangle}{3 \cdot 10^{-26}\mathrm{cm}^2\mathrm{s}^{-1}} \right) \left(\frac{m_\chi}{\mathrm{GeV}} \right)^{3/2}$$

- In the inelastic scenario, thermodynamic equilibrium is not reached, so C_A depends on:
  - DM mass m_χ and mass splitting δ
  - scattering cross section $\sigma_{\chi p}$
  - model of DM interaction with matter
- C_A is found using a numerical solution of the Boltzmann equation for DM density in the Sun.

Evolution of dark matter in the Sun

- The problem is isotropic $\Rightarrow$ everything depends on E and L.

$$E = \left(\frac{1}{2}w_\chi^2 + \phi(r)\right)\left(\frac{1}{2}v_{esc}^2\right)^{-1} \qquad L = \frac{|\vec{r}\times\vec{w}|}{R_\odot v_{esc}} = l \cdot L_{max}(E)$$

  where $v_{esc} = \sqrt{2\phi(r = R_\odot)}$, $R_\odot$ is the Sun's radius, $L_{max}(E)$ is the maximum angular momentum at energy E.
- The evolution equation for DM density $f(E,l)$ has the form:

$$\frac{\partial f(E,l)}{\partial t} = C(E,l) + \int dE'dl' S(E,l,E',l') f(E',l') - f(E,l) - \int A(E,l,E',l') f(E',l') dE'dl'$$

  where C is the capture rate, S is the collision matrix, A is the annihilation matrix.

Evolution of dark matter in the Sun

- The capture rate C and collision matrix S are defined as sums over all types of nuclei in the Sun

$$C(E,l) = \sum_{\alpha} C_{\alpha}(E,l) \quad S(E,l,E',l') = \sum_{\alpha} S_{\alpha}(E,l,E',l')$$

- We accounted for the most abundant nuclei in the Sun: H^1, He^3, He^4, C^{12}, O^{16}, Ne^{20}, Na^{23}, Mg^{24}, Al^{27}, Si^{28}, S^{32}, Ca^{40}, Fe^{56}, Ni^{58}.
- The concentration of nuclei in the Sun, temperature, density were determined from the AGS09met model [8].

Numerical solution methods

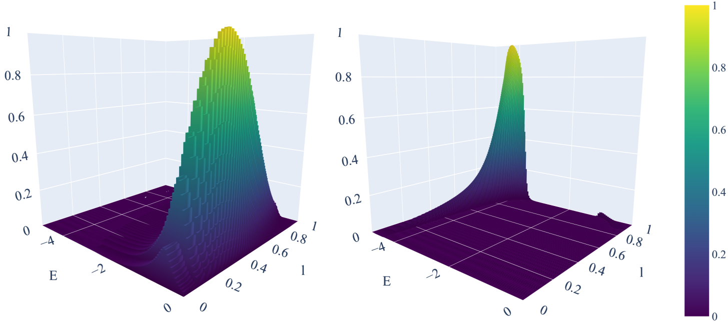
- To find C_A, we only need to solve the linear equation without accounting for the annihilation term.
- The first solution method is discretization in E and L (or l) and solving the linear equation:

$$\frac{\partial N_i}{\partial t} = \frac{1}{T_{\chi p}} \left(N_{\odot} c_i + \sum_j s_{ij} N_j \right)$$

(we factored out $T_{\chi p}^{-1} = \sigma_{\chi p} \langle n_{nuc} \rangle v_{esc}$ for dimensionlessness).

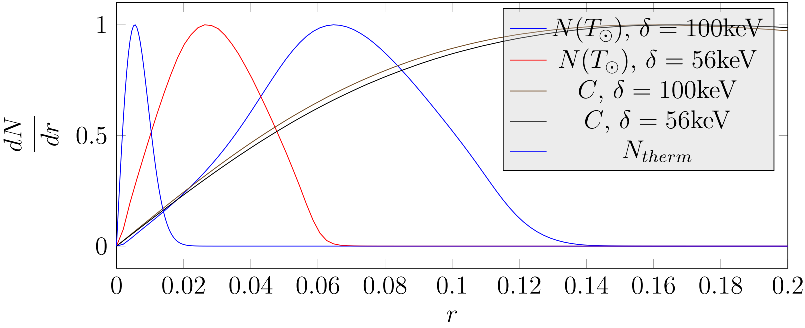
- Another method is to model the evolution of a large number of DM particles in the Sun using Monte Carlo, and reconstruct the DM density $f(E, l)$ from them.
- Both methods give the same result, which validates the correctness of the implemented algorithms.

Solution example



Distribution $f(E, l) = \frac{d^2 N}{dE\,dl}$ of dark matter in the Sun for $m_\chi = 100\,\mathrm{GeV}$, $\delta = 100\,\mathrm{keV}$. Left — initial distribution of captured particles, right — final distribution after evolution.

Solution example



Initial C and final $N(T_\odot)$ radial distributions of dark matter particles in the Sun for $m_\chi = 100\,\mathrm{GeV}$ and $\sigma_{\chi p} = 10^{-42}\mathrm{cm}^2$

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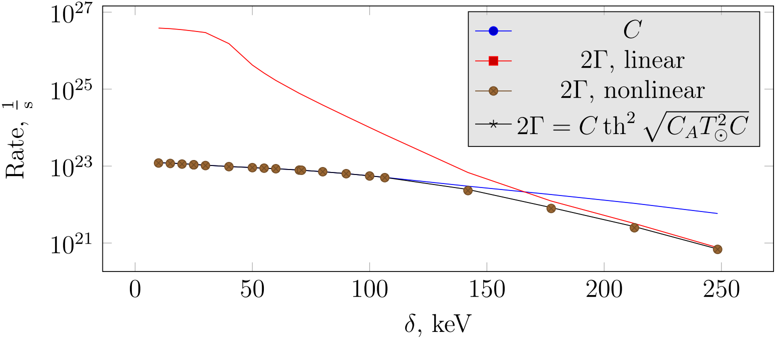
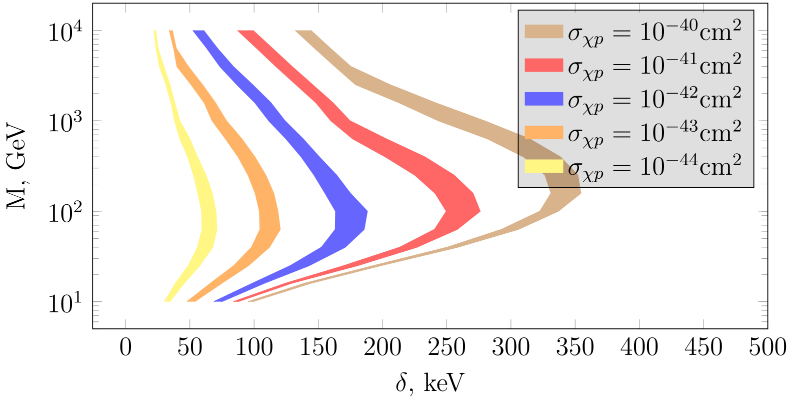
Capture and annihilation equilibrium



Dependence on δ of capture C and annihilation rate Γ for $m_\chi = 100\,\mathrm{GeV}$. Red — linear equation solution, large points — full equation solution, black — using Eq. (1)

Capture-annihilation equilibrium

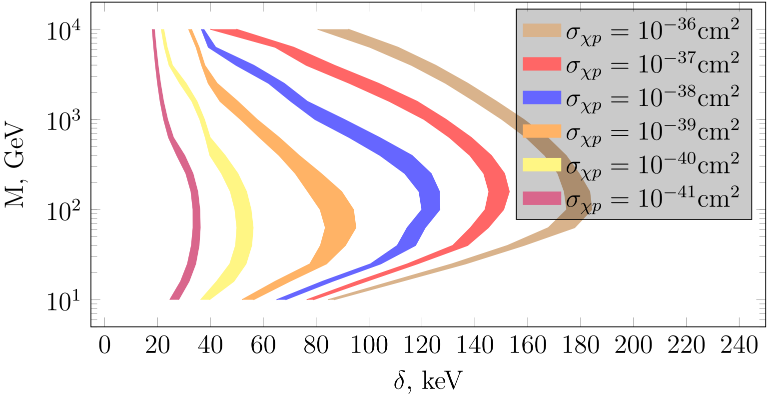
- Of interest are the values of m_χ and δ at which the assumption of equilibrium between annihilation and capture ($\Gamma = \frac{1}{2}C$) ceases to be valid.



Parameter region m, δ where equilibrium between Γ and C is reached:
$2\Gamma \in [C/\sqrt{2}, \sqrt{2}C]$

Capture-annihilation equilibrium

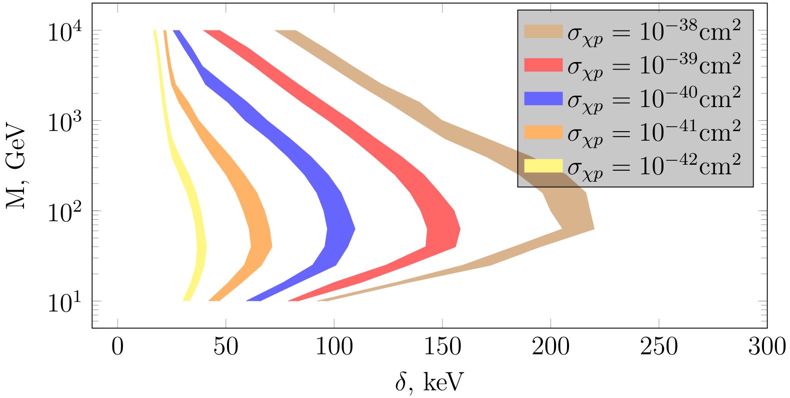
- Similar plots can be constructed for other types of interactions.



Parameter region m, δ where equilibrium between Γ and C is reached for spin-dependent dark matter (operator O_4)

Capture-annihilation equilibrium

- Similar plots can be constructed for other types of interactions.

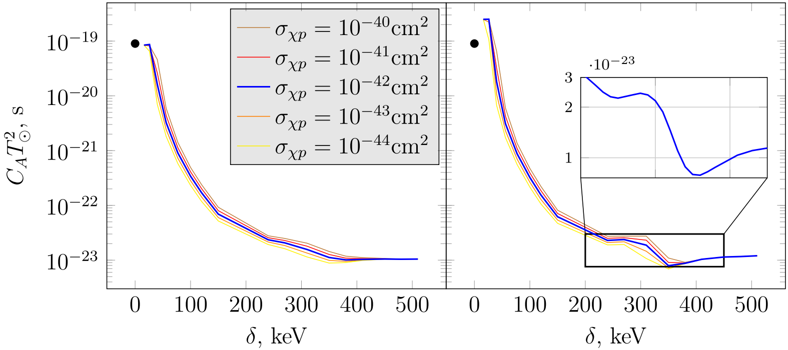


Parameter region m, δ where equilibrium between Γ and C is reached for magnetic dark matter

Annihilation coefficient

- The annihilation coefficient $C_A T_\odot^2 = 2\Gamma(t = T_\odot)/C^2$ is found from the solution of the linear equation.

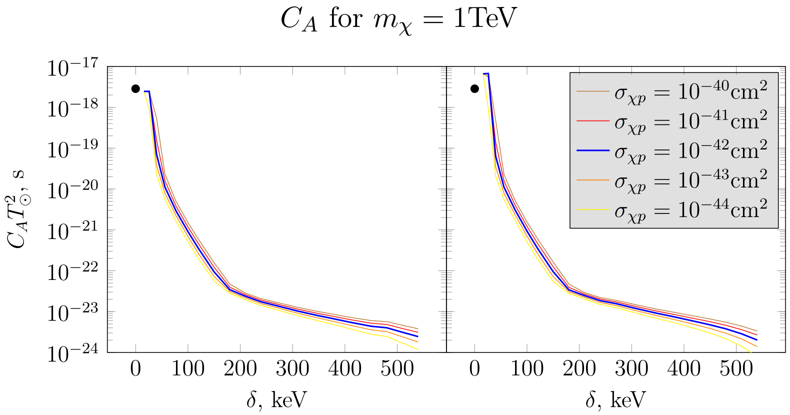
C_A for $m_\chi = 100$GeV



DM annihilation coefficient for $m_\chi = 100\,$GeV. Left assumes the excited state χ^* decays slowly, right — decays quickly.

Annihilation coefficient

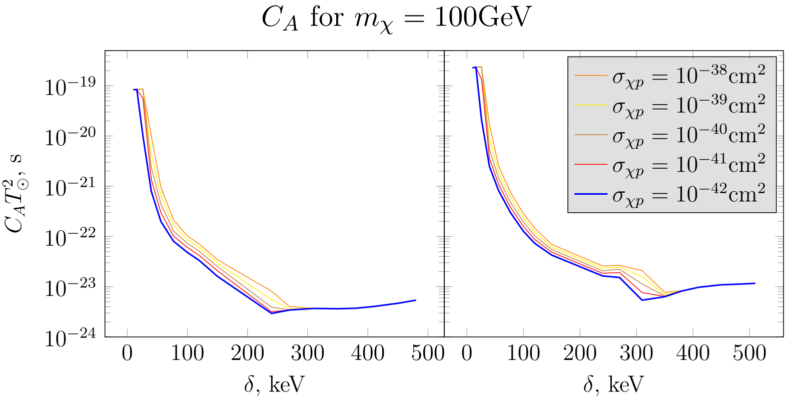
- The annihilation coefficient $C_A T_\odot^2 = 2\Gamma(t = T_\odot)/C^2$ is found from the solution of the linear equation.



DM annihilation coefficient for $m_\chi = 1$ TeV. Left assumes the excited state χ^* decays slowly, right — decays quickly.

Annihilation coefficient

- The annihilation coefficient C_A for other interaction types.

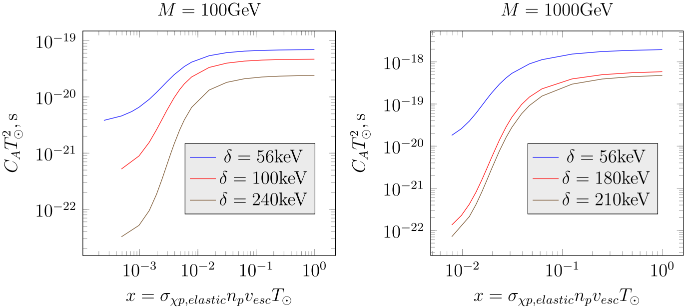


DM annihilation coefficient for $m_\chi = 100\,$GeV for spin-dependent (operator O_4) (left) and magnetic (right)

Small elastic contribution

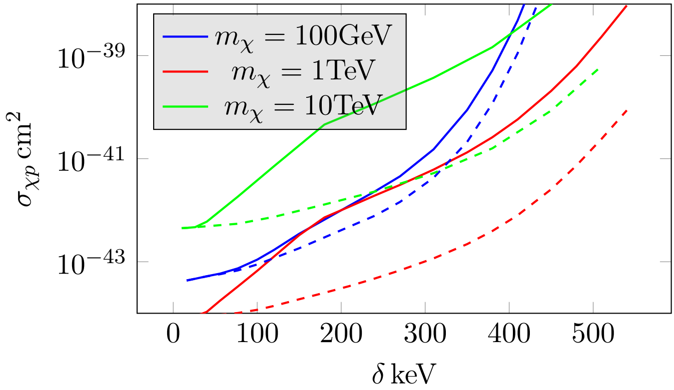
- Consider the case where inelastic scattering $(\chi + p \to \chi^* + p)$ dominates, but there is also a small elastic contribution $\sigma_{\chi p,\text{elastic}}$ from the process $(\chi + p \to \chi + p)$. For large $\sigma_{\chi p,\text{elastic}}$ there will be fully elastic thermalization, for small values — fully inelastic. The intermediate region is as follows:

$$10^{-52}\text{cm}^2 \cdot \frac{m_\chi}{\text{GeV}} \gtrsim \sigma_{\chi p,\text{elastic}} \gtrsim 10^{-51}\text{cm}^2 \cdot \frac{m_\chi}{\text{GeV}}$$



Constraints

- We derive constraints on $\sigma_{\chi p}$ from the neutrino signal using equation (2).



Constraints from IceCube neutrino signal [9], dahed — if DM has fully thermalizes.

Conclusion

- To obtain correct constraints on the cross section $\sigma_{\chi p}$, we determined the dark matter distribution in the Sun by solving the Boltzmann equation and computed the annihilation coefficient for inelastic dark matter for several models.
- Small elastic contributions $\sigma_{\chi p,\text{elastic}}$ can increase C_A, strengthening constraints — pure inelastic DM represents the worst-case scenario. We obtain the $\sigma_{\chi p,\text{elastic}}$ boundary of regions where thermalization is fully elastic/inelastic.
- For future: self-scattering processes such as $\chi + \chi \to \chi^* + \chi^*$ or $\chi + \chi \to \chi + \chi$ could increase C_A and warrant future investigation.

Thank you for your attention!

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Capture Formulas and Collision Matrices

- The capture rate and scattering matrix on nucleus α have the form:

$$C_\alpha(E,l) = \int 4\pi r^2 dr \frac{w\bar{f}(v)}{v} dv \frac{d\Omega_{in}}{4\pi} dR_\alpha(\vec{w} \to \vec{w}') \delta(E - E(r,\vec{w}'))\delta(l - l(r,\vec{w}'))$$

$$S_\alpha(E,l,E',l') = \int_{r(t)=r_{min}}^{r(t)=r_{max}} \frac{dt}{T} dR_\alpha(\vec{w} \to \vec{w}') \delta(E - E(r(t),\vec{w}'))\delta(l - l(r(t),\vec{w}'))$$

where

$$dR_\alpha(\vec{w} \to \vec{w}') = \left[\int d^3u\, n_\alpha(r)|\vec{w} - \vec{u}| f_\alpha(\vec{u}) \frac{d^3\sigma_{\chi\alpha}}{d^3\vec{w}'}\right] d^3\vec{w}'$$

Magnetic Dark Matter

- Magnetic dark matter has the following interaction [7]:

$$\mathcal{L}_{int} = \frac{\mu_\chi}{2}\bar{\Psi}_1 \Sigma_{\mu\nu} \Psi_2 F^{\mu\nu} + \frac{\mu_\chi}{2}\bar{\Psi}_2 \Sigma_{\mu\nu} \Psi_1 F^{\mu\nu} \qquad (3)$$

where $\Psi_{1,2}$ are Majorana spinors of states 1 and 2, and $\Sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu \gamma_\nu]$.

- The non-relativistic operator of interaction with the nucleon is then the following:

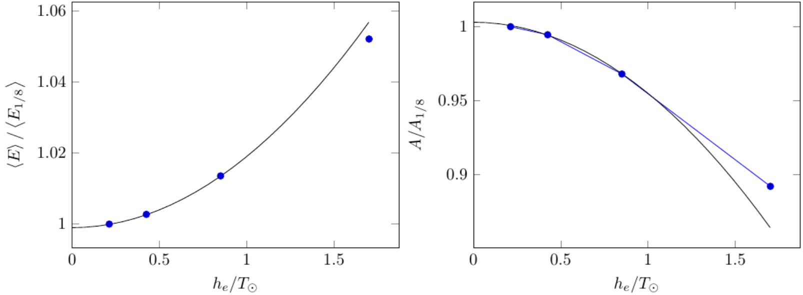
$$\hat{V} = \frac{Q_N e \mu_\chi}{2m_\chi}\hat{O}_1 + \frac{2g_N e \mu_\chi}{m_N}\hat{O}_4 - \frac{2Q_N e \mu_\chi m_N}{q^2}\hat{O}_5 - \frac{2g_N e \mu_\chi m_N}{q^2}\hat{O}_6$$

where

$$\hat{O}_5 = i\vec{S}_\chi \cdot \left(\frac{\vec{q}}{m_N} \times \vec{v}^{\perp N}_{inel}\right) \quad \hat{O}_6 = \left(\vec{S}_\chi \cdot \frac{\vec{q}}{m_N}\right)\left(\vec{S}_N \cdot \frac{\vec{q}}{m_N}\right) \qquad (4)$$

Convergence of Numerical Schemes

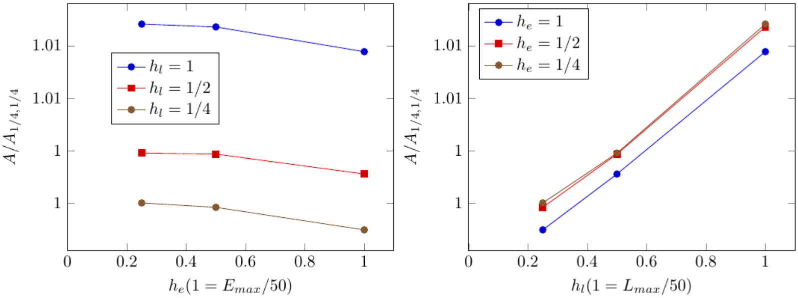
Elastic case



Dependence of physical quantities (average energy, and annihilation rate) on grid step for $m_\chi = 100\text{GeV}$

Convergence of Numerical Schemes

Inelastic case



Dependence of annihilation rate on grid step in E and l for $m_\chi = 100\text{GeV}$

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