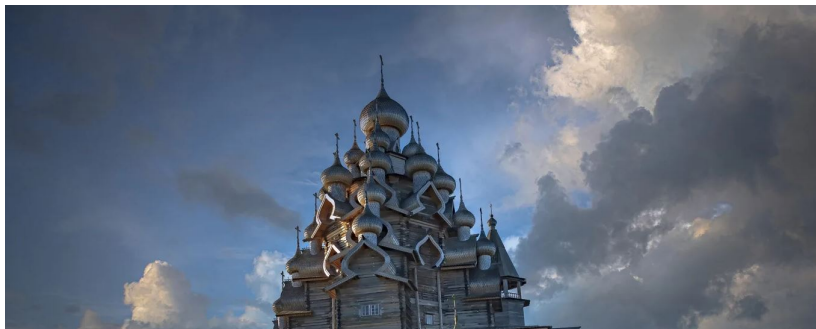


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Approximate renormalization group invariants and the Yukawa unification

Based on the paper K.D.Krylov, D.M.Rystsov, K.S., ArXiv:2604.23329.

Supersymmetry and Grand Unification

The detailed analysis of quantum numbers for various fields indicates that the **Standard Model (SM)** can be a low-energy remnant of a certain theory with a wider gauge symmetry. Such theories are usually called "**Grand Unified Theories**". The simplest of them is based on the gauge group $SU(5)$

H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974), 438.

An interesting prediction of such models is the **unification of the gauge couplings** at a certain scale. **This is not true for the Standard Model**, but in its supersymmetric extensions the running couplings really meet in a single point,

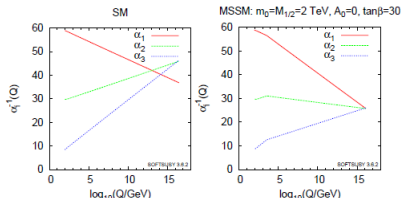


Figure 94.1: Running couplings in SM and MSSM using two-loop RG evolution. The SUSY threshold at 2 TeV is clearly visible on the MSSM side. (We thank Ben Allanach for providing the plots created using SOFTSUSY [61].)

J. R. Ellis, S. Kelley, D. V. Nanopoulos, Phys. Lett. B **260** (1991), 131;
U. Amaldi, W. de Boer, H. Furstenau, Phys. Lett. B **260** (1991), 447;
P. Langacker, M. X. Luo, Phys. Rev. D **44** (1991), 817.

The Minimal Supersymmetric Standard Model (MSSM)

The MSSM is the simplest supersymmetric extension of the Standard Model. It is a gauge theory with the group $SU_3 \times SU_2 \times U_1$ and softly broken supersymmetry. Consequently, there are 3 gauge coupling constants e_3 , e_2 , and e_1 in the MSSM (their number is equal to the number of factors in the gauge group). Quarks, leptons, and Higgs fields are components of the chiral matter superfields:

Superfield	SU_3	SU_2	$U_1 (Y)$	Superfield	SU_3	SU_2	$U_1 (Y)$
$3 \times Q$	$\bar{3}$	2	$-1/6$	$3 \times N$	1	1	0
$3 \times U$	3	1	$2/3$	$3 \times E$	1	1	-1
$3 \times D$	3	1	$-1/3$	H_d	1	2	$1/2$
$3 \times L$	1	2	$1/2$	H_u	1	2	$-1/2$

where for the superfields which include left quarks and leptons we use the brief notations

$$Q = \begin{pmatrix} \tilde{U} \\ \tilde{D} \end{pmatrix}; \quad L = \begin{pmatrix} \tilde{N} \\ \tilde{E} \end{pmatrix}.$$

Unlike SM, its supersymmetric extensions (like MSSM) necessarily contain at least two doublets of the Higgs superfields H_d and H_u .

The MSSM superpotential

The MSSM contains **three gauge couplings**

$$\alpha_3 = \frac{e_3^2}{4\pi}; \quad \alpha_2 = \frac{e_2^2}{4\pi}; \quad \alpha_1 = \frac{5}{3} \cdot \frac{e_1^2}{4\pi}$$

corresponding to the subgroups $SU(3)$, $SU(2)$, and $U(1)$, respectively. (The factor $5/3$ in the coupling constant α_1 is introduced in order that the unification of couplings has the form $\alpha_1 = \alpha_2 = \alpha_3$.) There are also **dimensionless Yukawa couplings** $(Y_U)_{IJ}$, $(Y_D)_{IJ}$, and $(Y_E)_{IJ}$ (which are 3×3 matrices) inside **the superpotential**

$$\begin{aligned} W = & (Y_U)_{IJ} \left(\tilde{U} \tilde{D} \right)_I^a \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{u1} \\ H_{u2} \end{pmatrix} U_{aJ} \\ & + (Y_D)_{IJ} \left(\tilde{U} \tilde{D} \right)_I^a \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} D_{aJ} \\ & + (Y_E)_{IJ} \left(\tilde{N} \tilde{E} \right)_I \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix} E_J \\ & + \mu (H_{u1} \ H_{u2}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_{d1} \\ H_{d2} \end{pmatrix}. \end{aligned}$$

Moreover, the superpotential includes a term with **the parameter μ** , which has the **dimension of mass**.

Gauge symmetry breaking in MSSM

The lowest scalar components of the Higgs superfields H_u and H_d (denoted by h_u and h_d , respectively) acquire **the vacuum expectation values**

$$(h_u)_0 = \begin{pmatrix} 0 \\ v_u \end{pmatrix}; \quad (h_d)_0 = \begin{pmatrix} v_d \\ 0 \end{pmatrix},$$

which **break the gauge symmetry**,

$$SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{em}.$$

After that, the superpotential produces **the mass matrices for fermions**, which are given by the expressions

$$M_u = v_u Y_U; \quad M_d = -v_d Y_D; \quad M_e = -v_d Y_E.$$

The usual masses are equal to the absolute values of the eigenvalues of these matrices and **have the hierarchical structure**.

Instead of v_u and v_d it is more convenient to use the parameters

$$v \equiv \sqrt{v_u^2 + v_d^2} = \sqrt{\frac{2m_z^2}{e_1^2 + e_2^2}} \approx 174. \text{ GeV}; \quad \text{tg } \beta \equiv \frac{v_u}{v_d}.$$

At present, the value of $\text{tg } \beta$ is unknown.

MSSM can also be considered as a low-energy remnant of a certain (softly broken) supersymmetric Grand Unified theory. An interesting prediction of such theories is **the existence of certain equations relating the Yukawa couplings**, which, in turn, lead to the relations between elementary particle masses. For instance, in the simplest $SU(5)$ model the invariant $\bar{5} \times 10 \times \bar{5}$ produces the relation $Y_E = (Y_D)^T$. This implies that a certain scale

$$m_d = m_e; \quad m_s = m_\mu; \quad m_b = m_\tau.$$

However, **this prediction cannot be satisfied**. This can be seen with the help of a certain approximate **renormalization group invariant (RGI)**. By definition, approximate RGIs are almost independent of scale. (Sometimes it is possible to construct exact RGIs, which do not depend on scale in all orders.) In the SM/MSSM it is possible to construct **the approximate RGI from the masses of down quarks and charged leptons**,

$$\frac{d}{d \ln \mu} \left(\frac{m_e m_s}{m_d m_\mu} \right) \approx 0; \quad \frac{m_e m_s}{m_d m_\mu} \approx \frac{1}{9} \cdot 0.866 \approx \frac{1}{9}.$$

Taking into account that **this expression remains almost constant throughout the whole evolution from the electroweak scale up to the scale of Grand Unification**, we see that **the simplest equality $Y_E = (Y_D)^T$ cannot be satisfied** for any values of the SM/MSSM parameters.

A way to solve this problem is to consider more complicated models leading, for example, to the Georgi and Jarlskog textures

H. Georgi, C. Jarlskog, Phys. Lett. B **86** (1979), 297.

If the Yukawa matrices for the down quarks and charged leptons are chosen in the form

$$Y_D = \begin{pmatrix} 0 & B & 0 \\ B & A & 0 \\ 0 & 0 & C \end{pmatrix}; \quad Y_E = \begin{pmatrix} 0 & B & 0 \\ B & -3A & 0 \\ 0 & 0 & C \end{pmatrix},$$

then for $B \ll A$ we obtain

$$\frac{m_e m_s}{m_d m_\mu} \approx \frac{1}{9}.$$

The factor -3 can be obtained either if the Higgs superfield comes from the representation $\underline{45}_h$ of the group $SU(5)$ and the Yukawa interaction is based on the invariant $5 \times \overline{10} \times \underline{45}_h$, or with the help of the nonrenormalizable interaction

$$\frac{1}{M} \cdot 5 \times \overline{10} \times 5_h \times 75_H,$$

where 75_H acquires vev breaking $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$.

Approximate RGs and the Yukawa relations

In this talk we consider (softly broken) supersymmetric extensions of the Standard Model. The aim is to analyze which Yukawa relations could be satisfied for a certain choice of the parameters and which cannot.

For this purpose, we will construct some approximate RGs containing Yukawa couplings. Using the (approximate) scale independence of these expressions, it is possible

- 1) to express them in terms of masses, $\tan\beta$, etc. at low energies;
- 2) to guess their values at the unification scale with the help of arguments based on the group theory.

Comparing the results, we will try to estimate values of the unknown parameters

$$\tan\beta; \quad \left| \frac{(Y_U)_{33}}{(Y_D)_{33}} \right|; \quad \left| \frac{(Y_U)_{22}}{(Y_D)_{22}} \right|, \quad \text{etc.}$$

Doing this, we will consider only the Yukawa couplings of the third and second generations and neglect mixing.

In general we will assume that the models under consideration can have a different field content compared to the MSSM, but will neglect the Yukawa interaction of the possible exotic superfields.

For simplicity, we assume that masses of all superpartners are equal. The renormalization group equations are taken in the two-loop approximation for the gauge couplings and in the one-loop approximation for the Yukawa couplings both below and above the supersymmetric threshold.

The first approximate RGI

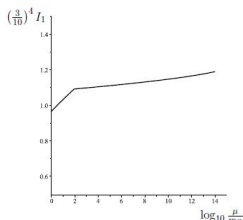
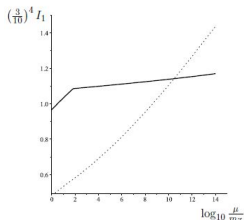
Let us construct **expressions which remain almost constant** throughout the whole renormalization group evolution. The first example of such an expression is

$$I_1 = \left| \left(\frac{(Y_U)_{33}}{(Y_D)_{33}} \right)^3 \left(\frac{(Y_D)_{22}}{(Y_U)_{22}} \right)^5 \right|.$$

The **derivative** of the logarithm of this expression with respect to $\ln \mu$ **does not contain the (large) couplings α_3 , $[(Y_U)_{33}]^2$, and $[(Y_D)_{33}]^2$** and is given by

$$\frac{d \ln I_1}{d \ln \mu} = \frac{\alpha_1}{5\pi} + \frac{1}{8\pi^2} [(Y_E)_{33}]^2 + \dots$$

The plot of I_1 as a function of scale for **the MSSM with $\text{tg } \beta = 28.2$** is presented below **on the left**. A similar plot for **the extension of the MSSM containing superfields forming three $SU(5)$ representations $5 + \bar{5}$ ($\text{tg } \beta = 28.2$)** is presented below **on the right**.



The second approximate RGI

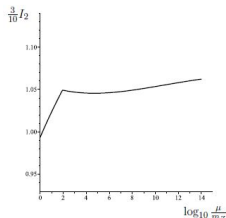
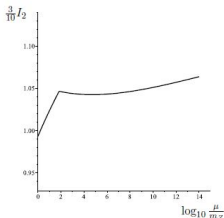
It is also desirable to construct such an RGI that can be estimated at low energies without involving $\text{tg } \beta$. For this purpose **let us assume that at the unification scale**

$$|(Y_U)_{33}| = x |(Y_D)_{33}|,$$

where x is a certain real number, and **the absolute value of $|(Y_U)_{33}| - x |(Y_D)_{33}|$ remains small** throughout the whole renormalization group evolution. Under this assumption it is possible to obtain that **the expression**

$$I_2 = \left| \frac{(Y_U)_{33}}{(Y_D)_{33}} \cdot \frac{(Y_D)_{22}}{(Y_U)_{22}} \cdot \left(\frac{(Y_E)_{33}}{(Y_D)_{33}} \cdot \frac{3(Y_D)_{22}}{(Y_E)_{22}} \right)^{\frac{2(x^2-1)}{x^2+3}} \right|$$

slightly depends on scale and is therefore an approximate RGI. The plots of I_2 for **the MSSM** and its extension with $n_5 = 6$ ($\text{tg } \beta = 28.2$) are presented below



Using RGEs for estimating ratios of the Yukawa couplings

Neglecting the non-supersymmetric evolution below the supersymmetric threshold, **it is possible to roughly estimate** values of I_1 and I_2 ,

$$I_1 \approx \text{tg}^2 \beta \left(\frac{m_t}{m_b} \right)^3 \left(\frac{m_s}{m_c} \right)^5; \quad I_2 \approx \frac{m_t}{m_b} \cdot \frac{m_s}{m_c} \cdot \left(\frac{m_\tau}{m_b} \cdot \frac{3m_s}{m_\mu} \right)^{\frac{2(x^2-1)}{x^2+3}}.$$

On the other side, **at the unification scale** M_X **we assume that**

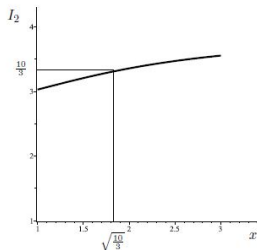
$$|(Y_U)_{33}| = x |(Y_D)_{33}|; \quad |(Y_E)_{33}| = |(Y_D)_{33}|; \quad |(Y_E)_{22}| = 3 |(Y_D)_{22}|.$$

This implies that

$$\left| \frac{(Y_D)_{33}}{(Y_U)_{33}} \right|_{M_X} = x; \quad \left| \frac{(Y_D)_{22}}{(Y_U)_{22}} \right|_{M_X} = \frac{1}{x} I_2.$$

The plot of I_2 as a function of the parameter x is presented on the right. From this plot we see **two interesting options**,

$$1) x = 1, \quad I_2 = 3; \quad 2) x = \sqrt{\frac{10}{3}}, \quad I_2 = \frac{10}{3}.$$

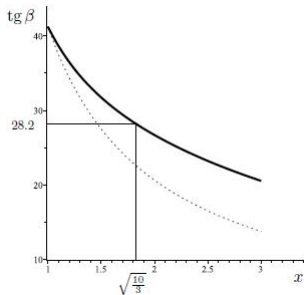


Using RGs for estimating values of the MSSM parameters

Taking into account that I_1 and I_2 slightly depend on scale and eliminating the ratio $|(Y_D)_{22}/(Y_U)_{22}|_{M_X}$, **it is possible to estimate the value of $\text{tg } \beta$ as a function of x ,**

$$\text{tg } \beta \approx \frac{1}{x} \cdot \frac{m_t}{m_b} \cdot \left(\frac{m_\tau}{m_b} \cdot \frac{3m_s}{m_\mu} \right)^{\frac{5(x^2-1)}{x^2+3}}.$$

The plot of this function is presented on the right together with the tree curve $\frac{1}{x} \cdot \frac{m_t}{m_b}$ denoted by dots.



Using this equation for **two** above **options** we obtain

1) $x = 1$; $I_2 = 3$; $\text{tg } \beta \approx 41.3$;

$$|(Y_U)_{33}| = |(Y_D)_{33}| = |(Y_E)_{33}|; \quad 3 |(Y_U)_{22}| = |(Y_D)_{22}| = \frac{1}{3} |(Y_E)_{22}|;$$

2) $x = \sqrt{\frac{10}{3}}$; $I_2 = \frac{10}{3}$; $\text{tg } \beta \approx 28.2$;

$$\sqrt{\frac{3}{10}} |(Y_U)_{33}| = |(Y_D)_{33}| = |(Y_E)_{33}|; \quad \sqrt{\frac{10}{3}} |(Y_U)_{22}| = |(Y_D)_{22}| = \frac{1}{3} |(Y_E)_{22}|.$$

Equations relating the Yukawa couplings appear in theories with wide gauge symmetry. The matter is that the **wide gauge invariance requires that the coefficients of various terms in the action should be related to each other**. For instance, the $SO(10)$ invariant $\overline{16} \times \overline{16} \times 10$ leads to the relations

$$Y_U = Y_D = Y_E = -Y_\nu,$$

which appear in the option **1) for the third generation**. However, we did not manage to find an invariant producing the relation

$$3 |(Y_U)_{22}| = |(Y_D)_{22}| = \frac{1}{3} |(Y_E)_{22}|.$$

However, for the option **2)** it is possible to construct an invariant producing the Yukawa relations presented above. It is the E_6 invariant $\overline{27} \times \overline{27} \times \overline{351}'$. It is convenient to write it in terms of parts corresponding to the subgroup $SO(10) \times U(1) \subset E_6$,

$$\begin{aligned} \overline{27} \times \overline{27} \times \overline{351}' &= P^2 A + \frac{4}{\sqrt{10}} P P_i A_i + \sqrt{2} P P^a A_a + \frac{1}{\sqrt{2}} P_i P_j A_{ij} \\ &+ \sqrt{2} P_i P^a A_{ia} + \frac{1}{4 \cdot 5!} P^a (\Gamma_{ijklm} B)_{ab} P^b A_{ijklm} + \frac{1}{4\sqrt{5}} P^a (\Gamma_i B)_{ab} P^b A_i. \end{aligned}$$

It is derived by requiring **the manifest $SO(10) \times U(1)$ invariance** and **the invariance under a certain hidden symmetry**.

The Yukawa relations for the third generation

For the **third generation** we take

$$\begin{aligned} \overline{27}|_{E_6} &\rightarrow 10|_{SO(10)} \rightarrow 5|_{SU(5)} \rightarrow D_3, L_3; & \overline{27}|_{E_6} &\rightarrow \overline{16}|_{SO(10)} \rightarrow \overline{10}|_{SU(5)} \rightarrow Q_3, U_3, E_3; \\ \overline{351}'|_{E_6} &\rightarrow 126|_{SO(10)} \rightarrow \overline{5}|_{SU(5)} \rightarrow (H_u)_3; & \overline{351}'|_{E_6} &\rightarrow \overline{144}|_{SO(10)} \rightarrow 5|_{SU(5)} \rightarrow (H_d)_3; \end{aligned}$$

E_6	$Y_3 \cdot \overline{27} \times \overline{27} \times \overline{351}'$		
$SO(10)$	$\overline{16} \times \overline{16} \times 10$	$\frac{1}{4} Y_3 \cdot \overline{16} \times \overline{16} \times 126$	$\sqrt{2} Y_3 \cdot 10 \times \overline{16} \times \overline{144}$
$SU(5)$	$\overline{10} \times \overline{10} \times \overline{5}$	$5 \times \overline{10} \times 45$	$-\frac{1}{\sqrt{10}} Y_3 \cdot \overline{10} \times \overline{10} \times \overline{5}$
MSSM		$(Y_U)_{33} = -\frac{1}{\sqrt{3}} Y_3$	$(Y_E)_{33} = (Y_D)_{33} = -\frac{1}{\sqrt{10}} Y_3$

The result reads as

$$Y_3 = -\sqrt{3} (Y_U)_{33} = -\sqrt{10} (Y_E)_{33} = -\sqrt{10} (Y_D)_{33}.$$

The Yukawa relations for the second generation

Similarly, for **the second generation** we take

$$\begin{aligned} \overline{27}|_{E_6} &\rightarrow \overline{16}|_{SO(10)} \rightarrow 5|_{SU(5)} \rightarrow D_2, L_2; & \overline{27}|_{E_6} &\rightarrow \overline{16}|_{SO(10)} \rightarrow \overline{10}|_{SU(5)} \rightarrow Q_2, U_2, E_2; \\ \overline{351}'|_{E_6} &\rightarrow 10|_{SO(10)} \rightarrow \overline{5}|_{SU(5)} \rightarrow (H_u)_2; & \overline{351}'|_{E_6} &\rightarrow \overline{126}|_{SO(10)} \rightarrow 45|_{SU(5)} \rightarrow (H_d)_2; \end{aligned}$$

E_6	$Y_2 \cdot \overline{27} \times \overline{27} \times \overline{351}'$			
$SO(10)$	$\frac{1}{4\sqrt{5}} Y_2 \cdot \overline{16} \times \overline{16} \times 10$	$\frac{1}{4} Y_2 \cdot \overline{16} \times \overline{16} \times 126$	$10 \times \overline{16} \times \overline{144}$	
$SU(5)$	$-\frac{1}{\sqrt{10}} Y_2 \cdot \overline{10} \times \overline{10} \times \overline{5}$	$2Y_2 \cdot 5 \times \overline{10} \times 45$	$\overline{10} \times \overline{10} \times \overline{5}$	$5 \times \overline{10} \times 5$
MSSM	$(Y_U)_{22} = -\frac{1}{\sqrt{10}} Y_2$	$\frac{1}{3} (Y_E)_{22} = -(Y_D)_{22} = \frac{1}{\sqrt{3}} Y_2$		

In this case we obtain

$$Y_2 = -\sqrt{10} (Y_U)_{22} = \frac{1}{\sqrt{3}} (Y_E)_{22} = -\sqrt{3} (Y_D)_{22}.$$

Running of the Yukawa couplings for the MSSM

Thus, we obtained that the Yukawa relations for the option 2) could be derived from certain invariants. That is why below we will consider this option. **It is convenient to define new Yukawa couplings**

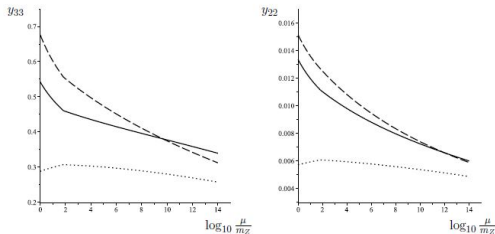
$$\begin{aligned}(y_U)_{33} &= \sqrt{\frac{3}{10}} |(Y_U)_{33}|; & (y_D)_{33} &= |(Y_D)_{33}|; & (y_E)_{33} &= |(Y_E)_{33}|; \\(y_U)_{22} &= \sqrt{\frac{10}{3}} |(Y_U)_{22}|; & (y_D)_{22} &= |(Y_D)_{22}|; & (y_E)_{22} &= \frac{1}{3} |(Y_E)_{22}|,\end{aligned}$$

(solid lines) (dashed lines) (dotted lines)

so that the unification conditions take the form

$$(y_U)_{33} = (y_D)_{33} = (y_E)_{33}; \quad (y_U)_{22} = (y_D)_{22} = (y_E)_{22}.$$

The renormalization group running of these couplings for the MSSM with $\text{tg}\beta = 28.2$ is presented in the figure. We see that **no Yukawa unification occurs in this case**. However, one can try to modify the theory in order to achieve the Yukawa unification.



How to modify the MSSM to achieve the Yukawa unification?

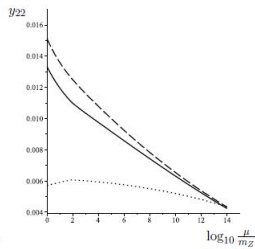
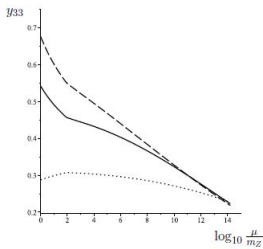
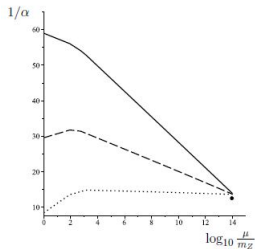
Modifications of the MSSM **should not destroy the unification of the gauge couplings**. This can be achieved **if new matter superfields form total $SU(5)$ multiplets**

D. Ghilencea, M. Lenzagorta and G. G. Ross, Nucl. Phys. B **511** (1998), 3;
G. Amelino-Camelia, D. Ghilencea and G. G. Ross, Nucl. Phys. B **528** (1998), 35.

We considered a modification of the MSSM ($\tan\beta = 28.2$) containing n_5 superfields forming the representations $\mathbf{5}$ and $\bar{\mathbf{5}}$ of the group $SU(5)$. We also took

$$M_{\text{SUSY}} = 10^{3.9} \text{ GeV}; \quad M_{\text{doublets}} = 10^{4.7} \text{ GeV}; \quad M_{\text{triplets}} = 10^{5.1} \text{ GeV}.$$

For the anomaly cancellation n_5 should be even and numbers of $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations should be equal. The unification of the Yukawa couplings (for the third and second generations) occurs if $n_5 = 6$ (equivalently, **there are 3 additional representations $\mathbf{5} + \bar{\mathbf{5}}$**).



A hint of the E_6 symmetry?

As is well known, **the superfields of a single generation** (including the right neutrinos) can be placed into the $SU(5)$ representation $1 + 5 + \overline{10}$,

$$\phi = N; \quad \phi_\alpha = \begin{pmatrix} D_1 \\ D_2 \\ D_3 \\ \tilde{E} \\ -\tilde{N} \end{pmatrix}; \quad \phi^{\alpha\beta} = \begin{pmatrix} 0 & U_3 & -U_2 & \tilde{U}^1 & \tilde{D}^1 \\ -U_3 & 0 & U_1 & \tilde{U}^2 & \tilde{D}^2 \\ U_2 & -U_1 & 0 & \tilde{U}^3 & \tilde{D}^3 \\ -\tilde{U}^1 & -\tilde{U}^2 & -\tilde{U}^3 & 0 & E \\ -\tilde{D}^1 & -\tilde{D}^2 & -\tilde{D}^3 & -E & 0 \end{pmatrix}.$$

According to the branching rule

R. Slansky, Phys. Rept. **79** (1981), 1

$$\overline{16} \Big|_{SO(10)} = 1(5) + 5(-3) + \overline{10}(1) \Big|_{SU(5) \times U(1)}$$

corresponding to the embedding $SU(5) \times U(1) \subset SO(10)$, they could form the spinor representation $\overline{16}$ of the group $SO(10)$. Similarly, **the additional superfields could form 3 representations 10 of the group $SO(10)$** ,

$$10 \Big|_{SO(10)} = 5(2) + \bar{5}(-2) \Big|_{SU(5) \times U(1)}.$$

A hint of the E_6 symmetry?

Therefore, it seems possible to unify 3 generations and $n_5 = 6$ additional superfields in three E_6 representations $\overline{27}$ according the branching rule

$$\overline{27} \Big|_{E_6} = 1(-4) + 10(2) + \overline{16}(-1) \Big|_{SO(10) \times U(1)}$$

corresponding to the embedding $SO(10) \times U(1) \subset E_6$. Certainly, in this case an $SO(10)$ singlet should be included to each generation,

$$3 \times \overline{27} \Big|_{E_6} \rightarrow \underbrace{3 \times 1}_{3 \text{ singlets}} + \underbrace{3 \times (5 + \bar{5})}_{\text{additional superfields producing the Yukawa unification}} + \underbrace{3 \times (1 + 5 + \overline{10})}_{3 \text{ ordinary generations}} \Big|_{SU(5)}$$

Thus, the Yukawa unification may indicate the possible underlying E_6 gauge symmetry in high energy physics, first discussed in

F. Gursev, P. Ramond, P. Sikivie, Phys. Lett. B **60** (1976), 177.

In this case one could expect existence of (rather large number of) new particles with masses of the order 10^2 TeV.

Nevertheless, a phenomenologically satisfactory model has not so far been constructed.

- For supersymmetric extensions of the Standard Model it is possible to construct some expressions that receive small quantum corrections and remain almost constant throughout the whole evolution from low energies to the unification scale.
- These renormalization group invariants (RGIs) can be used for estimating certain parameters, like ratios of the Yukawa couplings at the unification scale. Their values sometimes impose severe restrictions on the Grand Unified theories.
- Analyzing values of approximate RGIs we suggested two possible Yukawa relations for the third and second generations. One of them can be derived using group theory methods from the E_6 invariant $\overline{27} \times \overline{27} \times \overline{351}'$.
- For the MSSM the unifications of the Yukawa couplings does not take place for both options under consideration.
- For the extension of the MSSM with additional superfields forming 3 $SU(5)$ representations $5 + \overline{5}$ the Yukawa unification really occurs.
- Combining additional superfields with ordinary matter of 3 generations (and adding the relevant singlets) we obtain the field content which may follow from a theory with the gauge group E_6 .
- How to construct a phenomenologically satisfactory model is an open problem.
- Nevertheless, it is possible to expect existence of (a rather large number of) new particles with masses of the order of tens or hundreds of TeV.

Thank you for the attention!