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Cosmological Bootstrap Approach to the Scalar Bispectrum in a Non-Singular Cosmological Model

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Motivation for a Bouncing Universe

- The main objective of cosmology is to construct a self-consistent model of the Universe. The first such framework was the hot Big Bang model.
- However, this model faces several conceptual problems, such as the flatness problem, the horizon problem, baryon asymmetry, and the origin of large-scale structure. To address these issues, the inflationary model was proposed.
- Nevertheless, like any cosmological model based on general relativity, inflation, when extrapolated backwards in time, is geodesically incomplete according to the Hawking–Penrose singularity theorems. Therefore, it cannot by itself be regarded as a complete theory of cosmological evolution.

Primordial Singularity Problem

- The key tool behind the Hawking–Penrose singularity theorems is the Raychaudhuri focusing equation for a null geodesic congruence:

  $$\frac{d\theta}{d\lambda} = -R_{\mu\nu}\ell^{\mu}\ell^{\nu} + \ldots$$

  where θ is the expansion scalar of the null geodesic congruence.
- If the convergence condition

  $$R_{\mu\nu}\ell^{\mu}\ell^{\nu} \geq 0 \tag{1}$$

  is satisfied, gravity focuses null geodesics, which leads to geodesic incompleteness under the assumptions of the singularity theorems.
- Using Einstein's field equations, condition (1) can be rewritten as a condition on the matter stress-energy tensor:

  $$\Theta_{\mu\nu}\ell^{\mu}\ell^{\nu} \geq 0.$$
- Therefore, avoiding the initial singularity usually requires either exotic matter violating the energy conditions, or an alternative route: a modification of gravity.

Model Lagrangian

After performing a conformal transformation with the factor $\Omega = e^{-\mu\phi}/\sqrt{\hat{g}}$, we obtain a G-inflation model which, in the Einstein-frame variables, is described by the action [arXiv:2410.10742]

$$\mathcal{S} = \int d^4x\sqrt{-g}\,[G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4 R]\,, \tag{2}$$

with

$$G_2 = \frac{c_2 e^{-2(\mu-1)\phi}}{\hat{g}} + \frac{\sqrt{2X}\,d_2 e^{(1-\mu)\phi}}{\sqrt{\hat{g}}} - \frac{\sqrt{2X}\,c_3(1+2\mu)e^{(1-\mu)\phi}}{\sqrt{\hat{g}}} - 6d_3\mu X - 6\mu^2 X,$$

$$G_3 = \frac{1}{2}(d_3 + 2\mu)\ln X,$$

$$G_4 = \frac{M_{\rm pl}^2}{2}.$$

where $M_{\rm pl}$ is the reduced Planck mass, $\mu > 1$, $0 < \chi < 1$, $c_2, d_2, c_3, d_3, \hat{g}$ are model parameters, ϕ is a real scalar field, and X is the kinetic term.

Correlation functions

- In cosmology, we can measure only properties of late-time correlation functions, such as the spectral tilt and non-Gaussianities. In this work, we compute the bispectrum of curvature perturbations ζ

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3).$$

- There is a large variety of inflationary models, and one of the key modern problems is to identify those consistent with observational data.
- In the 2010-2018 the Planck and BICEP/Keck telescope constrained the scalar power spectrum. The next step is to study non-Gaussianities also weak constraints from Planck/BICEP and tensor perturbations.
- The standard way to compute such late-time correlation functions is the Schwinger–Keldysh, or in-in, formalism. We first use this method to derive the scalar three-point function explicitly.

Schwinger-Keldysh Formalism

To compute late-time correlation functions in cosmology, we use the Schwinger-Keldysh, or in-in, formalism.

Unlike the usual S-matrix approach, the in-in formalism computes expectation values at a finite time:

$$\langle \Omega | \mathcal{O}(t) | \Omega \rangle ,$$

where $|\Omega\rangle$ is the interacting vacuum and $\mathcal{O}(t)$ is an operator built from cosmological perturbations.

At tree level, expanding the time-evolution operator in the Dyson series gives

$$\begin{aligned} \langle \Omega | \zeta^3(t) | \Omega \rangle &\simeq -i \int_{t_0}^{t} dt' \langle 0 | [\zeta^3(t), H_{\text{int}}(t')] | 0 \rangle \\ &= i \int_{t_0}^{t} dt' \left\langle 0 \middle| \left[\zeta^3(t), \int d^3x \, \mathcal{L}_{\text{int}}(t') \right] \middle| 0 \right\rangle . \end{aligned}$$

Here $\mathcal{L}_{\text{int}}$ is the interaction Lagrangian.

Scalar Cubic Lagrangian

Expanding the action (2) up to third order in perturbations around the de Sitter background, we obtain the scalar-sector interaction Lagrangian in terms of conformal time

$$\mathcal{L}_{\zeta\zeta\zeta} = a^4 \mathcal{G}_S \left[\frac{C_1}{6a^3 H}\zeta'^3 + \frac{C_2}{a^2}\zeta'^2\zeta + \frac{2C_3 c_s^2}{a^2}\zeta(\partial_i\zeta)^2 + \right.$$
$$\left. + \frac{2C_4}{a^2}\zeta'\partial_i\zeta\partial_i\psi + \frac{2C_5}{a^2}\partial^2\zeta(\partial_i\psi)^2 \right],$$

where ζ is the curvature perturbation, $\psi = \partial^{-2}\zeta'$, H is the Hubble parameter, $\partial^2 \equiv \delta^{ij}\partial_i\partial_j$, and C_i are given in the appendix.

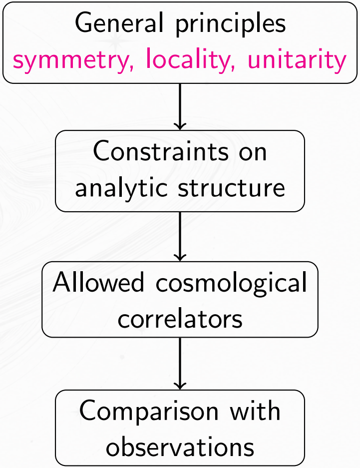
Scalar Bispectrum from the In-In Formalism

Evaluating all vacuum expectation values and using the additivity of the time integral, we obtain the scalar bispectrum

$$B_{\zeta\zeta\zeta} = \frac{H^4}{16 c_s^6 \mathcal{G}_S^2 k_T^3 e_3^3} \left(\frac{1-\chi}{\mu-\chi}\right)^4 \Bigg[C_1 \left(\frac{1-\chi}{\mu-\chi}\right) e_3^2 + C_2 k_T \left(k_T e_2^2 + e_2 e_3 - 2k_T^2 e_3\right)$$
$$+ C_3 k_T \left(k_T^2 - 2e_2\right)\left(k_T^3 - k_T e_2 - e_3\right)$$
$$+ \frac{1}{2} C_4 k_T \left(2k_T^5 - 7k_T^3 e_2 - 4k_T e_2^2 + 17 k_T^2 e_3 + 4 e_2 e_3\right)$$
$$+ C_5 k_T \left(2k_T^5 - 9k_T^3 e_2 + 15 k_T^2 e_3 + 4 k_T e_2^2 - 4 e_2 e_3\right) \Bigg].$$

Here k_T, e_2, and e_3 denote the elementary symmetric combinations of momenta, which will be introduced below in (3).

What Is a Bootstrap Approach?



General principles
symmetry, locality, unitarity

Constraints on analytic structure

Allowed cosmological correlators

Comparison with observations

Approaches to the cosmological bootstrap

Three complementary approaches can be distinguished in the cosmological bootstrap:

1. One may formulate a bootstrap equation, as in conformal field theory [arXiv:1811.00024].
2. In addition, one may formulate a set of rules, related to the symmetries of the model, imposed on the coefficients of the cosmological wavefunction [arXiv:2009.02898]

$$\Psi[\phi_0, \tau_0] = \exp\left[-\sum_{n=2}^{\infty}\frac{1}{n!}\int\frac{\prod_{m=1}^{n} d^3k_m}{(2\pi)^{3n}}\delta^{(3)}\left(\sum_{m=1}^{n}k_m\right)\psi_n(k_1,\ldots,k_n,\tau_0)\prod_{m=1}^{n}\phi(k_m)\right].$$

3. However, the most convenient approach for studying non-Gaussianities is to construct rules that constrain the possible form of the correlation functions themselves [arXiv:2010.12818]. We stick to this approach.

Rule 1

Due to homogeneity, isotropy, and scale invariance, the most general form of the bispectrum must be

$$B = \sum_{\text{contractions}} (\text{polarization factor}) \times (\text{trimmed bispectrum})$$
$$= \sum_{\text{contractions}} \left[\varepsilon^{h_1}(\mathbf{k}_1)\varepsilon^{h_2}(\mathbf{k}_2)\varepsilon^{h_3}(\mathbf{k}_3)\mathbf{k}_1^{\alpha_1}\mathbf{k}_2^{\alpha_2}\mathbf{k}_3^{\alpha_3}\right] \times \mathcal{B}(k_1, k_2, k_3),$$

where the indices $\alpha_1, \alpha_2, \alpha_3$ specify how many times each momentum enters the contractions.

Moreover, the trimmed bispectrum must be a homogeneous function of the momentum magnitudes

$$\mathcal{B}(\lambda k_1, \lambda k_2, \lambda k_3) = \lambda^{-(6+\alpha_{\text{tot}})}\mathcal{B}(k_1, k_2, k_3),$$

where $\alpha_{\text{tot}} = \alpha_1 + \alpha_2 + \alpha_3$.

Rule 2

Computing the three-point correlation function in the weak-coupling regime at tree level using the Schwinger-Keldysh formalism, we conclude that the trimmed bispectrum must be expressible in terms of polynomials in the momenta

$$\mathcal{B} = \frac{\mathrm{Poly}_{\beta}(k_1, k_2, k_3)}{\mathrm{Poly}_{6+\alpha_{\mathrm{tot}}+\beta}(k_1, k_2, k_3)}.$$

This follows from the polynomial momentum dependence of the interaction vertices and from the analytic form of the de Sitter mode functions:

$$\zeta(\mathbf{k}, \eta) \propto \frac{H}{\sqrt{2c_s k^3}}\Big[(1 + ic_s k\eta)\mathrm{e}^{-ic_s k\eta} a_{\mathbf{k}} + (1 - ic_s k\eta)\mathrm{e}^{ic_s k\eta} a^{\dagger}_{-\mathbf{k}}\Big].$$

Rule 3

Since we consider bosons, their correlation function must obey Bose–Einstein symmetry. We can therefore introduce a new symmetric basis

$$\begin{aligned} e_1 &= k_1 + k_2 + k_3 \equiv k_T, \\ e_2 &= k_1 k_2 + k_2 k_3 + k_3 k_1, \\ e_3 &= k_1 k_2 k_3. \end{aligned} \tag{3}$$

This decomposition is unique!

Rule 4

The requirement of unitarity gives a strong constraint on the form of the n-point correlation function

$$\lim_{k_T \to 0} \mathcal{B}_n = 2(-1)^n H^{2n+N-4} (N+n-4)! \, \mathrm{Re} \left[\frac{i A_n'(c_s)}{(-i c_s k_T)^{N+n-3}} \prod_{a=1}^{n} \left(\mathcal{N}^2 \frac{i + c_s k_a \eta_0}{2 k_a^2} \right) \right],$$

where N is the total derivative order in the cubic action, with $N+n>3$, H is the Hubble constant, c_s is the sound speed, $A_n'(c_s)$ is the flat-space amplitude with the momentum-conservation factor omitted, and

$$\mathcal{N}^2 \equiv \left(\frac{1-\chi}{\mu-\chi} \right)^2 \frac{1}{\mathcal{G}_S}.$$

Rule 5 I

The simplest way to define locality is through the Lagrangian. For example, $\pi\Box\pi$ is a local term, whereas $\pi\frac{1}{\Box}\pi$ is non-local.
However, this definition is not always sufficient. For example, the field equation

$$-(\Box + M^2)\phi + \frac{\lambda}{2}\pi^2 = 0$$

corresponds to the Lagrangian

$$\mathcal{L}_{\rm local} = -\frac{1}{2}\pi(\Box + m^2)\pi + \frac{\lambda^2}{8}\left(\frac{1}{M^2}\pi^4 - \pi^2\frac{\Box}{M^4}\pi^2 + \dots\right),$$

which appears local, although it can actually be written in the form

$$\mathcal{L}_{\rm non-local} = -\frac{1}{2}\pi(\Box + m^2)\pi + \frac{\lambda^2}{8}\pi^2\frac{1}{\Box + M^2}\pi^2.$$

Rule 5 II

These considerations motivate a more useful definition of locality. We call a theory local if the average of quantum fluctuations over the background can be represented as a functional Taylor series. This imposes another constraint on the form of the n-point correlation function

$$\lim_{q\to 0}\frac{B_n(q,k_1,\dots,k_n)}{P(q)}<\infty.$$

Note that this is only a necessary condition for locality of the theory!

Rule 6

In single-clock cosmologies, the time dependence of the inflaton ϕ spontaneously breaks dilatations and special conformal transformations. By Goldstone's theorem, this leads to a soft boson; using the Ward–Takahashi identities, one can derive the Maldacena consistency relation that the bispectrum must satisfy

$$\frac{\langle \zeta_{\mathbf{q}} \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle'}{\mathcal{P}_\zeta(q)} \xrightarrow[q \to 0]{} -\left[3 + \mathbf{k}_1 \cdot \partial_{\mathbf{k}_1} + \mathbf{k}_2 \cdot \partial_{\mathbf{k}_2}\right] \langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle' = (1 - n_s) \mathcal{P}_\zeta(k_1),$$

where ζ is the curvature perturbation.

Bootstrap ansatz

1. From the first bootstrap rule, the three-point function is the product of a trimmed bispectrum and a polarization factor. In the case of $\langle\zeta\zeta\zeta\rangle$, the polarization factor is equal to 1.
2. From the second bootstrap rule, the trimmed bispectrum can be represented as a ratio of polynomials in the momenta of the modes under consideration. Analyzing single-clock cosmologies, we arrive at the most general form of the trimmed bispectrum

$$\mathcal{B}_{\zeta\zeta\zeta} = \mathcal{P}_\zeta^2 \frac{1}{e_3^3 k_T^p} \sum_{l=0}^{\left\lfloor \frac{p+3}{3} \right\rfloor} \sum_{j=0}^{\left\lfloor \frac{p+3-3l}{2} \right\rfloor} C_{(p+3-2j-3l),j,l}\, k_T^{p+3-2j-3l} e_2^j e_3^l .$$

Locality and the ultraviolet limit

Analyzing the cubic action [arXiv:1301.5721], we find the bispectrum structure in the basis introduced by the third rule

$$\mathcal{B}_{\zeta\zeta\zeta} = \mathcal{P}_\zeta^2 \frac{C_0 e_3^2 + \widetilde{C}_0 e_2^3 + C_1 k_T e_2 e_3 + C_2 k_T^2 e_2^2 + C_3 k_T^3 e_3 + C_4 k_T^4 e_2 + C_6 k_T^6}{e_3^3 k_T^3}.$$

- The fourth rule fixes the terms with the highest-order pole as $k_T \to 0$.
- We note that the term with $\widetilde{C}_0$ must be excluded by the locality test, namely rule five.

Soft limit I

- To apply soft theorems, consider the symmetric soft limit

$$k_1 \equiv k_{\mathrm{soft}}, \qquad k_2 = k_3 \equiv k$$

and introduce a small expansion parameter

$$\varepsilon = \frac{k_{\mathrm{soft}}}{k} \ll 1.$$

- In this limit, the symmetric polynomials take the form

$$k_T = k(2+\varepsilon), \qquad e_2 = k^2(1+2\varepsilon), \qquad e_3 = k^3\varepsilon.$$

Soft limit II

We substitute them into the bispectrum structure and expand in a Taylor series in ε:

$$\frac{\mathcal{B}_{\zeta\zeta\zeta}}{\mathcal{P}_\zeta(k)\mathcal{P}_\zeta(k_{\rm soft})} = \Bigg[\left(\frac{1}{2}C_2 + 2C_4 + 8C_6\right) + \\ +\varepsilon\left(\frac{1}{4}C_1 + \frac{7}{4}C_2 + C_3 + 5C_4 + 12C_6\right) + \mathcal{O}(\varepsilon^{-1})\Bigg].$$

- Next, we apply the consistency relation

$$\frac{\mathcal{B}_{\zeta\zeta\zeta}}{\mathcal{P}_\zeta(k)\mathcal{P}_\zeta(k_{\rm soft})} \to 1 - n_s, \qquad k_{\rm soft} \to 0.$$

Soft limit III

Comparing coefficients, we obtain two conditions:

$$\frac{1}{2}(C_2 + 4C_4 + 16C_6) = 1 - n_s,$$

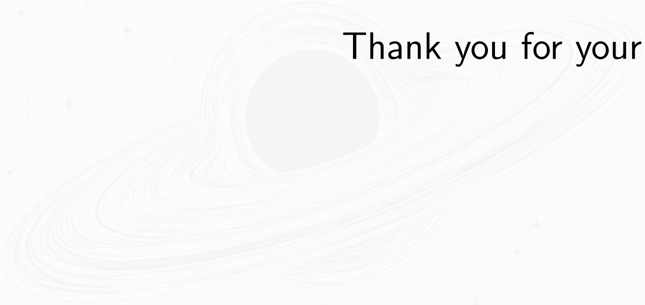
$$C_1 + 7C_2 + 4C_3 + 20C_4 + 48C_6 = 0.$$

New rule

- Thus, using the cosmological bootstrap method, we obtained the momentum structure of the three-point correlation function. However, it is not possible to determine the coefficients C_i from the bootstrap rules discussed above.
- It is therefore necessary to find a new rule. One possible way to do this is to use the effective field theory of cosmology in order to determine which of the coefficients are parametrically small.

Outlook

- We considered correlation functions at tree level. In future work, we plan to study the applicability of the cosmological bootstrap and its possible modifications for loop corrections.
- The results presented in this talk are intermediate. In our publication, we will also compute bispectra in the mixed sector and for three gravitons, $\langle\gamma\gamma\gamma\rangle$.
- The cosmological bootstrap has recently found broad applications in cosmological collider theory, and we would also like to pursue research in this direction.



Thank you for your attention!

Cubic Scalar Coefficients: C_1

$$
\begin{aligned}
C_1 = & -\frac{8\Sigma\mathcal{G}_T^3}{3\Theta^3\mathcal{G}_S} + \frac{2H}{\Theta\mathcal{F}_S}\left[\frac{2\Sigma\mathcal{G}_T^3}{\Theta^2} + \frac{3\mathcal{G}_T^3}{\Theta\mathcal{F}_S}(\mathcal{G}_S - 2\mathcal{F}_S) + \frac{9M_{\mathrm{pl}}^2}{\Theta}\mathcal{G}_T(2\mathcal{G}_T - \mathcal{G}_S)\right] \\
& + 2H\left[\frac{2(\Sigma - X\Sigma_X)\mathcal{G}_T^3}{\Theta^3\mathcal{G}_S} + \frac{\Sigma\mathcal{G}_T}{\Theta^2}\left(\frac{3\mathcal{G}_T}{\mathcal{G}_S} - 1\right) + \frac{3\mathcal{G}_T}{\Theta}\left(\frac{\mathcal{G}_S}{\mathcal{F}_S} + \frac{3\mathcal{G}_T}{\mathcal{G}_S} - 1\right) + \frac{3M_{\mathrm{pl}}^2}{\Theta}\left(\frac{3\mathcal{G}_T}{\mathcal{G}_S} - 2\right)\right] \\
& - \frac{6H^3M_{\mathrm{pl}}^2\mathcal{G}_S\mathcal{G}_T^3}{\Theta^3\mathcal{F}_S^2}.
\end{aligned}
$$

Cubic Scalar Coefficients: C_2 and C_3

$$C_2 = 3 + 3H\mathcal{G}_S\left(\frac{M_{\rm pl}^2}{2\Theta\mathcal{G}_T} - \frac{3\mathcal{G}_T}{2\Theta\mathcal{F}_S}\right) + \frac{3H^2\mathcal{G}_S}{\Theta\mathcal{F}_S}\left(\frac{2M_{\rm pl}^2\mathcal{G}_T}{\Theta} - \frac{\mathcal{G}_T^3}{2\Theta\mathcal{F}_S}\right) + \frac{3H^3M_{\rm pl}^2\mathcal{G}_S\mathcal{G}_T^3}{2\Theta^3\mathcal{F}_S^2}.$$

$$C_3 = \frac{M_{\rm pl}^2}{2\mathcal{F}_S} + H\left(\frac{(3\mathcal{G}_S - 2\mathcal{G}_T)\mathcal{G}_T}{4\Theta\mathcal{F}_S} - \frac{M_{\rm pl}^2\mathcal{G}_S}{4\Theta\mathcal{G}_T}\right) + \frac{H^2\mathcal{G}_S}{\Theta\mathcal{F}_S}\left(\frac{\mathcal{G}_T^3}{4\Theta\mathcal{F}_S} - \frac{M_{\rm pl}^2\mathcal{G}_T}{\Theta}\right) - \frac{H^3M_{\rm pl}^2\mathcal{G}_S\mathcal{G}_T^3}{4\Theta^3\mathcal{F}_S^2}.$$

Cubic Scalar Coefficients: C_4 and C_5

$$C_4 = -\frac{\mathcal{G}_S}{4\mathcal{G}_T} + 3H\mathcal{G}_S\left(\frac{1}{4\Theta} - \frac{\mathcal{G}_T}{2\Theta\mathcal{F}_S}\right) + \frac{3H^2 M_{\rm pl}^2 \mathcal{G}_S\mathcal{G}_T}{2\Theta^2\mathcal{F}_S},$$

$$C_5 = \frac{3\mathcal{G}_S}{8\mathcal{G}_T} - \frac{3H\mathcal{G}_S}{8\Theta}.$$

The scalar kinetic coefficient is

$$\mathcal{F}_S = \frac{1}{a}\frac{d}{dt}\left(\frac{a\mathcal{G}_T^2}{\Theta}\right) - M_{\rm pl}^2.$$