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Non-minimal Effective Scalar-Tensor Gravity in the Early Universe

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The action

$$S=\int\sqrt{-g}\left[\left(\frac{2}{\kappa^2}+\alpha\phi^2\right)R+\kappa^2\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{1}{3!}\lambda\phi^3-\frac{1}{4!}\tilde{g}\phi^4\right]d^4x \quad (1)$$

where

- $\kappa^2=32\pi G$,
- ϕ is a scalar field,
- α and β are dimensionless constants,
- λ is a cubic scalar coupling with mass dimension,
- $\tilde{g}$ is a dimensionless quartic scalar coupling.

$$\frac{2}{\kappa^2}+\alpha\phi^2=\frac{1}{16\pi G_{\text{eff}}(\phi)}$$

B. Latosh, Eur. Phys. J. C 80, 845 (2020).

Bounce

FRW ansatz:

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2),$$

where the scale factor $a(t)$ and the scalar field $\phi(t)$ depend upon the time t only.

The Klein-Gordon equation takes the form:

$$\ddot{\phi} = 12\alpha\phi\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \beta\kappa^2\left(\frac{\dot{a}^2}{a^2}\ddot{\phi} + 2\frac{\dot{a}}{a}\frac{\ddot{a}}{a}\dot{\phi} + \frac{\dot{a}^3}{a^3}\dot{\phi}\right) - 3\frac{\dot{a}}{a}\dot{\phi} + \frac{1}{2}\lambda\phi^2 + \frac{1}{6}\tilde{g}\phi^3 = 0, \qquad (2)$$

and the Einstein field equations, respectively, take the form:

$$G_{00} = 3\frac{\dot{a}^2}{a^2}\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) + 6\alpha\frac{\dot{a}}{a}\phi\dot{\phi} - \frac{1}{4}\dot{\phi}^2 - \frac{9}{2}\beta\kappa^2\frac{\dot{a}^2}{a^2}\dot{\phi}^2 + \frac{1}{12}\lambda\phi^3 + \frac{1}{48}\tilde{g}\phi^4 = 0, \qquad (3)$$

$$G_{ii} = \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) + 2\alpha\left(\dot{\phi}^2 + \phi\ddot{\phi} + 2\frac{\dot{a}}{a}\phi\dot{\phi}\right) + \frac{1}{4}\dot{\phi}^2 - \beta\kappa^2\left(\frac{\ddot{a}}{a}\dot{\phi}^2 + 2\frac{\dot{a}}{a}\dot{\phi}\ddot{\phi} + \frac{1}{2}\frac{\dot{a}^2}{a^2}\dot{\phi}^2\right) +$$
$$+\frac{1}{12}\lambda\phi^3 + \frac{1}{48}\tilde{g}\phi^4 = 0. \qquad (4)$$

Bounce

$$a = const > 0, \quad \dot{a} = 0, \quad \ddot{a} > 0, \tag{5}$$

$$\phi = const, \quad \dot{\phi} = 0. \tag{6}$$

$$\phi = -4\frac{\lambda}{\tilde{g}}, \tag{7}$$

$$\ddot{\phi} = -\frac{8\lambda^3}{3\tilde{g}^2}\left(\frac{\frac{1}{\kappa^2} + 8\alpha\frac{\lambda^2}{\tilde{g}^2}}{\frac{1}{\kappa^2} + 8\alpha\frac{\lambda^2}{\tilde{g}^2}(1+12\alpha)}\right), \tag{8}$$

$$a > 0, \tag{9}$$

$$\ddot{a} = -\frac{16a\alpha\lambda^4}{3\tilde{g}^3}\left(\frac{1}{\frac{1}{\kappa^2} + 8\alpha\frac{\lambda^2}{\tilde{g}^2}(1+12\alpha)}\right) > 0. \tag{10}$$

The stability condition is: $\tilde{g} < 0$ (otherwise, the scalar field potential would be unbounded from below). The solution of (9)-(10) imposes the following constraints on the α parameter:

$$\alpha > 0, \tag{11}$$

$$-\frac{1}{24} - \frac{1}{24}\sqrt{1 - \frac{6\tilde{g}^2}{\kappa^2\lambda^2}} < \alpha < -\frac{1}{24} + \frac{1}{24}\sqrt{1 - \frac{6\tilde{g}^2}{\kappa^2\lambda^2}}. \tag{12}$$

This last condition imposes a narrow range on the values of α leading to negative G_{eff} values at late times (if a bounce exists).

$$\frac{2}{\kappa^2} + \alpha\phi^2 = \frac{1}{16\pi G_{\text{eff}}(\phi)} \Rightarrow \alpha > -\frac{\tilde{g}^2}{8\kappa^2\lambda^2}. \tag{13}$$

$$\boxed{\begin{array}{l}\lambda > 0, \alpha > 0 \Rightarrow \phi > 0, \ddot{\phi} < 0, \\ \lambda < 0, \alpha > 0 \Rightarrow \phi < 0, \ddot{\phi} > 0.\end{array}} \tag{14}$$

O. Zenin et al, Eur. Phys. J. C 86, 471 (2026).

Genesis + bounce

$$G_{00} = 3\frac{\dot{a}^2}{a^2}\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) + 6\alpha\frac{\dot{a}}{a}\phi\dot{\phi} - \frac{1}{4}\dot{\phi}^2 - \frac{9}{2}\beta\kappa^2\frac{\dot{a}^2}{a^2}\dot{\phi}^2 + \frac{1}{12}\lambda\phi^3 + \frac{1}{48}\tilde{g}\phi^4 = 0.$$

We consider Taylor series of the scalar field $\phi(0)$ near $t = 0$ applying the values of $\phi(0) = -4\frac{\lambda}{\tilde{g}}$ and $\ddot{\phi}(0) = \gamma$ (from the bounce analysis). The first field derivative vanishes at $t = 0$ therefore

$$\phi(t) \approx \phi(0) + \dot{\phi}(0)t + \frac{1}{2}\ddot{\phi}(0)t^2 \equiv -4\frac{\lambda}{\tilde{g}} + \frac{1}{2}\gamma t^2. \quad (15)$$

$$H(t) = \frac{At + Bt^3}{C - (A+D)t^2 - \frac{1}{2}Bt^4}, \quad (16)$$

where

$$A = 4\alpha\frac{\lambda}{\tilde{g}}\gamma > 0,$$

$$B = -\frac{1}{2}\alpha\gamma^2 < 0,$$

$$C = \frac{2}{\kappa^2} + 16\alpha\frac{\lambda^2}{\tilde{g}^2} > 0,$$

$$D = \frac{3}{2}\kappa^2\beta\gamma^2.$$

To reproduce this scenario, the initial conditions are chosen near $t = 0$, where the Hubble parameter is small and its time derivative is negligible: $H \approx 0$, $\dot{H} \approx 0$. Thus, we consider $H(t)$ Taylor series around $t = 0$ up to the third order.

$$H(t) \approx \frac{A}{C}t + \frac{A(A+D) + BC}{C^2}t^3, \quad (17)$$

The phase of flat spacetime:

$$96\alpha^2 + 8\alpha\left(1 + \frac{2}{3}\frac{\lambda^2}{\tilde{g}}\right) + \frac{\tilde{g}^2}{\lambda^2\kappa^2} \gg 0, \quad (18)$$

$$\alpha \gtrsim -\frac{1}{12} - \frac{1}{18}\frac{\lambda^2}{\tilde{g}} > 0 \Rightarrow \frac{\lambda^2}{\tilde{g}} \lesssim -\frac{3}{2}. \quad (19)$$

The expansion phase:

$$\beta > \frac{1}{6}\frac{\tilde{g}}{\lambda\kappa^2\gamma}\left(\frac{1}{\kappa^2} - 8\alpha\frac{\lambda^2}{\tilde{g}^2}\right), \quad (20)$$

$$\alpha > \frac{1}{8}\frac{\tilde{g}^2}{\lambda^2\kappa^2}. \quad (21)$$

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Bounce + inflation

$$H(t) \approx \frac{A}{C}t + \frac{A(A+D)+BC}{C^2}t^3,$$

This function has extrema at:

$$t_1 = -\sqrt{-\frac{1}{3}\frac{AC}{A(A+D)+BC}}, \quad t_2 = \sqrt{-\frac{1}{3}\frac{AC}{A(A+D)+BC}} \Rightarrow D < -\frac{BC}{A} - A \Longrightarrow \beta < \frac{1}{6}\frac{\tilde{g}}{\lambda\gamma\kappa^2}\left(\frac{1}{\kappa^2} - 8\alpha\frac{\lambda^2}{\tilde{g}^2}\right). \quad (22)$$

To realize an inflationary phase it is necessary that $H(t) \approx const$ in the vicinity of the point t_2. Therefore, $H(t)$ near this point must exhibit neither slope nor curvature which means that $\dot{H}(t_2) \approx 0$ and $\ddot{H}(t_2) \approx 0$.

$$\ddot{H}(t_2) = \sqrt{-\frac{12A[A(A+D)+BC]}{C^3}} \approx 0$$

$$\Rightarrow \left(\frac{2}{\kappa^2} + 16\alpha\frac{\lambda^2}{\tilde{g}^2}\right)^3 \gg -48\alpha^2\frac{\lambda}{\tilde{g}\gamma^3}\left(8\alpha\frac{\lambda^2}{\tilde{g}^2} + 3\kappa^2\beta\frac{\lambda}{\tilde{g}}\gamma - \frac{1}{\kappa^2}\right). \quad (23)$$

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Bounce + genesis + inflation

Let's decompose the Hubble function into a Taylor series in the vicinity of a point $t = 0$ up to the fifth order:

$$H(t) \approx \frac{A(A+D)+BC}{C^2}t^3 + \frac{\frac{1}{2}ABC + (A+D)(A[A+D]+BC)}{C^3}t^5, \qquad (24)$$

The resulting function exhibits an extrema at:

$$t_0^3 = 0,$$

$$t_{1,2} = \mp\sqrt{-\frac{3}{5}\frac{C(A[A+D]+BC)}{\frac{1}{2}ABC+(A+D)(A[A+D]+BC)}}, \qquad \Rightarrow A(A+D)+BC > 0 \Rightarrow \beta > \frac{1}{6}\frac{\tilde{g}}{\lambda\gamma\kappa^2}\left(\frac{1}{\kappa^2} - 8\alpha\frac{\lambda^2}{\tilde{g}^2}\right), \quad (25)$$

$$\Rightarrow \frac{1}{2}ABC + (A+D)(A[A+D]+BC) < 0 \qquad (26)$$

The first condition (25) is identical to (20) from Genesis. The value of t_2 is small. This requirement together with the genesis condition leads to:

$$-\frac{D}{C} \gg 1 \Rightarrow -D \gg C \Rightarrow D < 0 \qquad (27)$$

Thus the following constraints on β arise:

$$\frac{1}{6}\frac{\tilde{g}}{\lambda\gamma\kappa^2}\left(\frac{1}{\kappa^2} - 8\alpha\frac{\lambda^2}{\tilde{g}^2}\right) < \beta < 0. \qquad (28)$$

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Bounce + genesis + inflation

Analogously to the previous discussion ($\dot{H}(t_2) \approx 0$ and $\ddot{H}(t_2) \approx 0$).

$$\ddot{H}(t_2) = \frac{5}{C^3}\left(\frac{1}{2}ABC + (A+D)(A[A+D]+BC)\right)t_2 \approx 0, \qquad (29)$$

then

$$\sqrt{-\frac{27}{5}\frac{(A[A+D]+BC)^3}{C^3\left(\frac{1}{2}ABC + (A+D)(A[A+D]+BC)\right)}} \approx 0. \qquad (30)$$

Next, taking into account (18) and (27) the following constraints arise:

$$B \gg D \Rightarrow -\alpha \gg 3\kappa^2\beta, \qquad (31)$$

In summary, for the bounce + inflation scenario realization the conditions (23),(25) must be satisfied. Preferably, the parameter β is negative. If the genesis scenario is additionally considered, conditions (23), (28) and (31) are required.

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Stability Analysis

To check the stability near $t = 0$ consider small perturbations around a stationary solution. So the scalar field and the Hubble parameter represent the sum of their background values and small perturbations:

$$\begin{aligned} \phi(t) &= \phi_0 + \delta\phi(t), \\ \dot{\phi} &= \delta\dot{\phi}, \\ \ddot{\phi} &= \delta\ddot{\phi}, \end{aligned} \qquad (32) \qquad \begin{aligned} H(t) &= H_0 + \delta H(t), \\ \dot{H} &= \delta\dot{H}, \\ \ddot{H} &= \delta\ddot{H}. \end{aligned} \qquad (33)$$

$$\delta\ddot{\phi} + \left[\frac{\left(\lambda\phi_0 - \frac{1}{2}\tilde{g}\phi_0^2\right)\left(\frac{2}{\kappa^2} + \alpha\phi_0^2\right)}{\frac{2}{\kappa^2} + \alpha\phi_0^2 + 12\alpha^2\phi_0^2} \right] \delta\phi = 0, \quad \Rightarrow \frac{\left(\lambda\phi_0 - \frac{1}{2}\tilde{g}\phi_0^2\right)\left(\frac{2}{\kappa^2} + \alpha\phi_0^2\right)}{\frac{2}{\kappa^2} + \alpha\phi_0^2 + 12\alpha^2\phi_0^2} > 0. \quad (34)$$

The obtained condition is fully consistent with (14). However, considering (12) leads to tighter restrictions on the α possible values:

$$-\frac{1}{24} - \frac{1}{24}\sqrt{1 - \frac{6\tilde{g}^2}{\kappa^2\lambda^2}} < \alpha < -\frac{1}{4}\frac{\tilde{g}^2}{\lambda^2\kappa^2}. \quad (35)$$

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Intersection of cosmological scenarios

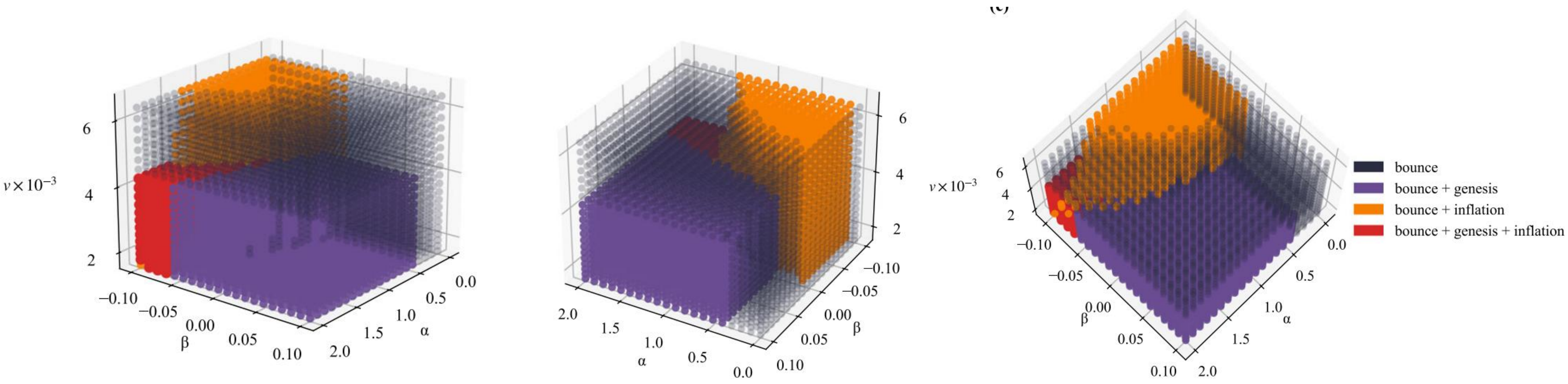

Fig. 1 Three-dimensional parameter space (α, β, v) for $\tilde{g} \in [-0.8, -0.4]$, where $v \equiv \dfrac{\tilde{g}^2}{\lambda^2\kappa^2}$. Other parameters are $\lambda = 1$ and $\kappa^2 = 32\pi$

Intersection of cosmological scenarios

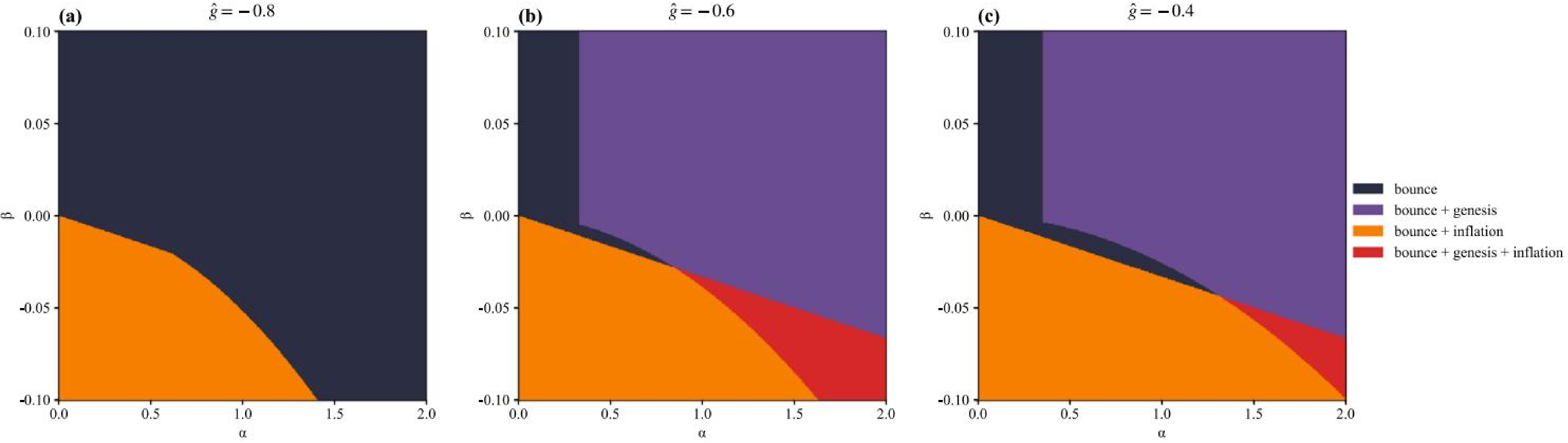

Fig. 2 Phase space classification in the (α, β) plane for three values of $\tilde{g}$: **a** $\tilde{g} = -0.8$, **b** $\tilde{g} = -0.6$, **c** $\tilde{g} = -0.4$, where $\upsilon \equiv \frac{\tilde{g}^2}{\lambda^2\kappa^2}$. Other parameters are $\lambda = 1$ and $\kappa^2 = 32\pi$

Discussion and Conclusions

- The bounce is realized under the condition: $\lambda > 0, \alpha > 0 \Rightarrow \phi > 0, \ddot{\phi} < 0,\ \lambda < 0, \alpha > 0 \Rightarrow \phi < 0, \ddot{\phi} > 0$. The bounce is also realized for α < 0. However, in this case, the range of he allowed α values is narrow and requires fine-tuning.
- The constraints for inflation are more stringent: the necessary condition for inflation is $\beta < \frac{1}{6}\frac{\tilde{g}}{\lambda\gamma\kappa^2}\left(\frac{1}{\kappa^2} - 8\alpha\frac{\lambda^2}{\tilde{g}^2}\right)$,

then the parameter β is negative (otherwise, a requirement for fine-tuning of the α parameter appears).

- The stability analysis of the equations in the vicinity of t = 0 shows that the model remains periodically stable.
- Thus, within the framework of the discussed scalar-tensor gravity model, it is possible to realize all three popular cosmological scenarios (bounce + inflation or bounce + genesis or bounce + genesis + inflation (when the role of genesis is to put the potential to a position for a slow roll start)). The conditions for the genesis and the bounce phases are identical. This means that whenever a bounce is possible it will always be followed by an inflationary or genesis phase (if β > 0 then the nflation is forbidden and genesis is automatically realized).

Thank you for your attention