

Domain walls and all that (Cosmic domain walls on a lattice)

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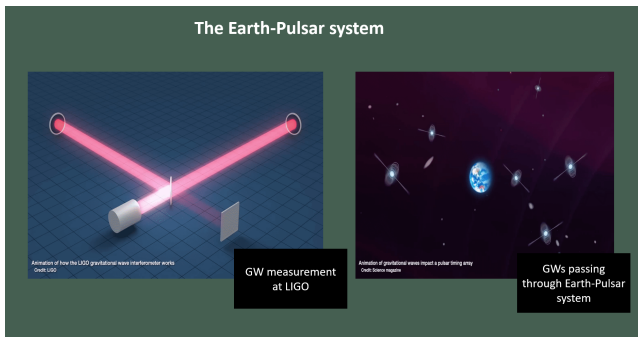
In collaboration with I. Dankovsky, E. Babichev, B. Gafarov, D. Gorbunov, R. Samanta,
M. Valencia-Villegas, A. Vikman

18-23 May 2026

Strong evidence of stochastic GW background has been reported: **NANOGrav**, **EPTA+InPTA**, **CnPTA**, **PPTA**

$$\Omega_{gw}(f) \simeq 5.8 \cdot 10^{-8} \cdot \left(\frac{f}{30 \text{ nHz}} \right)^n \quad n = 1.8 \pm 0.6$$

$$\Omega_{gw}(f) \equiv \frac{d\rho_{gw}}{\rho_{tot} d \ln f}$$



Domain walls arise in models with spontaneous breaking of discrete symmetries, e.g., Z_2 Zel'dovich, Kobzarev, and Okun'74

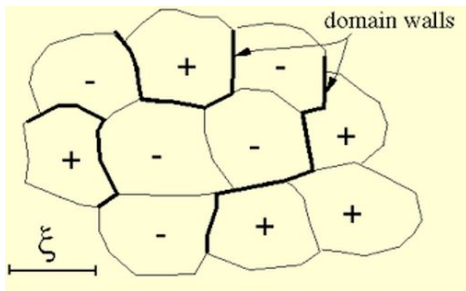
$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot (\chi^2 - v^2)^2}{4}$$

Static localized 1+1 solution

Kink $\chi(z) = v \cdot \tanh \left(\sqrt{\frac{\lambda}{2}} \cdot v \cdot z \right)$

Domain walls are embeddings of kinks into $1 + 3$

Domain walls separate regions, where $\chi = \pm v$



Domain wall tension:
$$\sigma_{wall} = \int_{-\infty}^{+\infty} dz' T_{00}(z') = \frac{2\sqrt{2\lambda}v^3}{3}$$

$$\delta_{wall} \sim \sqrt{\frac{2}{\lambda} \frac{1}{v}} \sim \frac{1}{m_\chi}$$

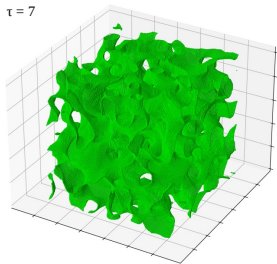
<http://www.ctc.cam.ac.uk/>

Evolution of domain walls: scaling

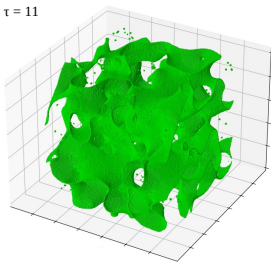
CosmoLattice

Figueroa, Florio, Torrenti, Valkenburg'20 '21

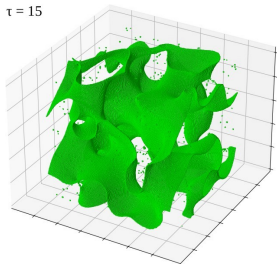
$\tau = 7$



$\tau = 11$



$\tau = 15$



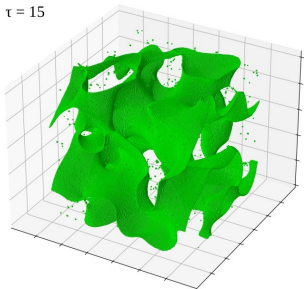
Independently of initial conditions

and independently of cosmological evolution

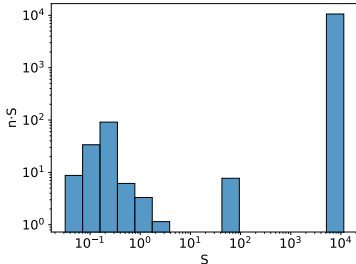
$$S \approx 2.4 \frac{V}{\tau} \quad \text{or} \quad S_{ph} \approx 2.4 \frac{V_{ph}}{l_{hor}} \implies$$

$$\rho_{wall} \approx 2.4 \frac{\sigma_{wall}}{l_{hor}}$$

$\tau = 15$



$\tau = 15$



$$r \equiv \frac{S(\text{closed walls})}{S(\text{long wall})} \lesssim 0.01$$

How does the system enter scaling?

Naive expectations: formation of closed walls \implies particle production *a la* the case of cosmic strings
Seems that it does not work!

The energy loss mechanism appears to be direct radiation from a long wall

Garagounis and Hindmarsh'02

Domain wall problem

$$\rho_{\text{wall}} \sim \frac{\sigma_{\text{wall}}}{l_{\text{hor}}} \sim \sigma_{\text{wall}} H \sim \sigma_{\text{wall}} \cdot \frac{T^2}{M_{\text{Pl}}} \quad \text{vs} \quad \rho_{\text{rad}} \sim T^4$$

$$\frac{\rho_{\text{wall}}}{\rho_{\text{rad}}} \propto \frac{1}{T^2(t)} \propto a^2(t) \implies$$

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Domain walls are typically very energetic and threaten standard cosmological evolution.

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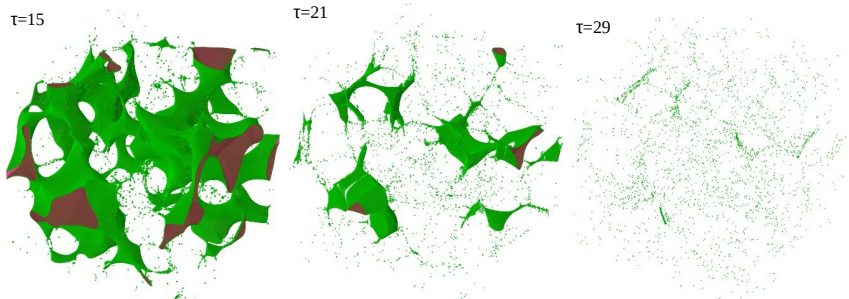
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Possible solution: explicitly break Z_2 -symmetry

$$\text{E.g., } V_{\text{breaking}}(\chi) = \epsilon v \chi^3 \implies V_{\text{bias}} = 2\epsilon v^3$$

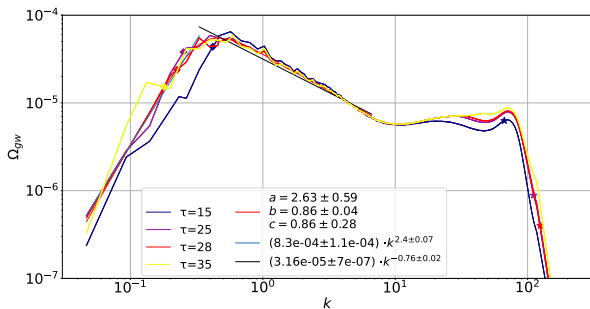
Annihilation of domain walls:
the network is dissected into small closed domain walls,
which get dissolved into scalar radiation



In first order phase transitions bubbles of true vacuum expand,
here bubbles of false vacuum shrink

Primordial black holes? [Kitajima'25](#) [Ferreira et al'24](#)

Spectrum of GWs from annihilating domain walls



Babichev et al'25

$$\epsilon = \lambda v / 20$$

IR slope: $\Omega_{gw}(k) \propto k^{2.4}$

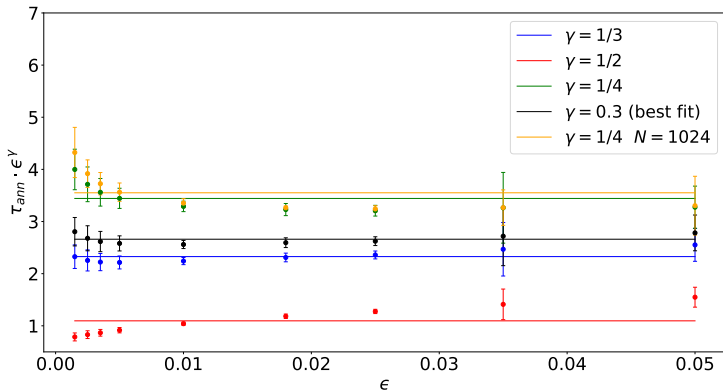
UV slope: $\Omega_{gw}(k) \propto k^{-0.74}$ vs $\Omega_{gw} \propto k^{-1.5}$ in the case of stable walls

Agrees with Kitajima et al'23, Cyr at al'24, disagrees with Notari et al'24

When do domain walls annihilate?

Common lore: $\rho_{wall} \sim \frac{\sigma_{wall}}{t_{ann}} \sim V_{bias} \sim 2\epsilon v^3 \implies \tau_{ann} \epsilon^{1/2} = \text{const}$

We have tested $\tau_{ann} \epsilon^\gamma = \text{const}$



$\gamma \approx 0.3$ better fits the data for domain wall annihilation

Fitting to PTA data

$$\Omega_{gw,peak} h_0^2 \simeq 1.4 \cdot 10^{-10} \cdot \lambda^{1.2} \left(\frac{\nu}{100 \text{ TeV}} \right)^{8.8} \cdot \left(\frac{10 \text{ keV}}{V_{bias}^{1/4}} \right)^{4.8} \cdot \left(\frac{100}{g_*(T_{ann})} \right)^{1/3} .$$

$$k_{peak} \simeq \frac{75 \text{ nHz}}{\lambda^{1/20} g_*^{1/12}(T_{ann})} \cdot \left(\frac{V_{bias}^{1/4}}{10 \text{ keV}} \right)^{1.2} \cdot \left(\frac{100 \text{ TeV}}{\nu} \right)^{0.7} .$$

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$$\gamma \rightarrow 1/2 \implies V_{bias}^{1/4} \simeq 10 \text{ MeV}$$

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$$\gamma \rightarrow 1/2 \implies V_{bias}^{1/4} \simeq 10 \text{ MeV}$$

$$\Omega_{gw}(k) = \Omega_{peak} \cdot \left(\frac{k}{k_{peak}} \right)^3 \quad k \ll k_{peak} \quad \text{Causality}$$

$n = 1.8 \pm 0.6$ 68% CL NANOGrav 15 yr

Melting domain walls

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda(\chi^2 - v^2(T))^2}{4} \quad v \propto T \propto \frac{1}{a}$$

$$\sigma_{wall} = \frac{2\sqrt{2\lambda}v^3}{3} \propto T^3$$

$$\rho_{wall} \simeq \sigma_{wall} H \propto T^5 \quad \frac{\rho_{wall}}{\rho_{rad}} \propto T(t) \propto \frac{1}{a(t)}$$

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Energy density of domain walls redshifts faster than radiation \implies **no domain wall problem**

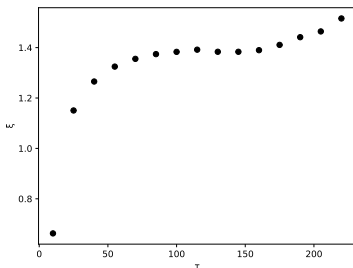
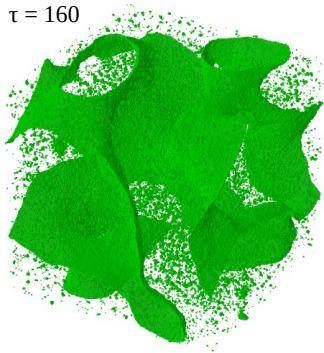
Vilenkin'81, Ramazanov et al'21, Babichev et al'21'23, Danovsky et al'24

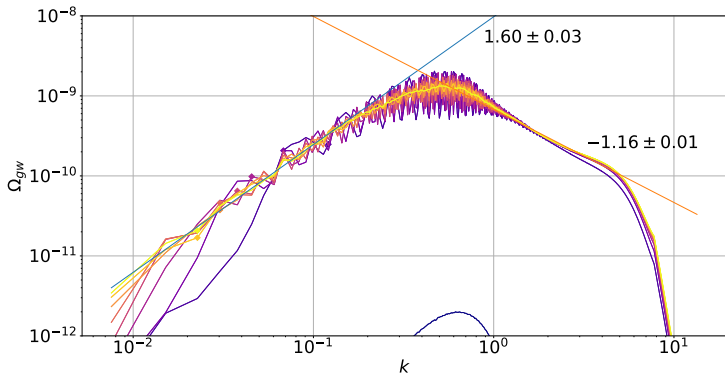
Evolution of melting domain in radiation dominated Universe is equivalent to evolution of constant tension domain walls in the Minkowski spacetime.

$$S \approx 2.8 \frac{V}{\tau}$$

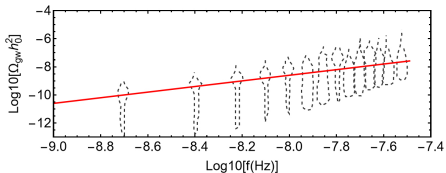
$$r = \frac{s(\text{closed walls})}{s(\text{long wall})} \simeq 0.15 - 0.3$$

$\tau = 160$





Very good agreement with the NANOGrav 15 yr $n = 1.8 \pm 0.6$



$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - \frac{\lambda \cdot \chi^4}{4} + \frac{g^2 \chi^2 \phi^\dagger \phi}{2}$$

χ is cold

ϕ is in thermal equilibrium with plasma

$$0 < g^2 \ll 1$$

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$$\langle \phi^\dagger \phi \rangle_T = \frac{\mathcal{N} T^2}{12} \implies V_{\text{eff}} = \frac{\lambda \cdot \chi^4}{4} - \frac{\mathcal{N} g^2 T^2 \chi^2}{24}$$

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$T \propto \frac{1}{a(t)} \implies Z_2$ -symmetry breaking at early times

$$v^2 = \frac{\mathcal{N} g^2 T^2}{12\lambda}$$

Fitting to NANOGrav

$$f_{peak} \simeq \frac{15 \text{ nHz} \sqrt{\mathcal{N}}}{g_*^{1/3}(T_{sc})} \cdot \left(\frac{g}{10^{-18}} \right)$$

$$\Omega_{gw,peak} h_0^2 \simeq \frac{5 \cdot 10^{-11} \mathcal{N}^4}{g_*^{7/3}(T_{sc}) \cdot \beta^2}$$

Vanilla region:

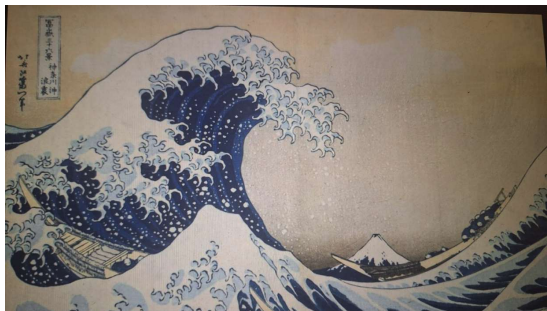
$$\beta \equiv \frac{\lambda}{g^4} \simeq 1$$

$$\mathcal{N} \gg 1$$

The field χ should be extremely weakly coupled!

Not an unfamiliar situation in physics, cf. axions, but we deal with a different group of underlying symmetries.

Where does the scaling come from? Shock waves



Scaling implies the release of energy. Emission of GWs by domain walls is insufficient \implies scaling is due to particle emission

Particles associated with the domain wall field χ are heavy $M = \sqrt{2\lambda}v!$

Shock formation is a violent process, which can make heavy particle emission possible. [Blanco-Pillado et al'25](#), [Babichev et al'26](#)

Thin planar domain walls: DBI

$$S_{NG} = -\sigma_{wall} \int d^3\zeta \sqrt{|\gamma|} \quad \gamma_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \zeta^a} \frac{\partial x^\nu}{\partial \zeta^b} .$$

Dirac-Born-Infeld (DBI) action for planar domain walls:

$$S = -\sigma_{wall} \int d\tau dx dy a^3(\tau) \sqrt{1 - 2X} \quad \text{stable} \quad X = \frac{1}{2} \eta^{ab} \partial_a z \partial_b z$$

$$S = -\sigma_{wall} \int d\tau dx dy \sqrt{1 - 2X} \quad \text{melting}$$

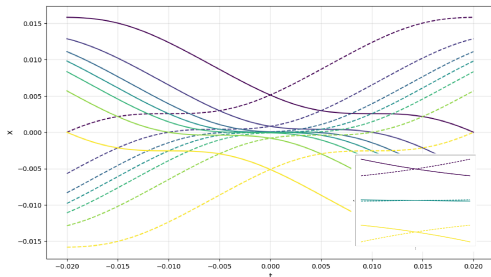
$$S = - \int d\tau dx dy a^3(\tau) \left[\sigma_{wall} \sqrt{1 - 2X} - \epsilon z \right] \quad \text{annihilating}$$

Settling to scaling can be associated with caustic formation in DBI

There are **NO** shocks in DBI in the case $c_s^2 > 0$

Generic waves propagate smoothly in DBI:

- in 2D Minkowski flat spacetime
earlier works: Blokhintsev and Orlov'53, Barbashov and Chernikov'67, Boillat'70, Deser et al'98
simple wave case: Mukohyama et al'16
- in expanding Universe
- in $D > 2$, at least in the case of spherical waves
- in the case of annihilating domain walls

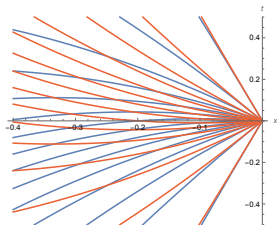


Shocks form in DBI, once $c_s \rightarrow 0$
Felder et al'02, Blanco-Pillado et al'25

DBI \approx pressureless perfect fluid near the shock

$$c_s = \sqrt{1 - 2X} = 0 \implies \sqrt{|\gamma|} = 0$$

In the case of cosmic strings $|\gamma| = 0$ corresponds to cusp formation.



Cosmic domain walls meet cosmic strings:
particle production takes place through cusp formation

- Evolution of domain walls has been investigated with a publicly available code CosmoLattice.
- Evolution is demonstrated to be remarkably universal.
- Low number of closed walls vs a long wall.
- Some peculiarities: plateau in the UV part of GW spectrum, annihilation law of domain walls....
- GWs in the case of melting domain walls: the IR spectral index of GWs is in excellent agreement with PTA data.
- Cusp formation on planar domain walls is likely to be the source of particle emission and hence scaling.

Thanks for your attention!!!