Entanglement viscosity and conformal quantum anomaly

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based on works:

[G. Prokhorov, 2601.02083 (2026)]

[G. Prokhorov, O. Teryaev, 2604.04222 (2026)]

[D. Lapygin, G. Prokhorov, O. Teryaev, V. Zakharov, PRD 112 (2025) 6, 065012]

[R. Khakimov, G. Prokhorov, O. Teryaev, in preparation]

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Contents

Part 1

Introduction

Minimal viscosity bounds

Hydrodynamics in linear gradients - corrections to EMT with dissipation:

$$T_{\mu\nu} = T^{\text{ideal}}_{\mu\nu} + T^{\text{diss}}_{\mu\nu} \qquad T^{\text{ideal}}_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - p g_{\mu\nu}$$

$$T^{\text{diss}}_{\mu\nu} = -\eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu - u_\mu u^\alpha \nabla_\alpha u_\nu - u_\nu u^\alpha \nabla_\alpha u_\mu) - \left(\zeta - \frac{2}{3}\eta\right) \nabla^\alpha u_\alpha (g_{\mu\nu} - u_\mu u_\nu) + \mathcal{O}(\nabla^2 u)$$

Bound inspired by string theory:

- **There are no completely ideal fluids!**
- It is believed that QGP near this limit
- **Does not cover case of Rindler space!**
- Can be derived within membrane paradigm

$$\boxed{\frac{\eta}{s} \geqslant \frac{1}{4\pi}}$$

KSS-bound

[Kovtun, Son, Starinets, PRL (2005)]

[Maulik Parikh and Frank Wilczek. Phys. Rev. D, 58:064011, 1998.]

- Another well-known bound for **bulk viscosity** also from holography:

[Buchel, PLB (2008), arXiv:0708.3459]

$$\boxed{\frac{\zeta}{\eta} \geqslant 2\left(\frac{1}{p} - c_s^2\right)}$$

Unruh effect

Formulation:

The **Minkowski vacuum** is perceived by an **accelerated** observer as a medium with a finite **Unruh temperature**

$$\boxed{T_U = \frac{a}{2\pi}}$$

[W. G. Unruh. Notes on black hole evaporation. Phys. Rev., D14:870, 1976.]

[Blasone, (2018), e-Print: 1911.06002]

Statement of the problem

- Does **Unruh radiation** have **viscosity**? How is it related to the **KSS bound**?
- Direct calculation for **quantum fluid** above the classical membrane.

First step (known):

[G. Chirco, C. Eling, and S. Liberati. Phys. Rev. D, 82:024010, 2010]

For massless scalar field - **finite shear viscosity**, saturates **KSS bound**

This work:

We generalize to **"any" theory**:

-- demonstrate the role of **Unitarity** and **Causality**

-- demonstrate the **relationship** with **irreversibility of renormalization group flows**

-- something more ...

[A. B. Zamolodchikov. JETP Lett., 43:730–732, 1986]

Part 2

Entanglement viscosity: universal formulas

Kubo formulas in curved spacetime

- Linear response theory allows us to write **Kubo formula** in **curved spacetime**:

  [Dam T. Son. Acta Phys. Polon. B, 39:3173–3182, 2008]

Shear viscocity:

$$\eta(x') = \mathrm{Im} \lim_{\omega \to 0^+} \frac{i}{\sqrt{g^{00}(x')}\omega} \int_{-\infty}^{\infty} dt\, \theta(t-t') \cdot e^{i\omega(t-t')} \int_V d^{d-1}x \sqrt{-g(x)}\, \langle [\hat{T}_{12}(x), \hat{T}_{12}(x')] \rangle$$

Bulk viscocity: similar formula with trace of EMT

- Apply Kubo formula to the ***d*-dimensional Rindler space:**

$$ds^2 = \rho^2 d\tau^2 - (dx^1)^2 - ... - (dx^{d-2})^2 - d\rho^2$$

- Consider **Minkowski vacuum state**:

  Thermal correlator corresponds to the **Minkowski vacuum** one (with a change of coordinates)

  [W. G. Unruh and N. Weiss. Phys. Rev., D29:1656, 1984]

Universal spectral representation

- **Universal spectral representation:**

[A. Cappelli, D. Friedan, and J. I. Latorre. Nucl. Phys. B, 352:616–670, 1991]
[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]
[S. N. Solodukhin. The a-theorem and entanglement entropy. 4 2013.]

$$\langle 0|\hat{T}_{\alpha\beta}(x)\hat{T}_{\rho\sigma}(x')|0\rangle_M = \frac{A_d}{(d-1)^2}\int_0^\infty d\mu\, c^{(0)}(\mu)\Pi^{(0)}_{\alpha\beta,\rho\sigma}(\partial')G_d(x-x',\mu) + \frac{A_d}{(d-1)^2}\int_0^\infty d\mu\, c^{(2)}(\mu)\Pi^{(2)}_{\alpha\beta,\rho\sigma}(\partial')G_d(x-x',\mu)$$

spectral function

scalar propagator

- Valid for “any” theory: different spins, conformal, nonconformal, interacting...
- For unitary theory: $c^{(0)}(\mu) \geq 0$ $c^{(2)}(\mu) \geq 0$
- Only $c^{(0)}(\mu)$ for 2d theory: positivity $c^{(0)}(\mu) \geq 0$ ⇒ irrevesibilty of renormalisation group flows

Unitarity provides irrevesibilty of renormalisation group flows

[A. B. Zamolodchikov. JETP Lett., 43:730–732, 1986]

Key integral: problem

- Use universal **spectral representaion**
- Use equation for (Whightman) scalar Green function: $\Box G_d(x-x') = -\mu^2 G_d(x-x')$
- Use **descent method:** $\int dx^{d-2} G^d(x) \to G^2(x)$

Main integral to be found:

$$\eta(\rho') = k_d \rho' \int d\mu \, \mu^4 c^{(2)}(\mu) \, \mathrm{Im} \lim_{\omega \to 0+} \frac{i}{\omega} \int_0^\infty d\tau \; e^{i\omega\tau}$$
$$\cdot \int_0^\infty d\rho \, \frac{\rho}{2\pi} \left\{ K_0 \left(\mu \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cosh(\tau - i\varepsilon)} \right) - (\tau \to -\tau) \right\}$$

- The central question is **how to find this integral?**

-- Includes non-tabular integrals ☹

-- Includes special divergent functions with a cut

-- Up to our knowledge, there is no description in the literature on how to find them.

Key integral: solution

- However, everything **becomes easy** in the **momentum representation**! ☺

$$\frac{1}{2\pi} K_0\left(\mu\sqrt{\rho^2+\rho'^2-2\rho\rho'\cosh(\tau-i\varepsilon)}\right) =$$

$$= \frac{1}{\pi^2}\int_0^\infty d\lambda\, K_{i\lambda}(\mu\rho)K_{i\lambda}(\mu\rho')\cosh(\pi\lambda - i\lambda(\tau-i\varepsilon))$$

[A P Prudnikov, Yu A Brychkov, and O I Marichev. Integrals and series: Special functions vol 2.]

Dependence on ρ and τ is **factorized**

- Now integral over the distance from the horizon can be easily found:

$$\int_0^\infty d\rho\, \rho K_{i\lambda}(\mu\rho) = \frac{\pi\lambda}{2\mu^2\sinh(\pi\lambda/2)}$$

- Integral over τ leads to frequency parts of delta function – principal value part are real and drops out

$$\eta = \frac{k_d\rho'}{4}\lim_{\omega\to 0^+}\frac{1}{\omega}\int_0^\infty d\mu\, c^{(2)}(\mu)\mu^2\int_0^\infty d\lambda\,\frac{\lambda K_{i\lambda}(\mu\rho')}{\sinh(\pi\lambda/2)}$$

$$\cdot\left\{e^{\pi\lambda}\delta(\omega-\lambda) + \cancel{e^{-\pi\lambda}\delta(\omega+\lambda)} - \cancel{e^{\pi\lambda}\delta(\omega+\lambda)} - e^{-\pi\lambda}\delta(\omega-\lambda)\right\}$$

Result: final universal formulas

- **Entanglement** (local) **shear viscosity** of **Unruh** radiation:

$$\boxed{\eta(\rho) = k_d \rho \int_0^\infty d\mu\, c^{(2)}(\mu)\mu^2 K_0(\mu\rho)}$$

The calculation for **bulk viscosity** is very similar:

$$\boxed{\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2} \int_0^\infty d\mu\, c^{(0)}(\mu)\mu^2 K_0(\mu\rho)}$$

- Valid in general case (conformal, nonconformal, interacting…).
- $c^{(2)}(\mu)$ defines **shear viscosity**.
- $c^{(0)}(\mu)$ defines **bulk viscosity**.
- In accordance with particular direct calculations (scalar, Dirac, photon):

  [G. Chirco, C. Eling, and S. Liberati. Phys. Rev. D, 82:024010, 2010]

  [D. Lapygin, G. Prokhorov, O. Teryaev, V. Zakharov, Phys.Rev.D 112 (2025) 6, 065012]

Discussion: Unitarity *vs* Irreversibility

$$\eta(\rho) = k_d\rho \int_0^\infty d\mu\, c^{(2)}(\mu)\mu^2 K_0(\mu\rho)$$

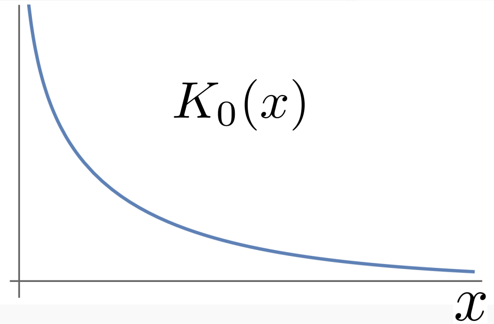
$$\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2} \int_0^\infty d\mu\, c^{(0)}(\mu)\mu^2 K_0(\mu\rho)$$

- **Unitarity:**

$$c^{(0)}(\mu) \geq 0 \qquad c^{(2)}(\mu) \geq 0$$

- Include modified Bessel function of the second kind:

$$K_0(x) > 0$$



$$\eta, \zeta \geq 0$$

$$\partial_\mu s^\mu \geq 0$$

- **Unitarity** underlies **thermal irreversibility** for a subsystem separated by a horizon
- **Analogy** with the **irreversibility** of **renormalization group flows** in 2d.

[Andrea Cappelli, Daniel Friedan, and Jose I. Latorre. Nucl. Phys. B, 352:616–670, 1991]

Part 3

Entanglement viscosity and conformal anomaly

New area: anomalous transport effects

New **(non-dissipative)** effects are predicted at the intersection of quantum field theory and gravity *(only some of them)*:

- Chiral Magnetic Effect **(CME)**:

  [K. Fukushima, D. E. Kharzeev, H. J. Warringa, PRD 78, 074033 (2008), 0808.3382]

$$\langle \hat{j}^\mu \rangle = C e^2 \mu_A B^\mu$$

- Axial Vortical Effect **(AVE)**:

  [D. T. Son, P. Surowka, PRL 103, 191601 (2009), 0906.5044]

$$\langle \hat{j}^\mu_A \rangle = C(\mu^2 + \mu_A^2)\omega^\mu$$

- Thermal part of AVE:

  [K. Landsteiner, E. Megias, F. Pena-Benitez, PRL 107, 021601 (2011), 1103.5006]

$$\langle \hat{j}^\mu_A \rangle \sim \mathcal{N} T^2 \omega^\mu$$

- **Kinematic Vortical Effect (KVE)**

  [G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, PRL 129, 151601 (2022), 2207.04449]

$$\boxed{\langle \hat{j}^A_\mu \rangle = (\lambda_1 \omega^2 + \lambda_2 a^2)\omega_\mu}$$

Associated with **axial anomaly** in the electromagnetic field:

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{Ce^2}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Associated with an **axial anomaly** in the **gravitational field:**

$$\boxed{\langle \nabla_\mu \hat{j}^\mu_A \rangle = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\rho\sigma}R_{\alpha\beta}{}^{\rho\sigma}}$$

Connection with **anomaly:**

$$\boxed{\lambda_1 - \lambda_2 = 32\mathcal{N}}$$

Entanglement viscosity and conformal anomaly

- **Conformal anomaly** effects are **not so well studied**.

[M. N. Chernodub, Phys. Rev. Lett. 117, 141601 (2016), arXiv:1603.07993 [hep-th]]
[C.-S. Chu and R.-X. Miao, Phys. Rev. Lett. 121, 251602 (2018), arXiv:1803.03068 [hep-th]]
[Yang, Gao, Liang, Prokhorov, Shi Pu, Teryaev, Zakharov, in preparation]

- Conformal gravitational anomaly in 4d:

$$\langle \hat{T}^{\mu}_{\mu} \rangle = a(-H + \frac{2}{3}\Box R) - bE_4 + c\,\Box R$$

$$\boxed{\eta = 8a\alpha^3}$$

- Entanglement viscosity – **novel anomalous effect** in systems with **extreme acceleration** α:

- “Cheshire cat’s grin” of conformal gravitational anomaly in flat spacetime:

Part 4

Novel sum rule for spectral densities

Modular Hamiltonian perturbation theory

- There is technique to find perturbatively effects of small angular deficit using modular Hamiltonian (which is actually a boost operator):

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]
[V. Rosenhaus and M. Smolkin. JHEP, 12:179, 2014.]

$$\lim_{\nu\to 1}\frac{\partial}{\partial\nu}\langle\hat{T}_{ij}\rangle = -\langle\hat{T}_{ij}\hat{K}_0\rangle$$

- Euclidean Rindler space - describes an accelerated system at a finite temperature

$$ds^2 = \rho^2 d\tau^2 + (dx^1)^2 + ... + (dx^{d-2})^2 + d\rho^2$$

It is also a conical space

- The derivative with respect to the angular deficit corresponds to the derivative with respect to temperature:

$$\nu = \frac{2\pi T}{a} \quad \Rightarrow \quad \lim_{\nu\to 1}\frac{\partial}{\partial\nu} \quad \leftrightarrows \quad \lim_{T\to T_U}\frac{\partial}{\partial T}$$

Anisotropy of pressure: Modular Hamiltonian

- Using the results of Smolkin&Solodukhin, one can write the derivatives of pressure with respect to temperature:

$$\langle \hat{T}^{\mu}_{\nu} \rangle (a,T) = \mathrm{diag}(\varepsilon, -p_{\perp}, ..., -p_{||})$$

$$\frac{\partial p_{\perp}}{\partial T}\Big|_{a} = \frac{-2\pi\rho A_d}{(d-1)^2\Gamma(d)} \int_0^{\infty} d\mu\, \mu^2 K_0(\mu\rho) \cdot \left\{ c^{(0)}(\mu) - (d-1)c^{(2)}(\mu) \right\} ,$$

$$\frac{\partial p_{||}}{\partial T}\Big|_{a} = \frac{2\pi A_d}{(d-1)^2\Gamma(d)} \int_0^{\infty} d\mu\, \mu K_1(\mu\rho) \cdot \left\{ c^{(0)}(\mu) + (d-1)(d-2)c^{(2)}(\mu) \right\}$$

- The key point – **different** formulas for **longitudinal** and **transverse pressures:**

In general, pressure can be anisotropic!

Pascal's law

- Let us require (or introduce a criterion) **isotropy** of **Unruh radiation**:

$$\boxed{\frac{\partial p_{||}}{\partial T} = \frac{\partial p_{\perp}}{\partial T}}$$

$$\Downarrow$$

- **Novel sum rule**, relating $c^{(0)}(\mu)$ and $c^{(2)}(\mu)$ spectral functions:

$$\boxed{\int_0^\infty d\mu \left\{ c^{(0)}(\mu)\mathcal{A}^{(0)}(\mu,\rho) + c^{(2)}(\mu)\mathcal{A}^{(2)}(\mu,\rho) \right\} = 0}$$

where:

$$\mathcal{A}^{(2)}(\mu,\rho) \;=\; -(d-1)\mu^2 K_0(\mu\rho) + (d-1)(d-2)\frac{\mu}{\rho}K_1(\mu\rho)$$

$$\mathcal{A}^{(0)}(\mu,\rho) \;=\; \mu^2 K_0(\mu\rho) + \frac{\mu}{\rho}K_1(\mu\rho)$$

- Postulation of **Pascal law** for Unruh radiation
- It is a criterion of **fluidity** (anisotropic pressure in solid body)
- Can be compared with the **Burkhardt-Cottingham sum rule**

[Hugh Burkhardt and W. N. Cottingham. Annals Phys., 56:453–463, 1970]

Direct verifiaction

- Can be **verified** in **conformal** case in **any dimensions**.
- Can be **verified** in **nonconformal** case: free massive Dirac field in any dimensions.

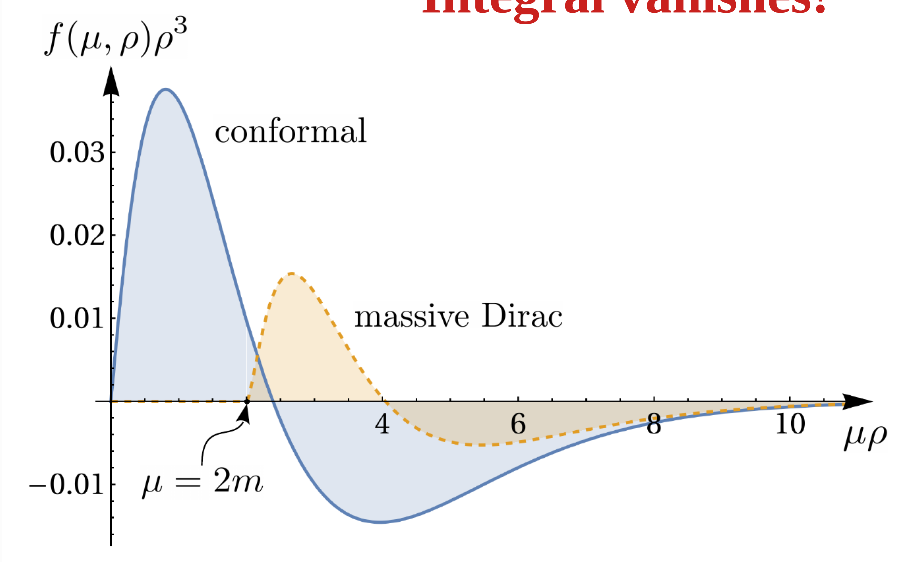
Spectral functions of **massive fermions**:

$$c^{(2)}(\mu,m) = \frac{2^{\lfloor d/2 \rfloor}(d-1)\Gamma\left(\frac{d}{2}\right)^2}{8\pi^d}\mu^{d-3}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{d-1}{2}} \cdot \left(1+\frac{8}{d-1}\cdot\frac{m^2}{\mu^2}\right)\theta(\mu-2m)\,.$$

$$c^{(0)}(\mu,m) = \frac{2^{\lfloor d/2 \rfloor}(d^2-1)\Gamma\left(\frac{d}{2}\right)^2}{2\pi^d}m^2\mu^{d-5}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{d-1}{2}} \cdot\theta(\mu-2m)\,,$$

[Andrea Cappelli, Daniel Friedan, and Jose I. Latorre. Nucl. Phys. B, 352:616–670, 1991]

Integral vanishes!



Scalar field:

- Satisfied for conformal scalar field
- Violated for massless nonconformal scalar field
- Open question - improved massive scalar field

- Anysotropic contribution $T_{\mu\nu} \sim a_\mu a_\nu$

[Frolov, V., & Serebryanyi, E. (1987), Phys. Rev. D, 35, 3779–3782]

- Surface terms in the modular Hamiltonian can be important

[Herzog, C., & Nishioka, T. (2016). JHEP, 12, 138.]

Part 5

Viscosity to entropy density ratio: local *vs* global

Viscosity to entropy density ratio: local vs global

- What is about **KSS bound** for entanglement viscosity?
- Central universal expression for entanglement shear viscosity:

$$\eta(\rho) = k_d \rho \int_0^\infty d\mu \, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

- **Heat capacity** from is the **temperature derivative** of **energy density**:

  - can be obtained via modular Hamiltonian perturbative approach

[M. Smolkin and S. N. Solodukhin, Phys. Rev. D 91, no.4, 044008 (2015)]

$$\frac{\partial \varepsilon}{\partial T}\Big|_a = \frac{2\pi\rho A_d}{(d-1)^2\Gamma(d)} \int_0^\infty d\mu \left[\mu^2 K_0(\mu\rho) + \frac{\mu}{\rho} K_1(\mu\rho)\right] \left\{c^{(0)}(\mu) + (d-1)(d-2)c^{(2)}(\mu)\right\}$$

use **isotropy sum rule**

$$c_V(\rho) = \frac{2\pi\rho A_d}{\Gamma(d)} \int_0^\infty d\mu \, c^{(2)}(\mu) \mu^2 K_0(\mu\rho)$$

only via $c^{(2)}(\mu)$

- **One can easily see:**

$$\boxed{\frac{\eta}{c_V} = \frac{1}{4\pi}}$$

- Valid in general case (conformal, nonconformal, interacting...)
- Obtained without holography

Viscosity to entropy density ratio: local vs global

- Can be transformed to the form with entropy density
- Use relation:

$$c_s^2(a,T) = \frac{\partial p}{\partial \varepsilon}\Big|_a = \frac{\partial p(a,T)/\partial T}{\partial \varepsilon(a,T)/\partial T} = \frac{s}{c_V}$$

we obtain:

$$\frac{\eta}{c_V} = \frac{1}{4\pi} \Rightarrow \boxed{\frac{\eta}{s} = \frac{1}{4\pi c_s^2}}$$

Compare with:

[Mauro Brigante, Hong Liu, Robert C. Myers, Stephen Shenker, and Sho Yaida. Phys. Rev. Lett., 100:191601, 2008]

$$\frac{\eta}{s} > \frac{16}{25 \cdot 4\pi}$$

- For exapmle, in conformal limit $c_s^2 = \frac{1}{3}$ then $\frac{\eta}{s} = \frac{3}{4\pi}$

- Relationship between **KSS bound** and **causality**:

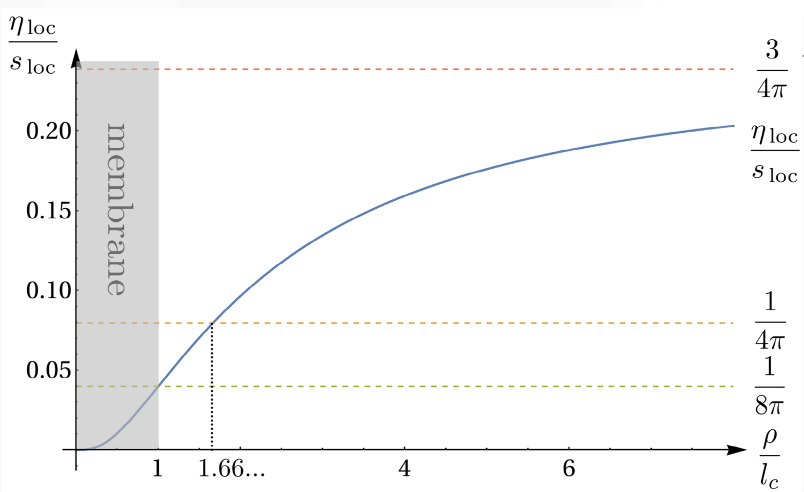
Causality: $c_s^2 < 1 \Rightarrow \frac{\eta}{s} > \frac{1}{4\pi}$

Viscosity to entropy density ratio: local *vs* global

- There are **local** and **global** quantities:

Local

- At certain distance ρ from the horizon
- We can now regularize integrals by stretched horizon $\rho > l_c$
- Any CFT in 4d: $\frac{\eta}{s} = \frac{1}{4\pi c_s^2}$



$$\frac{\eta_{\text{loc}}}{s_{\text{loc}}}(\rho) = f(\rho/l_c) = \frac{3\rho^4\left[\rho^4 + 4\rho^2 {l_c}^2 - 5{l_c}^4 - 4{l_c}^2(2\rho^2 + l_c^2)\ln\left(\frac{\rho}{l_c}\right)\right]}{4\pi(\rho^2 - l_c^2)^4}$$

- **Universal function** for any CFT in 4d

Global

$$\eta_{glob} = \int_{l_c}^{\infty} \eta(\rho)d\rho = \frac{C_T\pi^2}{480 l_c^2}$$

$$s_{glob} = \int_{l_c}^{\infty} s(\rho)d\rho = \frac{C_T\pi^3}{120 l_c^2}$$

$$\boxed{\frac{\eta_{glob}}{s_{glob}} = \frac{1}{4\pi}}$$

- **KSS bound** is **saturated globally** for any CFT in 4d

(all the previous discussion – for $l_c \to 0$ *)*

“Wandering” Planck constant

- In the original holographic derivation (or within membrane paradigm) of KSS bound the **viscosity** is **“classical”** – Planck constant comes from the **“quantum”** Bekenstein-Hawking **entropy**:

$$\left.\begin{aligned} &\eta \sim \mathcal{O}(\hbar^0) \\ &s \sim \frac{A}{\hbar G} \sim \mathcal{O}(\hbar^{-1}) \end{aligned}\right\} \qquad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

- In our case, the **viscosity** is **quantum:** determined by a **one-loop** diagram, contains Planck constant:

$$\left.\begin{aligned} &\eta_{\,\mathrm{loc}} \sim \frac{T_U^3}{\hbar^2} \\ &s_{\,\mathrm{loc}} \sim \frac{T_U^3}{\hbar^3} \end{aligned}\right\} \qquad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

However, the result is the same:
“Wandering” Planck constant

Higher spins

- **Negative viscosity** for **higher spins** (without supergravity)?

  [R. Khakimov, G. Prokhorov, O. Teryaev, in preparation]

- **Rarita-Schwinger-Adler** theory

$$\mathcal{L} = -\varepsilon^{\alpha\beta\mu\nu}\bar{\psi}_\alpha\gamma_5\gamma_\mu\partial_\nu\psi_\beta + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda$$

[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]

- We show that it has features of **conformal theory** on the one-loop level in the limit: $m \to \infty$ (unlike usual Rarita-Schwinger field)

- **Negative viscosity**

  Negative conformal **central charge**

  Entropy density is also **negative**

KSS-bound is **saturated**

$$\frac{\eta_{glob}}{s_{glob}} = \frac{1}{4\pi}$$

Part 5

Bulk viscosity

Viscosity to entropy density ratio: local vs global

- What is about **entanglement bulk viscosity**?

$$\zeta(\rho) = \frac{2k_d\rho}{(d-1)^2}\int_0^\infty d\mu\, c^{(0)}(\mu)\mu^2 K_0(\mu\rho)$$

- Use **isotropy sum rule:**

-- entropy density

-- heat capacity

Speed of sound

$$c_s^2(a,T) = \frac{s}{c_V}$$

- Use similar “spectral” formula for shear viscosity

- Everything is expressed through integrals with $c^{(0)}(\mu)$ and $c^{(2)}(\mu)$

Simple algebra

- **Bulk viscosity** saturates **known bound**:

$$\boxed{\frac{\zeta}{\eta} = 2\left(\frac{1}{d-1} - c_s^2\right)}$$

[Buchel, PLB (2008), arXiv:0708.3459]

- Obtained **without holography**

- Valid in general case (conformal, nonconformal, interacting...)

Part 6

Phenomenological applications

Phenomenological applications

- Are there any **phenomenological applications?**
- **New anomalous effect** in systems with **extreme acceleration**:

$$\eta, \zeta \ \sim \ a^3$$

extreme acceleration in HICs:

[G. Yu. Prokhorov, D. A. Shohonov, O. V. Teryaev, N. S. Tsegelnik, and V. I. Zakharov. Phys. Rev. C, 112(6):064907, 2025.]

- **Local ratio** with speed of sound:

expressed in terms of general hydrodynamic quantities ⇒ can be more general ⇒ **compare with QGP**

QGP viscosity from the experiments with HICs: $0.05 < \eta/s < 0.2$

[John W. Harris and Berndt Müller. "QGP Signatures" Revisited. Eur. Phys. J. C, 84(3):247, 2024]

Naive prediction of our formula:

$$\frac{\eta}{s} = \frac{1}{4\pi c_s^2} \Rightarrow c_s^2 \sim 1/3 \Rightarrow \eta/s \sim 0.24$$

Look on the Lattice!

Part 7 Conclusion

Key Results

- Thermal quantum radiation in a space with a horizon has an entanglement viscosity.
- We have derived universal formulas for entanglement viscosities via spectral densities.
- We have demonstrated a direct relationship between thermodynamic irreversibility and unitarity in this case. This is a direct analogy with the irreversibility of renormalization group flows.
- We have shown that the Unruh radiation isotropy conditions lead to a new sum rule relating different fundamental spectral functions.
- We have explicitly tested this sum rule for conformal and nonconformal theory (massive fermions) in any number of dimensions.
- Without using holography, we showed that globally for the conformal theory the KSS bound is saturated.
- However, locally the ratio contains the speed of sound.
- This demonstrates the relationship between the KSS bound and causality.
- Without using holography, we showed that the bulk entanglement viscosity of saturates another known bound.
- We have shown that the entanglement shear viscosity is a new anomalous transport effect associated with the conformal gravitational anomaly.
- The obtained relationships with the speed of sound are of interest from a phenomenological point of view: QCD on Lattice?

Summary & Conclusion

- **Universal formulas** for entanglement shear and bulk viscosities in terms of fundamental **spectral functions** are obtained:

  -- Valid for conformal and nonconformal case

  -- Demonstrate relationship: **Thermal irreversibility** $\Longleftrightarrow$ **QFT unitarity**

  -- Anology with irreversibility of **renormalization group flows**

  -- In accordance with direct calculations

- **Novel sum rule** for different spectral functions is obtained from **Pascal law** for Unruh radiation:

  -- Directly verified for **CFT** in d dimensions

  -- Directly verified for **nonconformal** case (massive fermions) in d dimensions

- It is shown (**without using holography**) that the entanglement viscosity **saturates known bounds**:

  -- The **KSS bound** is **globally saturated**
  -- The bound for bulk viscosity is saturated
  -- **Novel** formula for **local ratio** with speed of sound:

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi c_s^2}}$$

$$k_d = \frac{\pi^{d/2}}{(d+1)2^{d-1}\Gamma(d)\Gamma(d/2)}$$