QUARKS-2026

Neutrino oscillations and decoherence in astrophysical environments

Artem Popov, Alexander Studenikin, Alexander Tcvirov
Moscow State University

Supported by Russian Science Foundation
under grant No.24-12-00084

Outline of the talk

- Wave packet description of neutrino oscillations.
- Neutrino magnetic moments.
- Neutrino oscillations in a magnetic field: Dirac and Majorana.
- Coherence in oscillations of Dirac and Majorana neutrinos in astrophysical magnetic fields.

Neutrino oscillations and coherence

- Plane waves description is not applicable for the case of neutrino propagation on at large distances. Instead **wave packet** description must be adopted.
- Massive neutrino states propagating in vacuum obey the **dispersion relation** $E_i(p) = \sqrt{m_i^2 + p^2}$
- **Group velocities** that describe the motion of wave packets as whole are different for different massive neutrinos:

$$v_i(p) = \frac{\partial E_i(p)}{\partial p} = \frac{p}{\sqrt{m_i^2 + p^2}}$$

- Difference in the wave packets velocities results in the **wave packets separation** that is characterized by

$$\Delta v_{ij} \approx \frac{\Delta m_{ij}^2}{2p^2}$$

- When the distance between the massive neutrinos wave packets exceeds the wave packets size σ_x, one can expect **loss of coherence** and **damping of neutrino oscillations**.

$$\Delta v_{ij} t \sim \sigma_x$$

Neutrino flavour state evolution

$$P_{\alpha\beta}(L) = \sum_{i,j} U^*_{\beta i} U_{\alpha i} U_{\beta j} U^*_{\alpha j} \mathcal{I}_{ij}(L),$$ where $\mathcal{I}_{ij}(L) \sim$ overlap between the i-th and j-th neutrino states wave functions

At large distance (e.g. astrophysical neutrinos): $P_{\alpha\beta}(L) \to \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2$ (full loss of coherence)

Gaussian wave packets and coherence

- One dimensional **Gaussian wave packet** with average coordinate x at t=0 and average momentum p_0 are usually used:

$$|\nu_i\rangle = \int \frac{dk}{2\pi} e^{-iE_i(k)t} e^{-ikx} \frac{1}{N} \exp\left(-\frac{(k-p_0)^2}{4\sigma_p^2}\right) |k\rangle \otimes |i\rangle$$

where $\sigma_x \sigma_p = 1/2$.

- Massive neutrino states wave packets separation leads to the **exponential damping of neutrino flavour oscillations** at distances comparable to the ***coherence length***:

$$P_{\alpha\beta}(L) = \sum_{i,j} U_{\beta i}^* U_{\alpha i} U_{\beta j} U_{\alpha j}^* \exp\left(\frac{-2\pi i L}{L_{ij}^{osc}}\right) \exp\left(-\frac{L^2}{(L_{ij}^{coh})^2}\right), \quad L_{ij}^{osc} = \frac{4\pi p_0}{\Delta m_{ij}^2}, \quad L_{ij}^{coh} = \frac{4\sqrt{2}\sigma_x p_0^2}{|\Delta m_{ij}^2|}$$

- Wave packet widths $\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_p}$ are the *effective parameters* that depend on the neutrino production and detection processes. Bounds from the **reactor experiments**:
- **Daya Bay** collaboration: $10^{-11}\ \mathrm{cm} \lesssim \sigma_x \lesssim 2\ \mathrm{m}$ (F.P.An et al., Eur.Phys.J.C 77 (2017) 9, 606)
- Predicted **JUNO** sensitivity is an order of magnitude higher (J.Wang *et al.*, JHEP 06 (2022) 062)

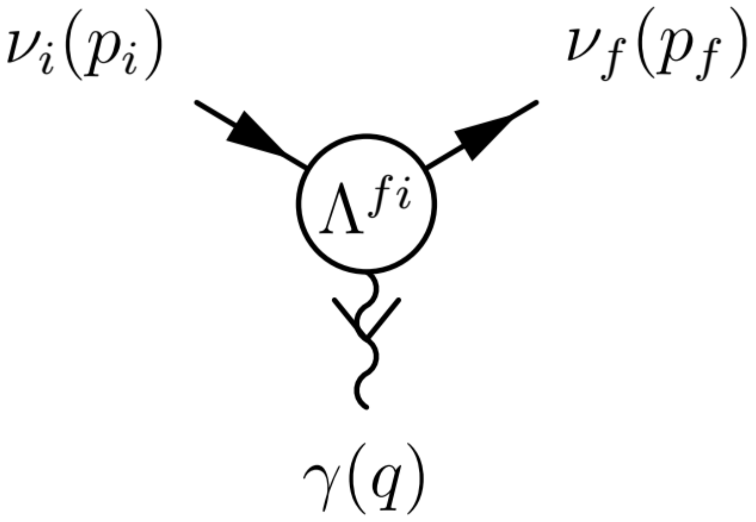
Wave packet description of neutrino evolution in external fields

- **Matter effects** in oscillations of solar and supernova neutrinos are considered in:
  J.Peltoniemi, V.Sipilainen, JHEP 06 (2000) 011;
  J.Kersten, A.Smirnov, Eur.Phys.J.C 76 (2016) 6, 339.

- **Collective effects** in oscillations of supernova neutrinos are considered within the wave packets approach in:
  E.Akhmedov, J.Kopp, M.Lindner, JCAP 09 (2017) 017.

- The wave packet treatment of neutrino oscillations accounting for their **interaction with a magnetic field** was developed for both Dirac and Majorana neutrinos in our papers:
  A.Popov, A.Studenikin, Phys.Rev.D 111 (2025) 12, 123001;
  A.Popov, A.Studenikin, A.Tcvirov, arXiv:2602.02804.

Neutrino electromagnetic properties



$$\mathcal{H}_{\text{em}}^{(\nu)}(x) = j_\mu^{(\nu)}(x) A^\mu(x) = \sum_{k,j=1}^{N} \overline{\nu_k}(x) \Lambda_\mu^{kj} \nu_j(x) A^\mu(x),$$

The vertex function is parametrized in terms of **charge, anapole, electric and magnetic form factors**:

$$\Lambda_\mu(q) = \left(\gamma_\mu - q_\mu \not{q}/q^2\right) \left[\mathfrak{f}_Q(q^2) + \mathfrak{f}_A(q^2) q^2 \gamma_5\right] - i\sigma_{\mu\nu} q^\nu \left[\mathfrak{f}_M(q^2) + i\mathfrak{f}_E(q^2)\gamma_5\right]$$

$\mathfrak{f}_M^{fi}(0) = \mu_{fi}$ - neutrino magnetic moments

C.Giunti, A.Studenikin, Rev.Mod.Phys. 87 (2015) 531;
C.Giunti, K.Kouzakov, Y.-F. Li, A.Studenikin, Ann.Rev.Nucl.Part.Sci. 75 (2025) 1, 1-33

Neutrino magnetic moments

Theory (Standard Model):

$$\mu_{ii}^D = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1\ eV}\right) \mu_B$$

K.Fujikawa, R.Shrock, *"The Magnetic Moment of a Massive Neutrino and Neutrino Spin Rotation"*, Phys.Rev.Lett. 45 (1980) 963

Experiment:

$$\mu_\nu < 6.4 \times 10^{-12}\ \mu_B$$

E.Aprile *et al.* [XENON collaboration], *"Search for New Physics in Electronic Recoil Data from XENONnT"*, Phys.Rev.Lett. 129 (2022) 16, 161805

Upper bounds from astrophysical neutrinos:
R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

$$\mu_\nu \lesssim 10^{-12} \mu_B$$

Majorana neutrinos

Dirac fermion

$$\Psi_D = \Psi_L + \Psi_R$$

Majorana fermion

$$\Psi_R = \Psi_L^c$$

A Majorana field can be written as $\Psi_M = \Psi_L + \Psi_L^c$

$\boxed{\Psi_M^c = \Psi_M}$ is satisfied for a Majorana field

Neutrino magnetic moments: Dirac and Majorana

CPT-invariance + hermicity:

- Magnetic moments matrix for **Dirac** neutrinos is **real and symmetric:**

$$\mu^D = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix}$$

- Magnetic moments matrix for **Majorana** neutrinos is **imaginary and asymmetric:**

$$\mu^M = \begin{pmatrix} 0 & i\mu_{12} & i\mu_{13} \\ -i\mu_{12} & 0 & i\mu_{23} \\ -i\mu_{13} & -i\mu_{23} & 0 \end{pmatrix}$$

- Thus, Dirac and Majorana neutrinos can be distinguished by their **electromagnetic properties.**

Flavour transitions in a magnetic field

- Neutrino interactions with a magnetic field modifies the patterns of the neutrino flavour transitions, and even induces neutrino-antineutrino transitions (in the Majorana case).

- The corresponding effects were studied in literature for various classes of astrophysical objects (Solar neutrinos, supernova neutrinos, AGN neutrinos, etc.)

- Decoherence effects due to the neutrino states wave packets separations were not accounted yet.

- The formalism involving wave packets to describe neutrino oscillations in magnetic fields were developed in our papers.

Neutrino interaction with a magnetic field

Dirac neutrino:

$$\mathcal{L}^{D}_{mag} = \sum_{i,k} \mu_{ik} \left[\overline{\nu^R_i} \boldsymbol{\Sigma} \boldsymbol{B} \nu^L_k + \overline{\nu^L_i} \boldsymbol{\Sigma} \boldsymbol{B} \nu^R_k \right]$$

Majorana neutrino:

$$\mathcal{L}^{M}_{mag} = \sum_{i,k} \mu_{ik} \left[\overline{(\nu^L_i)^C} \boldsymbol{\Sigma} \boldsymbol{B} \nu^L_k + \overline{\nu^L_i} \boldsymbol{\Sigma} \boldsymbol{B} (\nu^L_k)^C \right]$$

$$i, k = 1, 2, 3$$

For the Majorana case magnetic field induces neutrino-antineutrino transitions $\nu_\alpha \to \bar{\nu}_\beta$

Wave packet evolution in a magnetic field

- Single-particle wave function of *i*-th massive neutrino with helicity *h* is defined by

$$\nu_i^h(x) = \langle i| \otimes \langle 0|\hat{\nu}_i(x)|i, p_0, h, \sigma_p\rangle = \int \frac{dk}{2\pi} f(k, p_0, \sigma_p)\,\langle 0|\hat{\nu}_i(x)|k, h\rangle$$

where the wave packet state vector is

$$|i, p_0, h, \sigma_p\rangle = \int \frac{dk}{2\pi} f(k, p_0, \sigma_p)\,|i, k, h\rangle\,, \qquad |i, k, h\rangle = a_h^\dagger(k)\,|0\rangle \otimes |i\rangle$$

- For Majorana neutrinos, *h=-1* corresponds to neutrinos and *h=+1* to antineutrinos.

- Wave functions satisfy the modified Dirac equation

$$(i\gamma^\mu \partial_\mu - m_i)\nu_i^h(x) + \sum_j \mu_{ij} \boldsymbol{\Sigma} \boldsymbol{B} \nu_j^h(x) = 0.$$

We consider Gaussian wave packet

$$f(k, p_0, \sigma_p) = \frac{(2\pi)^{1/4}}{\sqrt{\sigma_p}} \exp\left(-\frac{(k - p_0)^2}{4\sigma_p^2}\right)$$

Wave packet evolution in a magnetic field

- Neutrino evolution in a magnetic field is described by the following Dirac equation:

$$(i\gamma^{\mu}\partial_{\mu} - m_i)\nu_i^h(x) + \sum_j \mu_{ij}\boldsymbol{\Sigma}\boldsymbol{B}\nu_j^h(x) = 0. \quad \textbf{(1)}$$

A.Popov, A.Studenikin, Phys.Rev.D 103 (2021) 11, 115027

- In the momentum space assuming slowly varying magnetic field, Equation (1) can be rewritten as

$$i\gamma^0\partial_t\nu_i(p,t) = (\gamma_3 p + m_i)\nu_i(p,t) + \sum_k \mu_{ik}\boldsymbol{\Sigma}\boldsymbol{B}(\langle x_i(t)\rangle)\nu_i(p,t) \quad \textbf{(2)}$$

A.Popov, A.Studenikin, Phys.Rev.D 111 (2025) 12, 123001

For ultra-relativistic neutrinos it is safe to assume that $< x_i > \approx ct$.

Equation (2) can be solved:

1. Analytically in the adiabatic case (slowly varying magnetic field),
2. Or numerically.

Dispersion relations: Dirac and Majorana

- Dirac neutrinos with diagonal magnetic moments matrix:

$$E_i^s(p) = \pm\sqrt{m_i^2 + p^2 + \mu_i^2 B^2 - 2s\mu_i\sqrt{m_i^2 B^2 + p^2 B_\perp^2}}$$

- Majorana neutrinos (two flavour case):

$$E_s(k) = \sqrt{p^2 + \frac{m_1^2 + m_2^2}{2} + 2sp\sqrt{\omega_B^2 + \omega_{vac}^2 + \frac{\omega_B^2(m_1+m_2)^2}{2k^2}}},$$

$$\omega_{vac}(k) = \frac{\Delta m^2}{4k}, \quad \omega_B = \mu B_\perp$$

- In other cases we study the dispersion relation numerically.

Adiabatic solution: Dirac neutrinos

- The probabilities of flavour conversions are:

$$P_{\alpha\beta}(L) = \frac{1}{2}\sum_{i=1}^{3} U_{\alpha i}^2 U_{\beta i}^2 \left[1 + 2\cos\left(\frac{2\pi L}{L_i^B}\right) D_i^B(L)\right] + 2\sum_{i>j} U_{\beta i}U_{\alpha i}U_{\beta j}U_{\alpha j} \cos\left(\frac{2\pi L}{L_{ij}^{vac}}\right)\cos\left(\frac{2\pi L}{L_i^B}\right)\cos\left(\frac{2\pi L}{L_j^B}\right) D_{ij}^{vac}(L),$$

$$D_{ij}^{vac}(L) = \exp\left(-\frac{L^2}{(L_{coh}^{ijss})^2}\right), \quad D_i^B(L) = \exp\left(-\frac{L^2}{(L_{coh}^{ii s\sigma})^2}\right)$$

where L_{osc} are oscillations lengths and L_{coh} are **coherence lengths**, $i, j = 1,2,3$ and $s, \sigma = \pm 1$.

$$L_{osc}^{ijss} = \frac{4\pi p}{\Delta m_{ij}^2} \quad \text{and} \quad L_{osc}^{ii-+} = \frac{\pi}{\mu_i B_\perp}$$

- Oscillations probability is a combination of oscillations on (1) vacuum frequencies $\omega_{ik}^{vac} = \frac{\Delta m_{ik}^2}{4p}$ and (2) magnetic frequencies $\omega_i^B = \mu_i B_\perp$.

(see A.Popov, A. Studenikin, Eur.Phys.J.C 79 (2019) 2, 144 and references therein)

Coherence lengths: Dirac neutrinos

- The probabilities of flavour conversions are:

$$P_{\alpha\beta}(L) = \frac{1}{2}\sum_{i=1}^{3} U_{\alpha i}^2 U_{\beta i}^2 \left[1 + 2\cos\left(\frac{2\pi L}{L_i^B}\right) D_i^B(L)\right] + 2\sum_{i>j} U_{\beta i}U_{\alpha i}U_{\beta j}U_{\alpha j}\cos\left(\frac{2\pi L}{L_{ij}^{vac}}\right)\cos\left(\frac{2\pi L}{L_i^B}\right)\cos\left(\frac{2\pi L}{L_j^B}\right) D_{ij}^{vac}(L),$$

$$D_{ij}^{vac}(L) = \exp\left(-\frac{L^2}{(L_{coh}^{ijss})^2}\right), \quad D_i^B(L) = \exp\left(-\frac{L^2}{(L_{coh}^{iis\sigma})^2}\right)$$

where L_{osc} are oscillations lengths and L_{coh} are **coherence lengths**, $i, j = 1,2,3$ and $s, \sigma = \pm 1$.

$$L_{coh}^{ijss} \approx \frac{4\sqrt{2}\sigma_x p^2}{\Delta m_{ij}^2},$$

(coincides with the expression for the coherence length for neutrino oscillations in vacuum)

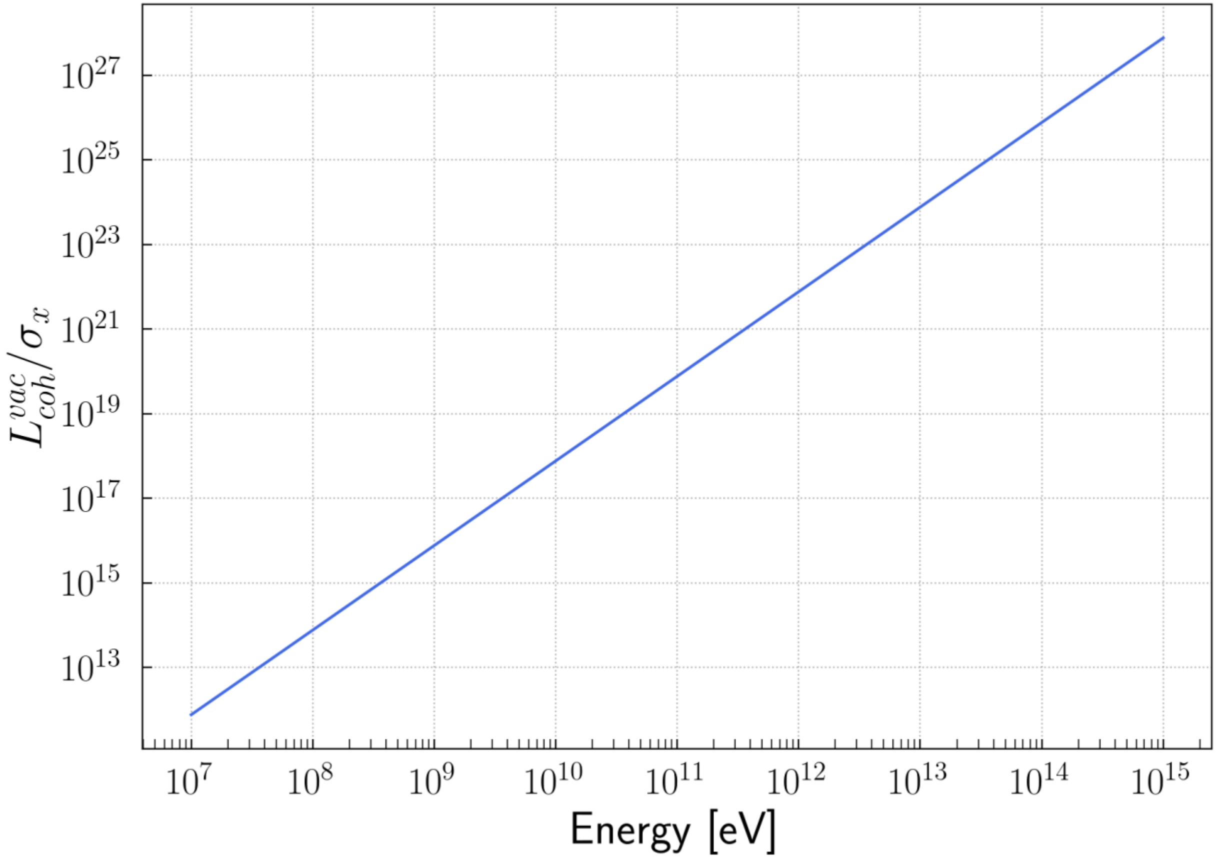
$$L_{coh}^{ii-+} \approx \frac{\sigma_x p^3}{\mu_i B m_i^2}.$$

A.Popov, A.Studenikin, Phys.Rev.D 111 (2025) 12, 123001

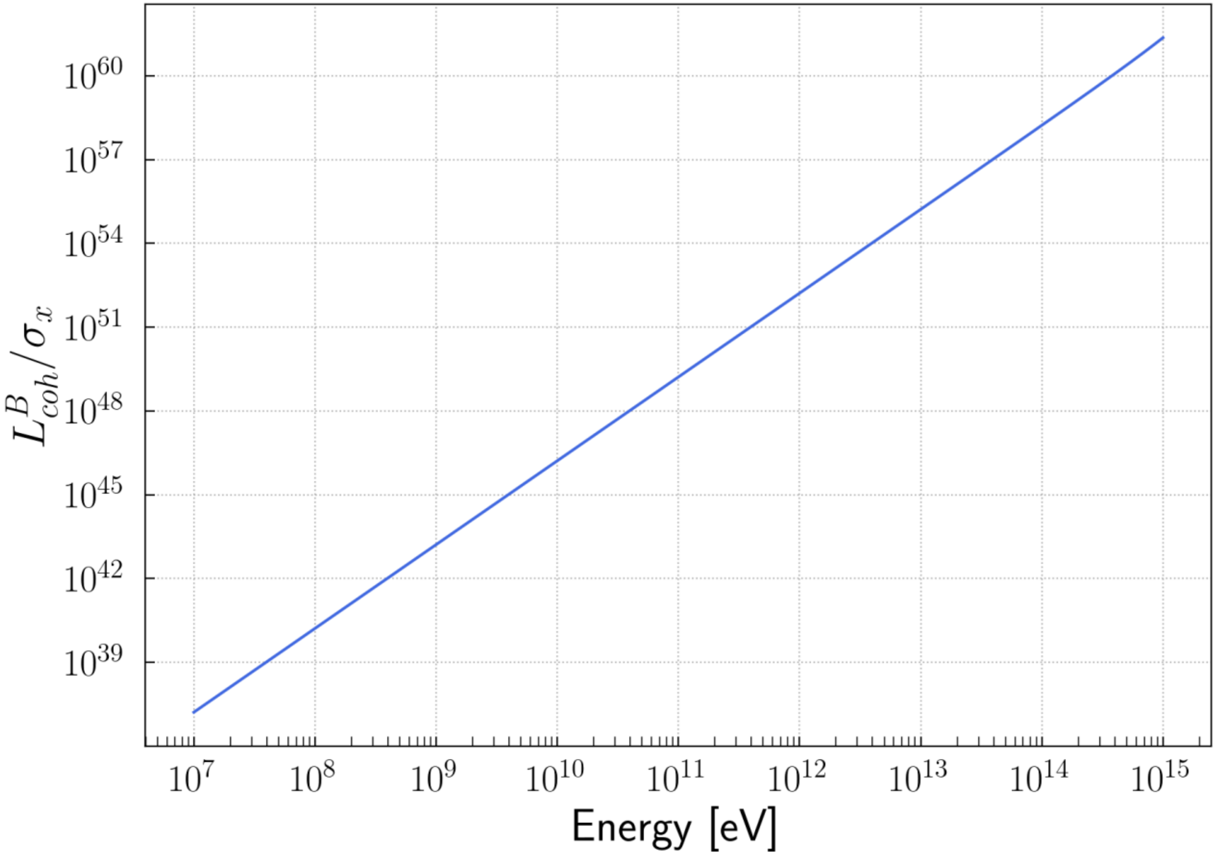
Coherence lengths: Dirac neutrinos

$\sigma_x \sim 10^{-17} \div 10^{-9}$ km for different neutrino production mechanisms.

$L_{coh} \sim E^2$



$L_{coh} \sim E^3$



$$P_{\alpha\beta}(L)\Big|_{L \gg L_{coh}^{vac}} = \sum_{i=1}^{3} |U_{\alpha i}|^2 |U_{\beta i}|^2 \cos^2(\pi L / L_i^B)$$ - **partial loss of coherence** is expected for Dirac neutrinos.

Adiabatic solution: Majorana neutrinos

- For illustrative purposes, we present the analytical solution for the case of two neutrino flavours. In the realistic three neutrino case the evolution equation is solved numerically.

$$P(\nu_e \to \nu_\mu; L) = \frac{\omega_{vac}^2}{2(\omega_{vac}^2 + \omega_B^2)} \sin^2 2\theta \left[1 + \cos\left(\frac{2\pi L}{L_{osc}}\right) \exp\left(-\frac{L^2}{L_{coh}^2}\right)\right],$$

$$P(\nu_e \to \bar{\nu}_\mu; L) = \frac{\omega_B^2}{2(\omega_{vac}^2 + \omega_B^2)} \left[1 + \cos\left(\frac{2\pi L}{L_{osc}}\right) \exp\left(-\frac{L^2}{L_{coh}^2}\right)\right],$$

$$P(\nu_e \to \bar{\nu}_e; L) = 0,$$

$$L_{osc} = \frac{2\pi}{\sqrt{\omega_B^2 + \omega_{vac}^2}} \qquad \omega_{vac} = \Delta m^2/4p_0 \qquad \omega_B = \mu B_\perp$$

- In the Majorana case, interaction with strong magnetic field induce **neutrino-antineutrino** transitions.
- Flavour transitions disappear in the strong field limit.

Coherence length: Majorana neutrinos

$$L_{coh} \approx \frac{4\sqrt{2}\sigma_x p_0^2}{\Delta m^2} \quad \text{if } \omega_{vac} \gg \omega_B,$$

$$L_{coh} \approx \frac{2\sqrt{2}\sigma_x \omega_B p_0^3}{(\Delta m^2)^2} \quad \text{if } \omega_{vac} \ll \omega_B$$

$$\omega_{vac} = \Delta m^2/4p_0$$

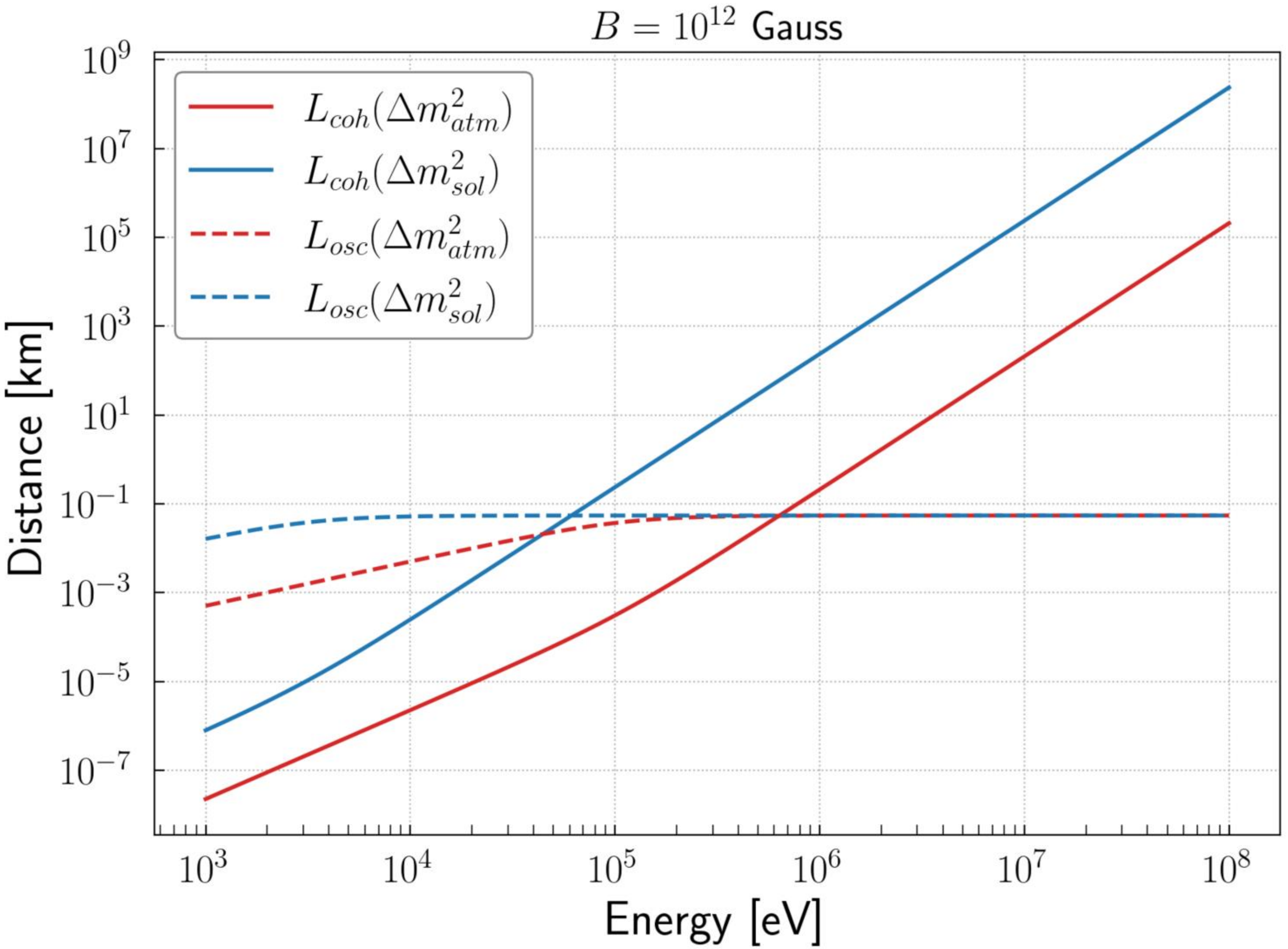
$$\omega_B = \mu B_\perp$$

- In the weak field limit the coherence length coincides with the vacuum coherence length.
- For the case of strong magnetic field, $L_{coh} \sim p_0^3$.
- This pattern remains intact for the realistic three neutrino case, which is confirmed by the numerical solution.
- The coherence length for Majorana neutrinos in the strong field limit is *much smaller* than the coherence length for oscillations on magnetic frequencies of Dirac neutrinos:

$$\frac{L_{coh}^M}{L_{coh}^D} \sim \frac{m^2}{\Delta m^2} \cdot \frac{(\mu B)^2}{\Delta m^2} \ll 1 \quad , \text{if } \omega_{vac} \ll \omega_B$$

A.Popov, A.Studenikin, A.Tcvirov, arXiv:2602.02804

Coherence length: Majorana neutrinos



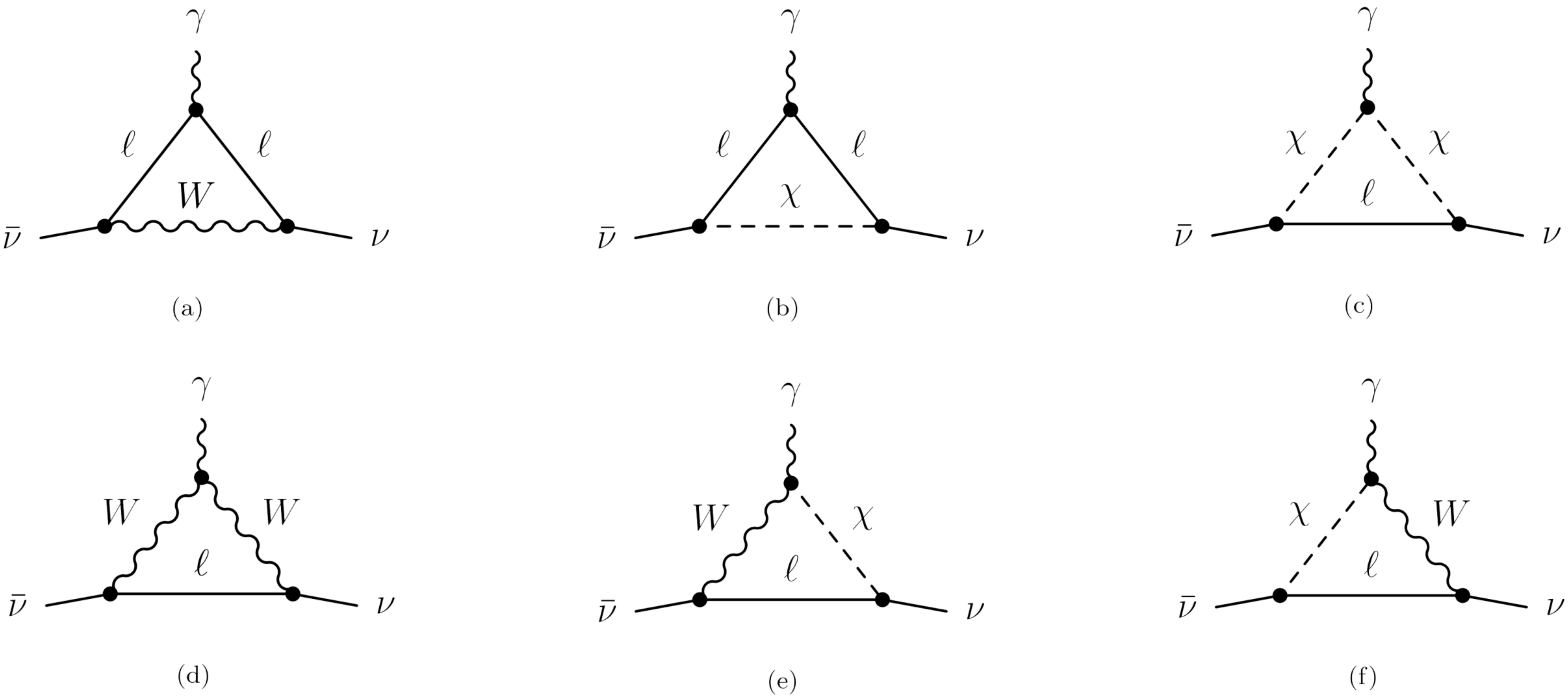
- $B = 10^{12}$ Gauss
- $\mu = 10^{-12}\mu_B$
- $\sigma_x = 10^{-12}$ cm
- For supernova neutrinos $E \sim 10$ MeV

Conclusions

- Neutrino oscillations in a magnetic field are considered accounting for decoherence effects due to the wave packets separation.
- The expressions for the coherence lengths are obtained for Dirac and Majorana neutrinos.
- Loss of coherence is expected for supernova neutrino energies.
- In the Dirac neutrinos case, partial decoherence appears, i.e. the decoherence between massive, but not helicity states, leading to partial damping of the neutrino flavour transitions.
- For the Majorana case, flavour transitions may disappear completely.

Backup

Neutrino magnetic moments



M.Dvornikov, A.Studenikin, *"Electric charge and magnetic moment of massive neutrino"*, Phys.Rev.D. (2004)

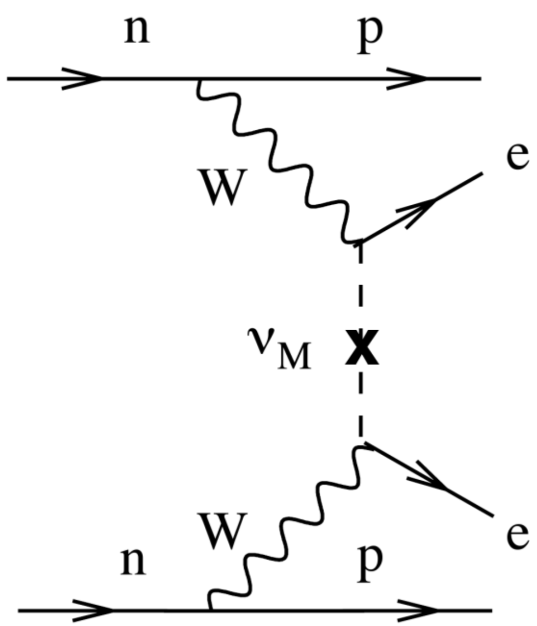
Majorana neutrinos mixing

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ik} = \cos\theta_{ik}$$
$$s_{ik} = \sin\theta_{ik}$$

Dirac CP-violating phase

Majorana CP-violating phases



- **Dirac CP-violating phase** can be measured by oscillatory experiments.
- Neutrinoless double beta decay experiments are potentially sensitive to the values of **Majorana CP-violating phases.**
- Majorana phases can be potentially probed by e/m properties (this talk).

G.C. Branco, R.Gonzalez Felipe, F.R. Joaquim, Rev.Mod.Phys. 84 (2012) 515-565

Neutrino interaction with supernova matter

$$\mathcal{L}^{M}_{mat} = -\sum_{\alpha} V^{(f)}_{\alpha} \left[\overline{\nu^L_\alpha} \gamma_0 \nu^L_\alpha - \overline{(\nu^L_\alpha)^c} \gamma_0 (\nu^L_\alpha)^c \right]$$

$$\mathcal{L}^{D}_{mat} = -\sum_{\alpha} V^{(f)}_{\alpha} \overline{\nu^L_\alpha} \gamma_0 \nu^L_\alpha$$

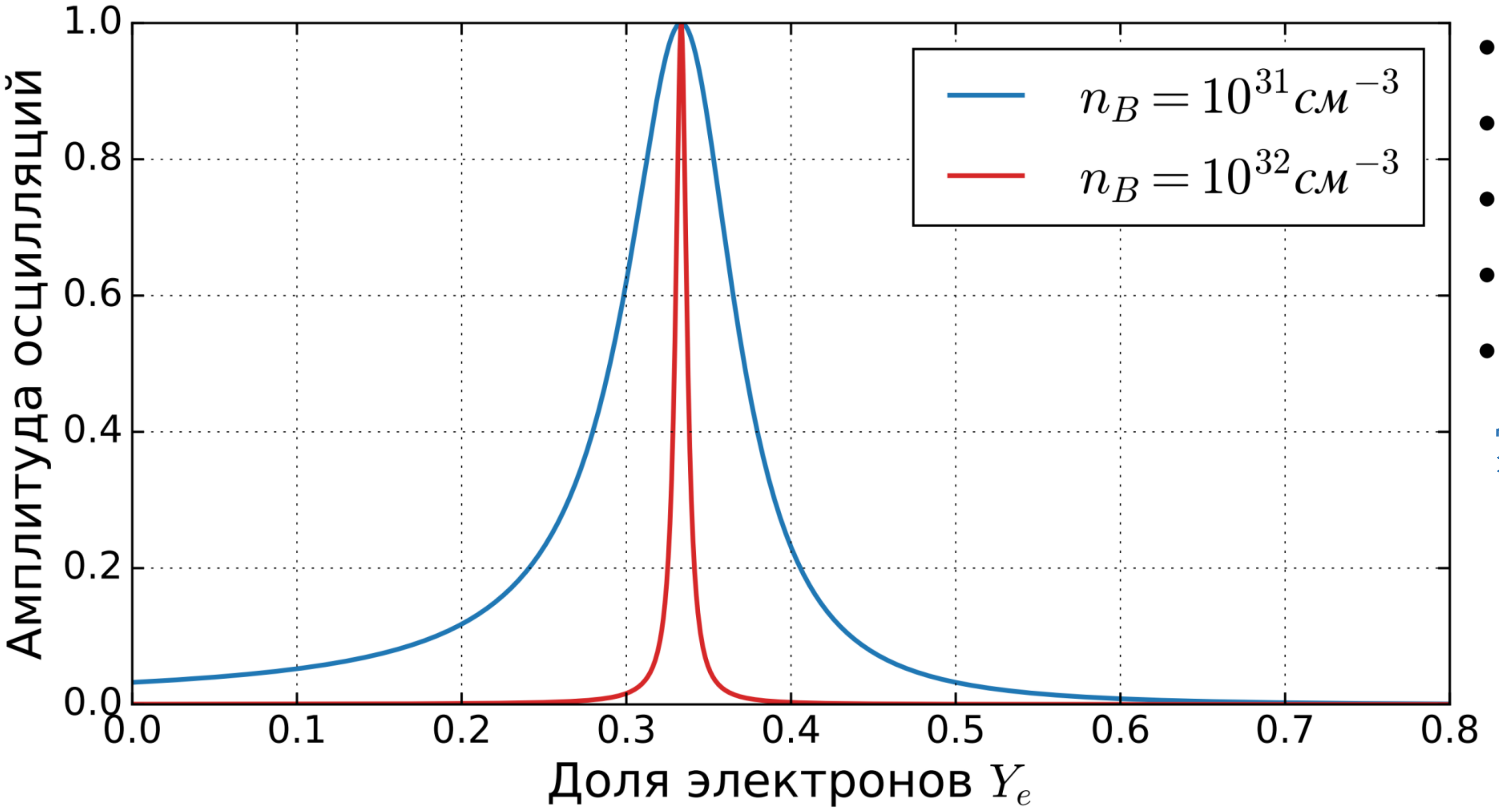
$$\alpha, \beta = e, \mu, \tau$$

$$V^{(f)} = \mathrm{diag}\left(\frac{G_F n_e}{\sqrt{2}} - \frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}} \right)$$

Wolfenstein potential

n_n, n_e are neutron and electron number densities of supernova environment

(1) Резонансное усиление осцилляций дираковских нейтрино



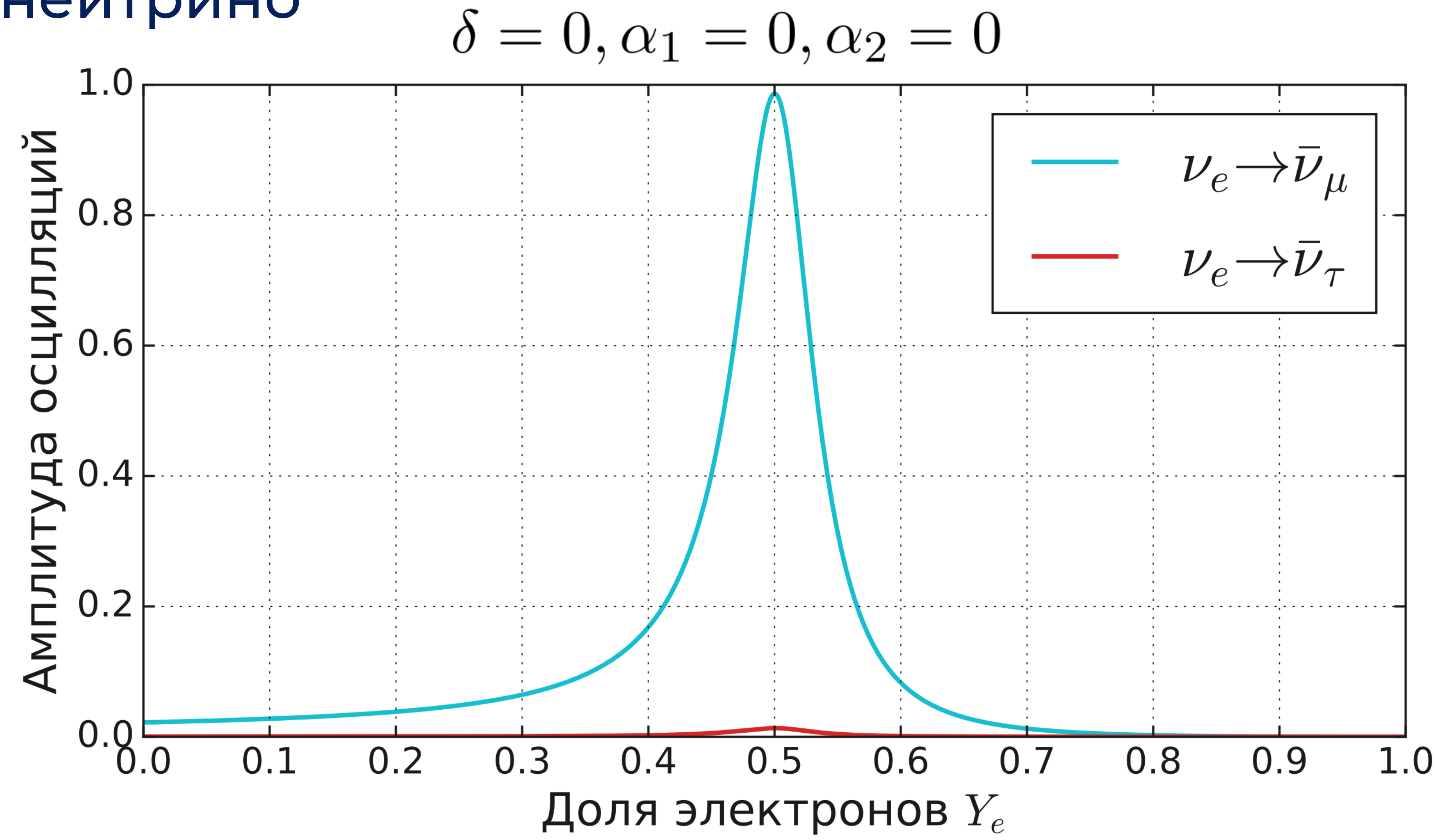
- $|\mu_{12}| = |\mu_{13}| = |\mu_{23}| = 10^{-12}\mu_B$
- $n_n = 10^{31}$ cm^{-3}
- $B = 10^{12}$ G
- $E = 10$ MeV
- $Y_e = n_e/n_B$, $n_B = n_n + n_p$

Доля электронов — Плотность барионов

При $Y_e = 1/3$ наблюдается резонансное усиление переходов $\nu_e^L \to \nu_e^R$

[1] М.Б. Волошин, М.И. Высоцкий, Л.Б. Окунь, ЖЭТФ 91, 754 (1986).
[2] E.Akhmedov, Phys. Lett. B 213, 64 (1988).
[3] C.-S.Lim, W.Marciano, , Phys.Rev.D37 (1988) 1368.

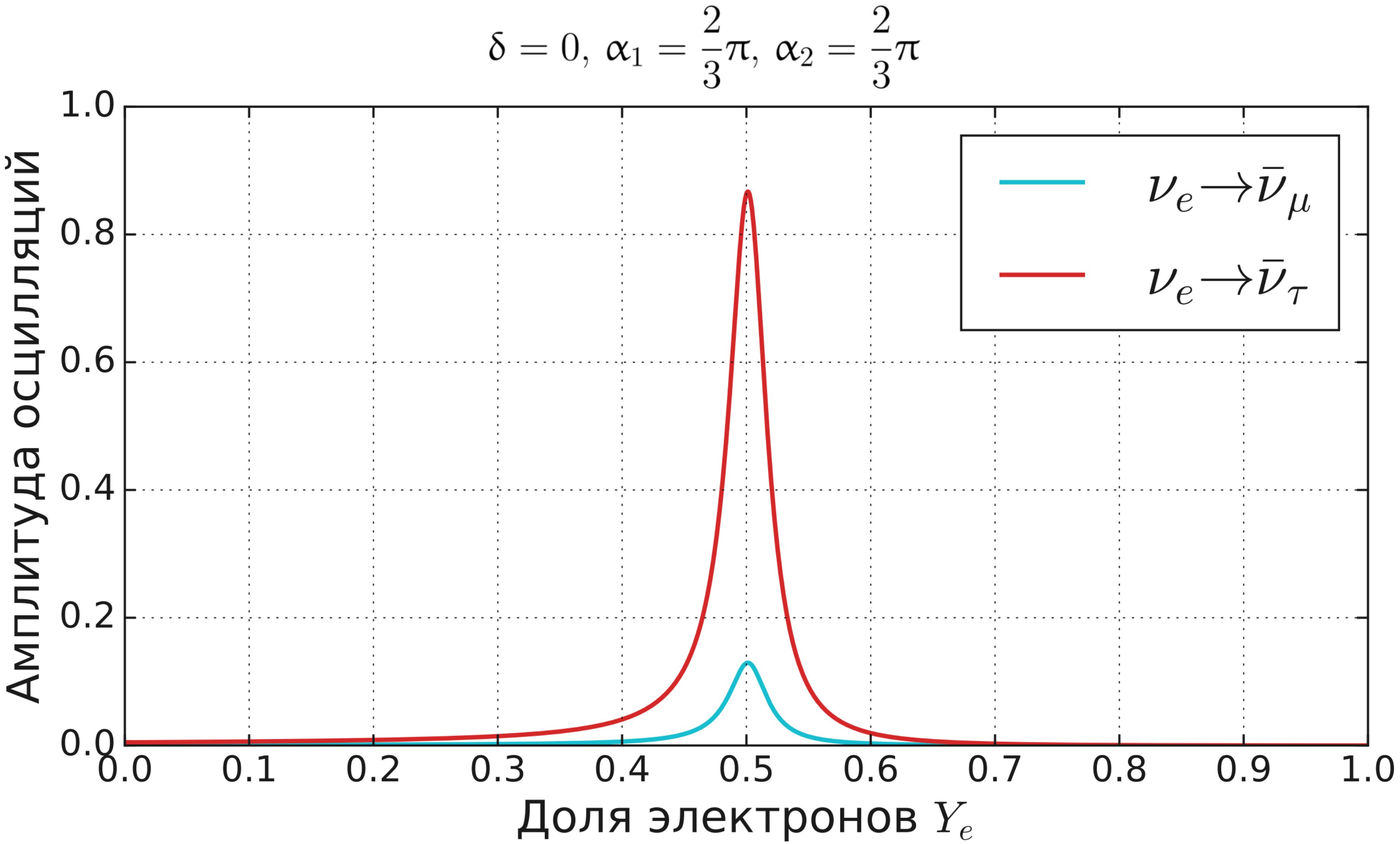
(2) Резонансное усиление осцилляций майорановских нейтрино



При $Y_e = 1/2$ наблюдается резонансное усиление переходов $\nu_e \to \overline{\nu_\mu}$

[1] E.Akhmedov, *"Resonant amplification of neutrino spin rotation in matter and the solar-neutrino problem"*, Phys. Lett. B 213, 64 (1988);
[2] C.-S.Lim, W.Marciano, *"Resonant spin-flavour precession of solar and supernova neutrinos"*, Phys.Rev.D37 (1988) 1368.

(2) Резонансное усиление осцилляций майорановских нейтрино



При $Y_e = 1/2$ наблюдается резонансное усиление переходов $\nu_e \to \bar{\nu}_\tau$

A.Popov, A.Studenikin, *"Manifestations of nonzero Majorana CP-violating phases in oscillations of supernova neutrinos"*, Phys.Rev.D 103 (2021) 11, 115027.