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Electromagnetic effects of gravitational shockwaves

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Gravitational geometric optics

• The analyses of Maxwell and Einstein equations shows that the basic concepts of geometric optics are common to gravitational and electromagnetic fields.

As in classical Maxwell's ED, the law of propagation of gravitational waves is determined by the eikonal equation

$$g^{\alpha\beta}\psi_{,\alpha}\psi_{,\beta} = 0 \tag{1}$$

The scalar function $\psi(x^\mu)$, the eikonal, determines the hypersurface of the gravitational wave front, $\psi(x^\mu) = 0$, and the trajectories of its propagation with the wave vector $k_\alpha = -\psi_{,\alpha}$.

Different solutions of the eikonal equation correspond to different types of wave fronts and types of gravitational waves. A distinction is made between plane and spherical gravitational waves.

In what follows I will discuss mainly plane-fronted waves.

Plane-fronted gravitational waves (pp-waves)

J. Ehler, W. Kundt'1962

For a pp-wave moving the x direction, from the left to the right, the space-time metric introduced by Ehlers and Kundt is

$$ds^2 = -dvdu - H(v, u, y)du^2 + dy_i^2 \,. \tag{2}$$

With null normals to constant u hypersurfaces, $l_\mu = -\delta^u_\mu$, the metric tensor can be written as $g_{\mu\nu} = \eta_{\mu\nu} - Hl_\mu l_\nu$, where $\eta_{\mu\nu}$ is the Minkowsky metric.

The properties of the wave are described by $H(v, u, y) = \chi(u)f(v, y)$, where $f(v, y)$ is called the 'profile' function. Evolution of the shock in the retarded time is determined by the 'signal' function $\chi(u)$. The integrated signal function

$$\bar{\chi}(u) = \int_{-\infty}^{u} \chi(u')du' \,. \tag{3}$$

has a normalization condition $\bar{\chi}(u) \to 1$ at $u \to \infty$.

Sandwich waves and impulsive limit

Let the signal function $\chi(u) \neq 0$ and arbitrary in some time interval, $0 < u < \delta$. At $u < 0$, $u > \delta$ the pp-wave metric coincides with the Minkowsky metric.

- Non-vanishing components of Einstein tensor are linear in $H(v, u, y) = \chi(u) f(v, y)$,

$$G_{uu} = \frac{1}{2}\partial_i^2 H\,, \quad G_{ui} = \partial_v \partial_i H\,, \quad G_{ij} = 2\delta_{ij}\partial_v^2 H\,. \tag{4}$$

Therefore $G_{\mu\nu}$ has a well-defined limit of extremely short signals, when $\chi(u) \to \delta(u)$ (impulsive limit).

- pp-waves are solutions of Einstein equations produced by sources with the stress-energy tensor

$$T_{\mu\nu} = \chi(u)\left(\sigma l_\mu l_\nu + j_i(l_\mu e_\nu^i + l_\nu e_\mu^i) + p\delta_{ij} e_\mu^i e_\nu^j\right)\,, \tag{5}$$

where

$$\sigma = f_{,ii}/\kappa\,, \quad j_i = -2f_{,vi}/\kappa\,, \quad p = 4f_{,vv}/\kappa\,, \quad \kappa = 16\pi G\,. \tag{6}$$

Vectors $l = \partial_v$, $e^i = \partial_i$ are tangent to the wave front $\mathcal{N}$ (null hypersurface of constant u).

Shock Gravitational Waves from extended sources

C.O. Luostó, N. Sanches, Nucl. Phys. B355 (1991), 231-249

- Shock Gravitational waves are solutions of Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}, \quad T_{\mu\nu} \sim \delta(u).$$

- Ultrarelativistic sources ($v \to c, m \to 0$), both point-like and extended, with energy density $\sigma(x_\perp)$, induce spaces with metrics that have the form of a shock gravitational wave which is an impulsive limit of PP waves

$$ds^2 = -du dv + f(y_\perp, v)\delta(u) du^2 + dy_\perp^2. \quad (7)$$

Ultrarelativistic source	$\sigma(y_\perp)$	profile function $f(y_\perp)$
Massless point particle	$E_p \delta(\rho)$	$-8GE_p \ln \rho$
Null straight cosmic string	$E_s \delta(y_1)$	$-8\pi G E_s \|y_1\|$
Domain wall	E_w	$-4\pi G E_w \rho^2$

E_p, E_s, E_w are energies of the sources, $\rho = \sqrt{y_\perp^2}$

- For an arbitrary profile function $f(y, v)$ we study the effects in linear with respect to f approximation.

Example: SGW of a Massless Particle

G. 't Hooft, PLB 198 (1987) 61-63; T. Dray, G. 't Hooft, NPB 253 (1985) 173-188

"The surprisingly simple geometric shape of a gravitational shock wave of massless particles in flat space can help us obtain a better understanding of gravitational interactions among particles at extreme energies. ...at extremely high energies interactions due to this shock wave will dominate over all quantum field theoretic interactions, ...".

"The scattering process of two pointlike particles at CM energies in the order of Planck units or beyond, is very well calculable using known laws of physics, because graviton exchange dominates over all other interaction processes. ",

$$S(k,p,E) \sim \delta(k_v - p_v)\frac{\Gamma(1-iB)}{\Gamma(iB)}t^{-1+iB}\frac{e^{\pi B}}{\cosh \pi B}$$

with

$$B = -2\pi G(s+m_1^2), \quad s=(k_1+k_2)^2, \quad t=(k_i-p_i)^2, \quad s+m_1^2 = -4k_vE$$

- Coinsides with Veneziano amplitude, obtained in the eikonal approximation of string theory . Classical scattering on SGW reproduces contribution of all ladder diagrams of graviton exchanges in quantum amplitudes. /in R. J. Eden, P. V. Landshoff, D. I. Olive et all., "The Analytic S-Matrix"/

Example: Null String

A. Schild. Classical Null Strings. Phys. Rev. D, 16:1722, 1977

A null cosmic string is a one-dimensional object whose points move along trajectories of light rays, orthogonally to the string itself (Schild equations).

From a massive cosmic string at rest along z-axis

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\Theta^2, \quad r^2 = x^2 + y^2$$

$\longrightarrow$ Aichelburg-Sexl boost

$$\cosh\chi = (1 - v^2/c^2)^{-1/2} \to \infty, \quad E = mc^2 \cosh\chi \to finite$$

$\longrightarrow$ Kerr-Schield metric

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi G E_s$$

ε - energy per unit length, $u = t - x$, $v = t + x$.

C. Barrabes, P.A. Hogan, W. Israel, Aichelburg-Sexl boost of domain walls and cosmic strings, Phys.Rev. D66 (2002) 025032.

Example: null brane (ultrarelativistic domain wall)

A.A.Zheltukhin. Yad. Phys. 48 (1988) 587-595

Multidimensional analogue of massless particle is a null brane with a action

$$S_M = \int d\tau d\sigma_1 \ldots d\sigma_d \, \frac{\det(\partial_\mu \vec{x} \cdot \partial_\nu \vec{x})}{2E}, \quad \mu, \nu = 0, 1 \ldots d$$

By choosing a suitable gauge, the equations of motion for null-branes can be linearized (for ordinary membranes, this is not the case).

- The simplest metric of space with an ultrarelativistic domain wall (null brane) in the plane (y, z)

$$ds^2 = -dudv - 4\pi G E_w (z^2 + y^2) \delta(u) du^2 + dy^2 + dz^2.$$

- Scattering amplitude C.O. Luostó, N. Sanches'1991: $S = \frac{2i}{\pi G s} \exp\left[\frac{2it}{Gs}\right]$

Null shells in GR

C. Barrabes, W. Israel, PRD 43 (1991) 1129-1142; E. Poisson, gr-qc/0207101 (2002)

• According to Barrabes-Israel formalism for null shells, the shock wave profile function (supertranslation function) f defines physical characteristics of a shock wave: surface energy σ, surface current j_i, isotropic surface pressure p. In this terms the stress-energy of the source reads

$$T_{\mu\nu} = \delta(u)\left(\sigma l_\mu l_\nu + j_i(l_\mu e^i_\nu + l_\nu e^i_\mu) + p\,\delta_{ij} e^i_\mu e^j_\nu\right) \quad , \tag{8}$$

where $l_\mu = -\delta^u_\mu$, $e^i_\mu = \delta^i_\mu$ are tangent vectors to a wave front $\mathcal{N}$.

• To satisfy at least the weak energy condition $T_{\mu\nu}u^\mu u^\nu \geq 0$ for any time-like vector u, necessary conditions are

$$p\sigma - j^2 \geq 0 \quad , \quad p > 0. \tag{9}$$

Surface continuity relations hold $\partial_v \sigma = -\partial^i j_i \quad , \quad \partial_v j_i = -\partial_i p \quad .$

Gluing (soldering) and characteristic problem

R.Penrose. The Geometry of Impulsive Gravitational Waves, in General relativity: Papers in honour of J.L. Synge, L. O'Raifeartaigh, ed. (1972), pp. 101–115.

In what follows we do not calculate the impulsive limit of pp-waves to obtain shockwaves with arbitrary profiles.

The procedure to construct a space-time which is locally Minkowski $R^{1,3}$ except a null hypersurface where the Riemann tensor has singular structure (shock wave) is based on the Penrose cut-and-glue approach [Penrose'1972] or its generalization called soldering [M.Blau, M. O'Loughlin'2016].
One cuts $R^{1,3}$ by a null hypersurface $u = 0$, denoted by $\mathcal{N}$, into two 'halfs', $\mathcal{M}^+$ $(u > 0)$ and $\mathcal{M}^-$ $(u < 0)$. Then $\mathcal{M}^\pm$ are "glued" again along $\mathcal{N}$ with the following identification of points:

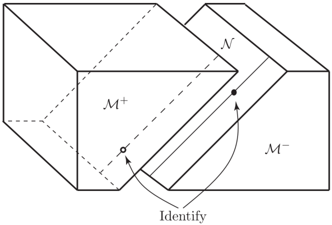
$$v_+ = v - f(v, y) \quad , \quad y^i_+ = y^i \quad . \tag{10}$$

Transformations (10) are called supertranslations.

They leave invariant metric on $\mathcal{N}$ and, thus, make an infinite dimensional group of isometries of $\mathcal{N}$, also known as Carroll transformations

“Identify in warped fashion by shunting along null generators.”(R. Penrose)

- A flat space $R_{1,3}$, $ds^2 = -du\,dv + (dx^i)^2$, is cut by null hypersurface $\mathcal{N}$, $u = 0$, which is interpreted as the front of a plane shock wave, $R_{1,3} \to R^+_{1,3} \bigcup R^-_{1,3}$, and glued back after supertranslation in $R^+_{1,3}$ with a profile function f.



Jumps of first derivatives of the metric on SW front

The behaviour of the Ricci tensor is related to jumps of first derivatives of the metric on the fronts of shockwaves. With the help of a null vector n at $\mathcal{N}$, such as $(n \cdot l) = -1$, $(n \cdot e_i) = 0$ one can define the transverse curvature

$$\mathcal{C}_{ab} = n_{\mu;\nu} e^{\mu}_{a} e^{\nu}_{a} \quad , \tag{11}$$

where $e_v = l, e_i = e^i$. Then

$$\sigma = -\frac{1}{8\pi}[\mathcal{C}_{ii}] \quad , \quad j_i = \frac{1}{8\pi}[\mathcal{C}_{iv}] \quad , \quad p = -\frac{1}{8\pi}[\mathcal{C}_{vv}] \quad , \tag{12}$$

$$[\mathcal{C}_{ab}] \equiv \mathcal{C}^{+}_{ab} - \mathcal{C}^{-}_{ab} \quad , \tag{13}$$

where $\mathcal{C}^{\pm}_{ab}$ correspond to $\mathcal{M}^{\pm}$,

$$[\mathcal{C}_{ab}] = \frac{1}{2}(\partial_u g_{ab}^{\ +} - \partial_a g_{ub}^{\ +} - \partial_b g_{ua}^{\ +}) \quad .$$

Soldering transformations

Coordinate transformations of $\mathcal{M}^+$ in a neighborhood of $\mathcal{N}$ are

$$x^\mu_+ = x^\mu - \zeta^\mu(x) \quad . \tag{14}$$

Since the transformation should not change the metric on $\mathcal{N}$ we require

$$\mathcal{L}_\zeta g^-_{\mu\nu}|_{u=0} = (\zeta_{\mu,\nu} + \zeta_{\nu,\mu})|_{u=0} = 0 \quad , \tag{15}$$

which yields:

$$\zeta^\mu(x) \simeq f\delta^\mu_v + \frac{u}{2}\eta^{\mu a} f_{,a} \quad , \tag{16}$$

$$u^+ = u(1+f_{,v}), \quad v^+ = v - f, \quad y^+_i = y_i - uf_{,i}/2 \quad . \tag{17}$$

In vicinity of $\mathcal{N}$ deviation of the metric from the flat one is

$$g^+_{\mu\nu} \simeq g^-_{\mu\nu} + \mathcal{L}_\zeta g^-_{\mu\nu} = \eta_{\mu\nu} + \zeta_{\mu,\nu} + \zeta_{\nu,\mu} \quad , \tag{18}$$

therefore

$$\partial_u g_{vv}^+(u,\mathbf{x})|_{u=0} = f_{,vv} \quad , \quad \partial_u g_{vi}^+(u,\mathbf{x})|_{u=0} = f_{,iv} \quad , \quad \partial_u g_{ij}^+(u,\mathbf{x})|_{u=0} = f_{,ij} \quad , \tag{19}$$

here $\mathbf{x} = v, y^i$. We call (17) soldering transformations.

Soldering transformations and Carroll transformations

One can introduce on $\mathcal{N}$ vector fields $V = V^a \partial_a$ and one-forms $\theta = \theta_a dx^a$, where $x^a = (v, y^i)$ [index a cannot be risen or lowered since the metric of $\mathcal{N}$ is degenerate]. The Carroll symmetries (supertranslations) are generated by the vector field on $\mathcal{N}$

$$\bar{\zeta}^a(\mathbf{x}) = f(\mathbf{x})\delta^a_v \quad , \quad \mathbf{x} = v, y^i \quad . \tag{20}$$

The supertranslations induce transformations of components of θ_a, V^a and make different representations of the Carroll group. They leave invariant forms like $\theta_a V^a$. One can also define direct products of the representations by considering tensor structures on $\mathcal{N}$ with arbitrary numbers of upper and lower indices $a, b, ...c$.

Soldering transformations map tangent (cotangent) spaces of $\mathcal{N}$ to themselves. Indeed, transformations of 4-dimensional one-forms θ_μ and vectors V^μ on $\mathcal{M}^+$ are determined by the Lie derivatives,

$$\mathcal{L}_\zeta V^\mu = \zeta^\nu \partial_\nu V^\mu - V^\nu \partial_\nu \zeta^\mu \quad , \tag{21}$$

$$\mathcal{L}_\zeta \theta_\mu = \zeta^\nu \partial_\nu \theta_\mu + \theta_\nu \partial_\mu \zeta^\nu \quad . \tag{22}$$

Interaction classical EM field with a SGW

- The solutions to the Maxwell equations in Minkowsky space-time,

$$\partial_\mu F^{\mu\nu} = j^\nu \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad \partial j = 0 \tag{23}$$

are determined by initial data at space-like hypersurface $t = 0$:

$$A_i \mid_{t=0} = a_i \quad , \quad F_{0i} \mid_{t=0} = \pi_i \quad , \quad i = x, y, z \quad .$$

- In a spacetime with a shock gravitational wave the initial data is set on shockwave fronts, which are null hypersurfaces $\mathcal{N}$ $(u = 0)$, and one encounters a characteristic Cauchy problem /R.K. Sachs'1962/

There are two types of EM problems to consider in a spacetime with a SGW :
(a) Scattering of EM wave by an extended ultra-relativistic object (null particle, null string or null brane), $j = 0$.
Here we expect different effects such as refraction, lensing, and Dopler effect /D.Fursaev, I.P., Phys.Rev.D 107 (2023), 0250183/

(b) Perturbations of EM source field by plane fronted shock gravitational wave:
They result at a flux at future null infinity, plane and spherical secondary shock waves in the field system itself / D.V. Fursaev, E.A. Davydov, I.P., V.A. Tainov, JHEP 2024 (11), 1-29

(a) Refraction of EM waves on a plane GSW front

Example: Consider monochromatic plane wave scatterring by a null string ($j = 0$).

Before scattering on the string (in the region $u < 0$) it is :

$$\bar{A}_\mu(\bar{x}) = \Re\,(\bar{E}_\mu\ e^{i\bar{k}\cdot\bar{x}}) \quad , \tag{24}$$

where $\bar{E}_\mu$ is some complex polarization vector, $\bar{k}^\mu \bar{E}_\mu = 0$. Bar denotes the incoming data. Other types of electromagnetic waves can be treated as a superposition of plane monochromatic waves.

The plane wave is refracted by the null string horizon. It leaves $\mathcal{H}$ with the transformed momentum and polarization

$$k^\mu = M^\mu_{\ \nu}(\omega)\,\bar{k}^\nu \quad , \quad E_\mu = M_\mu^{\ \nu}(\omega)\bar{E}_\nu . \tag{25}$$

If E and $\vec{k}$ are, respectively, the energy and the momentum of the incoming wave, the refraction angle $\varphi_{\rm refr}$ and the energy of the refracted wave are

$$\cos\varphi_{\rm refr} = \frac{(\vec{k}_-\vec{k})}{E_-E} = \frac{1}{EE_-}\left[E^2 + \frac{\omega^2}{2}(Ek^x - (k^x)^2) + \omega E k^y\right] \tag{26}$$

$$E_- = \left(1 + \frac{\omega^2}{2}\right)E - \frac{\omega^2}{2}k^x + \omega k^y \quad . \tag{27}$$

The refraction is absent only for the waves traveling along the string axis z when $k^x = k^y = 0$.

Interference wedge

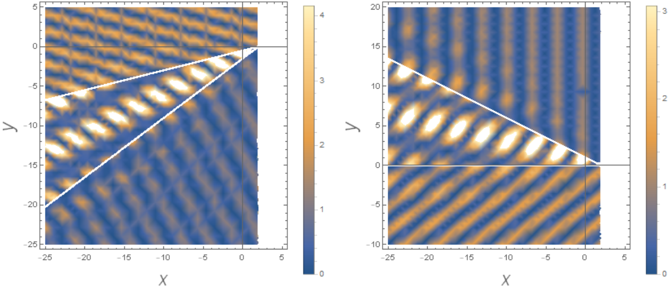


Fig. The energy density of a real scalar field is shown for the string at the moment t_0. The string is stretched along the z axis, orthogonal to plane of the Fig., and is located at $x = t_0 = 2$, $y = 0$; $\omega = 1/2$. For the left fig.: $k_v > 0$, $k_{-y} = k_y - 2\omega k_v > 0$. For the right fig.: $k_y = 0$, $\cos\phi_{int} = (1+\omega^2)^{-1/2}$.

The angle of the interference wedge

$$\cos\varphi_{\rm intf} = \frac{k_y(k_y - 2\omega k_v) + 4k_v^2}{(k_y^2 + 4k_v^2)^{1/2}((k_y - 2\omega k_v)^2 + 4k_v^2)^{1/2}} \quad . \tag{28}$$

$\varphi_{\rm intf} = O(\omega^2)$ at small ω.

(b) Perturbations of EM source field by plane fronted shock gravitational wave

Let $\bar{A}_\mu$ be a solution to the problem $\partial_\mu \bar{F}^{\mu\nu} = \bar{j}^\nu$ at $u < 0$.

We solve $\partial_\mu F^{\mu\nu} = j^\nu$ at $u > 0$ with initial conditions on null hypersurface $\mathcal{H}$.

The source of the electric type is an electric charge with a current

$$j_0 = e\delta^{(3)}(\vec{x} - \vec{x}_o) \quad , \quad j_i(x) = 0 \quad i = 1, 2, 3 \ .$$

In the absence of the SGW it has a field

$$\bar{A}_0(x) = e\phi(x), \qquad \bar{A}_i(x) = 0, \phi(x) = -\frac{1}{4\pi}\frac{1}{\sqrt{x^2 + (y-a)^2 + z^2}}.$$

The sources of the magnetic type are described by the current

$$j_0 = 0, \quad j_i(x) = \varepsilon_{ijk} M_j \, \partial_k \delta^{(3)}(\vec{x} - \vec{x}_o) \ .$$

where M_i is a magnetic moment. In the absence of a SGW the field is

$$\bar{A}_0(x) = 0, \quad \bar{A}_i(x) = \varepsilon_{ijk} M_j \partial_k \phi. \quad (\star)$$

The solution

One can write a solution in the form

$$A_\mu(x) = \bar{A}_\mu(x) + A_{S,\mu}(x) \ ,$$

$\bar{A}_\mu$ as a solution in the absence of the SGW, $-\infty < u < \infty$,
$A_{S,\mu}$ is a perturbation caused by the GSW and it is a solution to a homogeneous characteristic initial value problem

$$\partial_\mu F_S^{\mu\nu} = 0 \quad , \quad A_{S,b}(x) \mid_{\mathcal{H}} = a_{S,b}(\mathbf{x}) \quad ,$$

where $F_{S,\mu\nu} = \partial_\mu A_{S,\nu} - \partial_\nu A_{S,\mu}$.

Properties of the perturbation $A_{S,\mu}$:
i) vanishes in the limit $\omega \to 0$, where ω is parameter of the supertranslation ;
ii) depends on the choice of the source j_μ through initial data ;
ii) A geometrical interpretation of the perturbation is that at small ω it is the Lie derivative,

$$A_S = \mathcal{L}_\zeta \mathcal{A} \quad ,$$

generated by vector $\zeta^\mu(x) \simeq f\delta_v^\mu + \frac{u}{2} f_a \eta^{\mu a}$ associated to soldering transformations, which at $u = 0$ conside with Penrose supertranslations.

Example: perturbation of a Coloumb field caused by a null string

$$A_{S,v}(x) = \frac{1}{2}(\Phi_\omega(x) - \Phi(x)) \quad , \quad A_{S,y}(x) = -\omega\Phi_\omega(x) \quad , \quad A_{S,z}(x) = 0 \quad ,$$

where $\Phi(x) = \Phi_{\omega=0}(x)$. Component $A_{S,u}$ is determined by the gauge condition $\partial A_S = 0$.

$$\Phi_\omega(x) = -\frac{1}{8\pi^3}\int\limits_{S^2} d\Omega' \; \Re\left(\frac{\tilde{\Phi}_\omega(\Omega')}{x^\mu\, m_\mu + ia\varepsilon}\right), \quad \tilde{\Phi}_\omega(\Omega') \equiv \frac{\cos\varphi'}{g(\Omega',\omega)}.$$

$g(\Omega',\omega) = e^{i\theta'} + \omega\sin\theta'\cos\varphi'$,

$\varepsilon = 2\sin^2\theta'\cos\varphi'$.

The vector m_μ is null, $m^2 = 0$,

$m_u = 1 - \sin^2\theta'\cos^2\varphi'$,

$m_v = \sin^2\theta'\cos^2\varphi'$, $m_y = \sin 2\theta'\cos\varphi'$,

$m_z = \sin^2\theta'\sin 2\varphi'$.

The solution at $u > 0$ can be represented as

$$A_{S,\mu}(x) = -\frac{e}{8\pi^3}\int_{S^2} d\Omega' \; \Re\left[\frac{\beta_\mu(\Omega')}{x^\nu \; m_\nu(\Omega') + ia\varepsilon(\Omega')}\right],$$

$$\beta_v = -\frac{1}{2}\cos\varphi' \;\; \left(g^{-1}(\Omega',\omega) - g^{-1}(\Omega',0)\right), \quad \beta_u = \frac{m_y\beta_y - 2m_u\beta_v}{2m_v}$$

$$\beta_y = \cos\varphi' \;\omega g^{-1}(\Omega',\omega), \quad \beta_z = 0.$$

The power of the radiation emitted to $\mathscr{I}^+$

$$\partial_t E(R,t) = \int d\Omega\, \gamma^{AB} \dot{a}_A \dot{a}_B = \int d\Omega\, (\dot{a}_\mu \eta^{\mu\nu} \dot{a}_\nu) \quad ,$$

where $\eta_{\mu\nu}$ is the flat metric. The functions $\dot{a}$ are expressed in terms of real and imaginary parts of $\bar{\beta}_\mu$,

$$\dot{a}_\mu = -\frac{e}{2\sqrt{2}\pi^2\sqrt{1-n_x}}\, \frac{\left[U\, \Re\bar{\beta}_\mu + a\bar{\varepsilon}\, \Im\bar{\beta}_\mu\right]}{U^2 + a^2\bar{\varepsilon}^2}.$$

The flux density in the limit of low string energy, $\omega \ll 1$, reads

$$f_E(U,\Omega) \equiv \dot{a}_\mu \eta^{\mu\nu} \dot{a}_\nu = \frac{4n_y \dot{a}_y \dot{a}_v + 4(1+n_x)\dot{a}_v^2 + (1-n_x)\dot{a}_y^2}{1-n_x}.$$

Substituting $\dot{a}_\mu$, one arrives at

$$f_E(U,\Omega) = \frac{e^2\omega^2}{16\pi^4 a^2}\, \frac{1}{[U^2/a^2 + 2(1-n_x) - n_y^2]^2}$$
$$\times \left\{ \frac{[1 - n_x^2 - n_y^2(1-\bar{\varepsilon}^2)]U^2/a^2 + 2n_y(1-n_x-n_y^2)\bar{\varepsilon}^2\, U/a + (1-n_y^2)\bar{\varepsilon}^4}{2(1-n_x) - n_y^2} \right\},$$

where $\bar{\varepsilon}^2 = 2(1-n_x) - n_y^2$, $n_x = \cos\theta$, $n_y = \sin\theta\sin\varphi$.

Conclusion

- We have studied the following binary systems:

(1) Electromagnetic plane wave refracted by SGW front;
(2) EM source perturbed by a GSW of a null particle or null string.

- A planar gravitational shockwave creates two shocks in the field system. The first shock is a planar shockwave accompanying the initial gravitational wave. The second shock is a spherical scalar shockwave which appears when the gravitational wave hits the source. Both shocks are specified by jumps of components of the stress energy tensor of the field which are tangent to the wave fronts. This corresponds to θ-function-like non-analyticity of the background curvature.

- The analysis can be extended from the Maxwell theory to the linearized Einstein gravity .

- New mechanism of Fast Radio Bursts was proposed (talk by E. Davydov)


THANK YOU!