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PRAUTVINAULT, RF

QUARKS - 2026

XXIII INTERNATIONAL SEMINAR ON HIGH-ENERGY PHYSICS

Petrozavodsk , Russia

18-23 May 2026

On Sizes of Particles

(and related subjects)

A. Einstein
Eine neue Bestimmung der Moleküldimensionen. 1905.
A new determination of molecular dimensions. The size of the sugar molecule $C_{12}H_{22}O_{11}$: $r_M = 5 \cdot 10^{-8} cm$

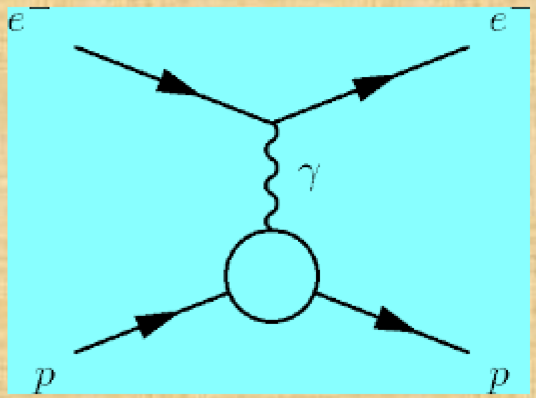
Rutherford (1911): the "distance of closest approach" of alpha particles to the nucleus

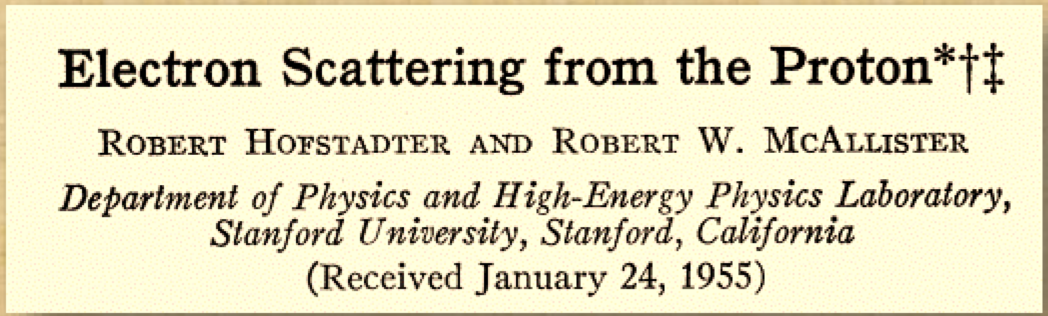
$\boldsymbol{Bohr}$ **(1913):** $\boldsymbol{r_H \leq 0.5 \cdot 10^{-8} cm}$

$\boldsymbol{r_{nucleus} \sim r_{nucleon} \leq 10^{-13} cm}$

1911-1955 : no direct manifestation of the proton size

1955:

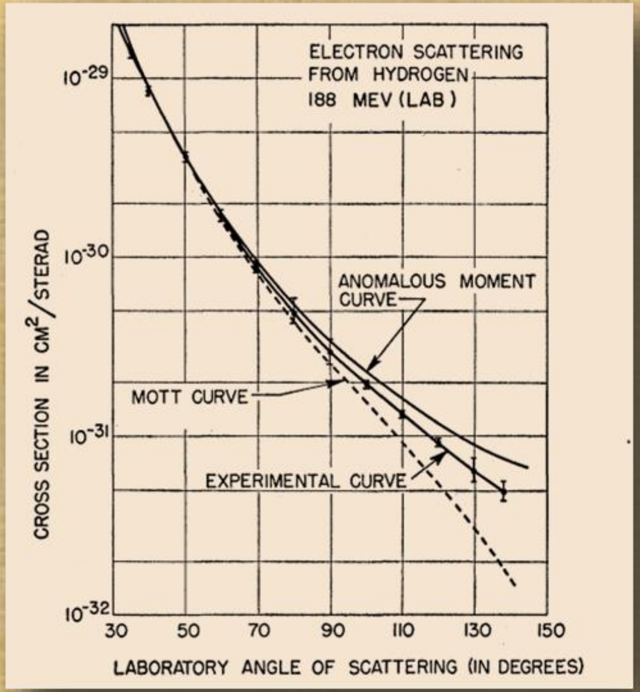




Electron Scattering from the Proton*†‡

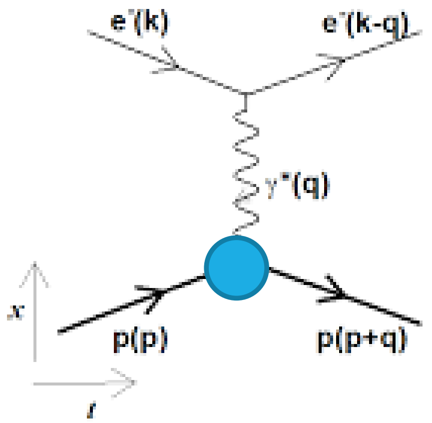
Robert Hofstadter and Robert W. McAllister
Department of Physics and High-Energy Physics Laboratory, Stanford University, Stanford, California

(Received January 24, 1955)



- "If the proton were a spherical ball of charge, this rms radius would indicate a true radius of $\boldsymbol{9.5 \times 10^{-14}}$ **cm...**" **(0.95 fm)**

Generally accepted definition of the proton charge radius



$$\langle p+q | J_\mu(0) | p \rangle = (2p_\mu + q_\mu) G(t)$$

$$\boldsymbol{G(t), t = q^2}$$

$$r_{ch}^2 = -6 \left[\frac{\partial G(t)}{\partial t} \right]_{t=0}$$

Is $\boldsymbol{r_{ch}}$ a radius?

$$r_{ch}^2 = \int d\boldsymbol{x}\boldsymbol{x}^2 \rho(\boldsymbol{x}), \quad \int d\boldsymbol{x}\rho(\boldsymbol{x}) = Q \ (charge)$$

$$\rho(\boldsymbol{x}) \notin \mathbf{R+}, r_{ch}^2 \neq \langle \boldsymbol{x}^2 \rangle$$

Proper length in relativity

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = c^2 d\tau^2 - dl^2$$

$$d\tau = \sqrt{g_{00}}dt + \frac{g_{0i}dx^i}{c\sqrt{g_{00}}}$$

$$dl^2 = (-g_{ik} + g_{0i}g_{0k}/g_{00})dx^i dx^k$$

$$dl^2 = -ds^2_{d\tau=0}$$

In relativity theory, we lose
the observer-independent notion of a distance.

What is r_{ch}^2?

$$r_{ch}^2 = \int d^4x \left[\frac{(px)^2}{m^2} - x^2\right] \frac{p_\mu}{2m^2} \left\langle 0 \left| \frac{\delta J^\mu(x)}{\delta \varphi^+(0)} \right| p \right\rangle = \left\langle\left\langle s^2 = \tilde{g}_{\alpha\beta}(p) x^\alpha x^\beta \right\rangle\right\rangle$$

Effective metric $g_{\alpha\beta}(p)$:
$\tilde{g}_{\alpha\beta}(\boldsymbol{p} = 0) = (\tilde{g}_{00} = 0, \tilde{g}_{0j} = 0, \tilde{g}_{ij} = \delta_{ij})$

$$\frac{\delta J^\mu(x)}{\delta \varphi^+(0)} = \theta(-x_0)[J^\mu(x), I(0)]$$

$$I(y) = (\partial^2 + m^2)\varphi(y)$$

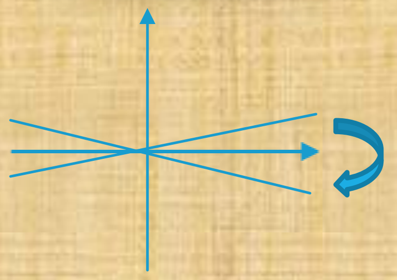
$p \to (m, \boldsymbol{p} = \boldsymbol{0})$ $\to$ $r_{ch}^2 = \int d\boldsymbol{x}\boldsymbol{x}^2 \rho(\boldsymbol{x}),$

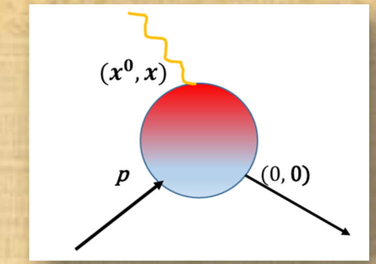
$$\rho(\boldsymbol{x}) = \int dx_0 \frac{1}{2m} \left\langle 0 \left| \frac{\delta J_0(x)}{\delta \varphi^+(0)} \right| \boldsymbol{p} = \boldsymbol{0} \right\rangle = \int dx_0 \frac{1}{2m} \vartheta(-x_0) \langle 0 | [J_0(x), I(0)] | \boldsymbol{p} = \boldsymbol{0} \rangle$$

$$\int d\boldsymbol{x}\, \rho(\boldsymbol{x}) = target\ charge$$

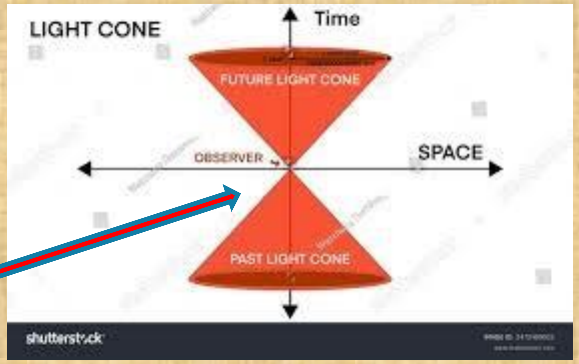
$$\left\langle 0 \left| \frac{\delta J^\mu(x)}{\delta \varphi^+(0)} \right| p \right\rangle \xrightarrow[c \neq 1,\, c = \infty]{} \sim \delta(t) f(\boldsymbol{x})$$

In the rest frame only!





$-|\boldsymbol{x}| \geq x_0$



What is r_{ch}^2?

Charge density

$$\rho(x) = \rho_+(x) - \rho_-(x)$$

$$r_{ch}^2 = \langle x^2 \rangle_+ - \langle x^2 \rangle_-$$

$$r_{ch}^2(proton) = (0.841 \ldots fm)^2$$

$$r_{ch}^2(neutron) = -\,0.116 \ldots (fm)^2$$

$$r_{ch}^2(proton) = \frac{2}{3}\int dx x^2\, \rho_u(x) - \frac{1}{3}\int dx x^2\, \rho_d(x)$$

$$r_{ch}^2(neutron) = \frac{2}{3}\int dx x^2\, \rho_u(x) - \frac{1}{3}\int dx x^2\, \rho_d(x)$$

Physical size of the nucleon?

$$\sqrt{\langle x^2 \rangle}(nucleon) \approx 0.77\ fm$$

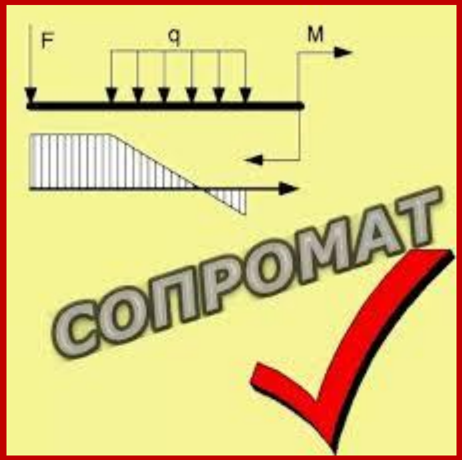
M.V. Polyakov

$$\langle p'|T_{ij}|p\rangle \rightarrow T_{ij}(\boldsymbol{x}) = \left(\frac{x_i x_j}{\boldsymbol{x}^2} - \frac{1}{3}\delta_{ij}\right) s(\boldsymbol{x}) + \delta_{ij} p(\boldsymbol{x})$$

The pressure distribution inside the proton

PRESSURE INSIDE HADRONS ?

"Strength of materials" inside the proton?



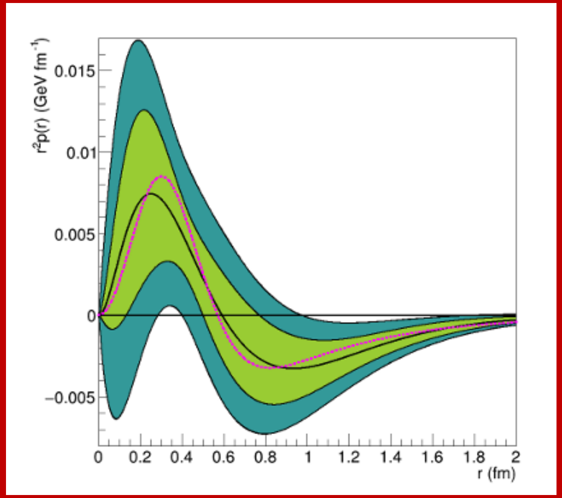
Interpreting hadronic matrix elements of the stress tensor in terms of the concepts of "pressure" and "shear forces" borrowed from continuum mechanics is far from a trivial step.

we treat the interior of a hadron as an elastic medium…

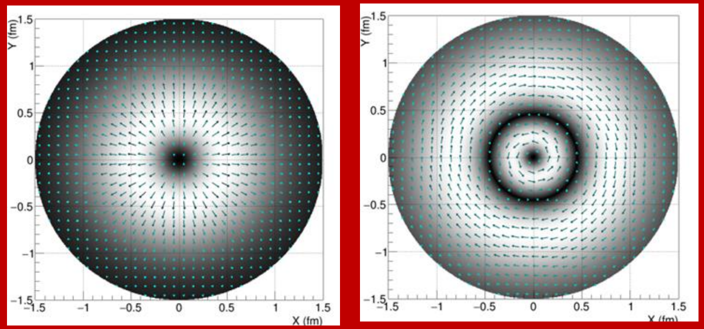
Mechanical stability…

Mechanical equilibrium…

analogy with continuum mechanics…



The distributions of pressure $r^2 p_q(r)$ (top) and shear stress $r^2 s_q(r)$ …



Proton in bench vice?

r from… to … ?

Gerald A. Miller
Phys. Rev. C **112**, 045204 (2025)

Impossibility of obtaining time-independent, three-dimensional, spherically symmetric densities of confined systems of relativistically moving constituents.

The use of the infinite momentum frame, light front formalism seems to be the only way to obtain densities consistent with the quantum mechanical definition of probability, the uncertainty principle and the principle of relativity.

Counterexample: charged scalar target ?

$$\rho(\boldsymbol{x}) = \int dx_0 \frac{1}{2m} \left\langle 0 \middle| \frac{\delta J_0(x)}{\delta\varphi^+(0)} \middle| \boldsymbol{p} = \boldsymbol{0} \right\rangle = \int dx_0\, F(x_0, {x_0}^2 - \boldsymbol{x}^2)\theta(-|x| - x_0)$$

$$\rho(\boldsymbol{x}) = \tilde{\rho}(|\boldsymbol{x}|)$$

Problems

* The main problem: the size of the habit of gluon fields in a hadron.

* Related problem: insufficiency of the descriptional requisite

$$\left\langle p \middle| \mathcal{O}^{ns,sea(glue)}_{\mu_1\mu_2....\mu_J} \middle| p \right\rangle = \Lambda_{\mu_1\mu_2....\mu_J}(p) f_J^{ns,sea(glue)}$$

$$f_1^{ns} = fixed$$

$$f_1^{sea(glue)} = N_g = \infty(?)$$

$$\langle x_q \rangle = \frac{1}{N_q} f_2^{ns}$$

$$\langle x_g \rangle \neq \frac{1}{N_g} f_2^{g} = \int dx\, x\, f^g(x)$$

$$\langle x_g \rangle = \int dx\, x\, w^g(x), \int dx w^g(x) = 1.$$

* No q.f.t. definition of the probability densities.

* One more problem.

"Hegerfeld theorem"

on incompatibility of the existence of spatially localized discrete particles with the combination of the principles of quantum mechanics and relativity.

Many thanx for your attention