Progress of polyvector Yang-Baxter deformations in context of holography

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Yang–Baxter Deformations and Holography

The bivector deformation is a hidden symmetry of the space of solutions of IIB supergravity:

$$
\begin{aligned}
(G+B)^{-1} &= (g+b)^{-1} + \beta, \\
\beta^{mn} &= k_a{}^m k_b{}^n \boxed{r^{ab}}
\end{aligned}
\tag{1}
$$

- $AdS_D \ltimes M_{10-D}$ are dual to deformations of conformal field theories
- The simplest and most studied case is $AdS_5 \ltimes S_5$
- The magic formula was first used for $AdS_5 \ltimes S_5$ and $\beta = \partial_\varphi \wedge \partial_\theta \in T^2_p S^5$

$$
\begin{tikzcd}
AdS_5 \times \mathbb{S}^5 \arrow[r, Leftrightarrow] \arrow[d] & \mathcal{N} = 4 \text{ SYM} \arrow[d] \\
\text{Deformed solution} \arrow[r, Leftrightarrow] & \begin{array}{c} \mathcal{N} = 1 \text{ SYM} \\ W = \mathrm{Tr}\left(e^{i\beta}\Phi_1\Phi_2\Phi_3 - e^{-i\beta}\Phi_1\Phi_3\Phi_2\right) \end{array}
\end{tikzcd}
$$

Yang–Baxter Deformations and Holography

- Main conclusion: a bivector deformation along Killing vectors on the sphere generates subspaces of conformal field theories
- But this magic formula works without conditions ONLY FOR COMPACT ABELIAN Killing vectors

$$[k_a, k_b] = 0$$

- What about the non-Abelian case?

Non-Abelian Deformations

- YB bivector deformation of an arbitrary solution with a trivial B-field
  [Bakhmatov, Colgain, Sheikh-Jabbari, Yavatanoo (2018)]

$$(G)^{-1} = (g + b)^{-1} + \beta \tag{2}$$

- It is necessary to define

$$\begin{aligned}
[k_a, k_b] &= f_{ab}{}^c k_c && \text{(symmetry algebra)} \\
\beta^{mn} &= k_a{}^m k_b{}^n r^{ab} && \text{(bivector ansatz);} \\
r^{b_1[a_1} r^{|b_2|a_2} f_{b_1 b_2}{}^{a_3]} &= 0 && \text{(classical Y–B equation);} \\
r^{b_1 b_2} f_{b_1 b_2}{}^a k_a{}^m = I^m &= 0 && \text{(unimodularity);}
\end{aligned} \tag{3}$$

In the case of compact isometries:

- Abelian $\mathfrak{u}(1)^n$: $\quad f_{ab}{}^c = 0 \quad \Longrightarrow \quad \forall r_{ab}$

- Non-Abelian (SU(N), SO(N), ...): $\quad r_{ab} \equiv 0$
  [Lichnerowicz, Medina (1988), Pop, Stolin (2007)]

The Simplest Proof

- Direct substitution of the new solution into the SUGRA equations of motion:

$$\mathrm{EoM}_{\mathrm{SUGRA}}\left[G + B = \left((g + b)^{-1} + \beta(x)\right)^{-1}\right] = 0 \tag{4}$$

- Works, but requires extremely strong calculations
- There are no clear ideas for generalization routes, because we have only checked the existing formula for the case generalized parameter: $\beta \implies \beta(x) = \rho^{ab} k_a \wedge k_b$
- This made it possible to obtain one of the results of the work, i.e. proving of the magic formula for an arbitrary initial B–field

$$(G + B)^{-1} = (g + b)^{-1} + \beta \tag{5}$$

Geometric Approach

- There exists a special 20-dimensional field theory Double Field Theory (DFT), which can be reduced to SUGRA by a special KK ansatz, eliminating half of the ”extra” degrees of freedom
- DFT theory

$$S_{DFT} = \int dx\, d\tilde{x}\, e^{-2d} \left(\frac{1}{8}\mathcal{H}^{MN}\partial_M \mathcal{H}^{KL}\partial_N \mathcal{H}_{KL} - \frac{1}{2}\mathcal{H}^{KL}\partial_L \mathcal{H}^{MN}\partial_N \mathcal{H}_{KM} - \right.$$
$$\left. -2\partial_M d\, \partial_N \mathcal{H}^{MN} + 4\mathcal{H}^{MN}\partial_M d\, \partial_N d \right), \; \mathbb{X}^M = (x^m, \tilde{x}_m). \tag{6}$$

- is clear invariant under the action of the magic formula (a 20x20 matrix with 10x10 blocks):

$$\mathcal{O}_\beta = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \tag{7}$$

- KK ansatz to 10-dimensional SUGRA

$$\frac{\partial}{\partial \tilde{x}} \cdots = 0, \mathcal{H}_{MN} = \begin{bmatrix} g + b\, g^{-1}\, b & g\, b \\ b\, g & g^{-1} \end{bmatrix}, d = \phi - \frac{1}{4}\log\, g \tag{8}$$

Geometric Approach

- One can deform a generalized metric by direct act of matrix view of magic formula

$$\mathcal{H}_{MN} = \mathcal{O}_M{}^{N'} \mathcal{O}_M{}^{N'} \mathcal{H}_{M'N'} \tag{9}$$

- This is equivalent to the simplest proof above

$$S_{DFT}[\mathcal{H}(\mathsf{b}, g, \varphi)] = S_{SUGRA}(\mathsf{b}, g, \varphi) \tag{10}$$

- Obvious way to generalizations: just change DFT to other theories, which include SUGRA in itself, and repeat calcs

$$\text{DFT} \to \text{ExFT}, \text{etc} \tag{11}$$

Yang-Baxter Equations from Supergravity

Consider the generalized frame

$$\mathcal{H}_{MN} = E_M{}^A E_N{}^B \mathcal{H}_{AB}, \quad E_M{}^A = \begin{bmatrix} e & b \\ 0 & e^{-1} \end{bmatrix} \tag{12}$$

The equations of 10-dimensional supergravity can be written in terms of $\mathcal{F}_{AB}{}^C$ and $\mathcal{F}_A$

$$\begin{gathered} [E_A, E_B]_C := \mathcal{L}_{E_A} E_B{}^M - \mathcal{L}_{E_B} E_A{}^M = \mathcal{F}_{AB}{}^C E_C{}^M, \mathcal{L}_{E_A} d = \mathcal{F}_A. \\ S_{SUGRA} = MQ^{AB} F_A F_B + MR^{ABCDEG} F_{ABC} F_{DEG} \end{gathered} \tag{13}$$

Require that $\mathcal{F}_{AB}{}^C$ be scalars with respect to the bivector transformation

$$\begin{gathered} E'_M{}^A = \mathcal{O}_{\beta M}{}^N E_N{}^A, \mathcal{L}_k E_M{}^A = 0 \\ \mathcal{F}'_{AB}{}^C \overset{!}{=} \mathcal{F}_{AB}{}^C \quad \Longrightarrow \mathrm{CYBE}[r] = 0, \\ \mathcal{F}'_A \overset{!}{=} \mathcal{F}_A \quad \Longrightarrow \mathrm{Uni}[r] = 0, \end{gathered} \tag{14}$$

[Bakhmatov, Musaev (2018), Borsato, Wulff (2020)]

Limit of the Computations

- This works only for a very special class of backgrounds, since it was required that $\mathcal{L}_k E_M{}^A = 0$
- This condition is very strong, and ordinary backgrounds generally do not satisfy it
- It's necessary to redo these computations for the case when $\mathcal{L}_k E_M{}^A \neq 0$
- For flux deformations of SUGRA in DFT, one can compute what the deformations look like under this assumption:

$$\begin{aligned} \delta F_{ABC} &= r^{ab} E_{MA} E_{NB} K_a^M \mathcal{L}_{bC}^N + r^{ab} E_{MC} E_{NA} K_a^M \mathcal{L}_{bB}^N + r^{ab} E_{MB} E_{NC} K_a^M \mathcal{L}_{bA}^N, \\ \delta F_A &= r^{ab} K_a^M \mathcal{L}_{bA}^N \eta_{MN}; \end{aligned} \tag{15}$$

Results

- One can take the SUGRA action, rewrite it in terms of fluxes, and try to deform them:

$$S'_{SUGRA} = \int d^{10}x e^{-2d} \Big[H^{AB} F_A F_B + F_{ABC} F_{DEF} \Big(\frac{1}{4} H^{AD} \eta^{BE} \eta^{CF} - \frac{1}{12} H^{AD} H^{BE} H^{CF} \Big) \Big].$$

$$F_A \to F_A + \delta F_A$$
$$F_{ABC} \to F_{ABC} + \delta F_{ABC}$$

  and try to find what conditions are needed for it to be equal to S_{SUGRA}.
- For the terms of the deformed action of order $r^{ab} r^{cd}$, the author showed that the CYBE and Uni equations are sufficient conditions for them to vanish, i.e.

$$\delta S_{SUGRA}[r^2, CYBE = 0, Uni = 0] = 0 \tag{16}$$

- It was also possible to show that the terms of the deformed action of order r^{ab} vanish for any B, moreover it's proportional to similar calculation for first order deformation in geometric approach

Conclusion

- Previous works showed that bivector deformations can work for a nontrivial algebra of Killing vectors, and outlined a path toward generalization.
- In my works, the formula for bivector deformations was fully proved, and the flux formalism of deformations was improved for a more general case
- The prospects for further improvement of the polyvector deformation method have been demonstrated; based on this, it is planned to use the improved flux formalism in the future to study polyvector deformations

Thank you for your attention!