

Anisotropic drag force in finite-density QGP from charged rotating black holes in AdS_5

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Finite-density rotating QGP and heavy probes

Finite density

The QCD phase diagram requires control over matter at non-zero conserved charge density.

$$\mu_B \neq 0$$

- RHIC Beam Energy Scan;
- NICA physics programme;
- holographically: bulk gauge field and chemical potential.

Rotation and anisotropy

Non-central collisions carry large angular momentum; the plasma can develop vorticity and a preferred axis.

$$\Omega, \omega_{\text{fluid}} \neq 0, \quad SO(3) \text{ broken}$$

- elliptic flow and anisotropic response;
- global hyperon polarization;
- holographic rotation: rotating black holes.

Heavy quarks are early-time real-time probes; here the observable is deterministic drag force on an external heavy quark.

Bzdak et al. Phys. Rept. 2020; STAR Nature 548 (2017); NICA physics programme; He–van Hees–Rapp 2023

Heavy-quark drag in a structured medium

For an isotropic plasma the drag force has the familiar viscous form

$$\vec{F}_{\text{drag}} = -\eta_D(T, \mu) \vec{v}$$

In a rotating anisotropic plasma at finite density, this need not remain true:

$$F_i = -\eta_{ij}(T, \mu, \Omega) v^j \quad + \quad \text{possible transverse components}$$

Question 1

How do rotational anisotropy and finite density modify drag?

Question 2

Can they be included together analytically?

Question 3

What is equilibrium for a heavy quark in such a plasma?

Holographic heavy-quark drag

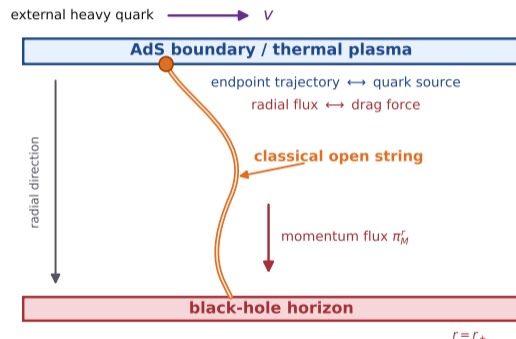
The standard real-time holographic construction represents an external heavy quark by the endpoint of an open string.

$$S_{\text{NG}} = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det g_{\alpha\beta}},$$

$$g_{\alpha\beta} = G_{MN} \partial_\alpha X^M \partial_\beta X^N$$

The drag force is the radial flux of spacetime momentum along the string:

$$\frac{dp_M}{dt} = -\frac{1}{2\pi\alpha'} \pi^r{}_M$$



Herzog–Karch–Kovtun–Kozcaz–Yaffe 2006; Gubser 2006; Casalderrey–Solana–Teaney 2006

Nambu–Goto equations and conserved fluxes

We look for stationary solutions of the **Nambu–Goto equations** in the CCLP geometry.

$$\begin{aligned}\nabla_\alpha P_M^\alpha &= 0, & P_M^\alpha &= -\frac{1}{2\pi\alpha'} G_{MN} \partial^\alpha X^N, \\ P_M^\alpha &= -\frac{1}{2\pi\alpha' \sqrt{-g}} \pi_M^\alpha, & \partial_\alpha \pi_M^\alpha &= 0\end{aligned}$$

In physical gauge,

$$(\sigma^0, \sigma^1) = (t, r),$$

the force on the endpoint is

$$\boxed{\frac{dp_M}{dt} = -\frac{1}{2\pi\alpha'} \pi_M^r}$$

Boundary value problem

1. choose endpoint trajectory;
2. solve NG equations in the bulk;
3. impose regularity at the worldsheet horizon;
4. evaluate conserved radial flux.

Charged rotating AdS₅ background

5d minimal gauged supergravity

$$S \sim \int \left[(R + 12g^2) \star 1 - \frac{1}{2} F \wedge \star F + \frac{1}{3\sqrt{3}} A \wedge F \wedge F \right]$$

- The CCLP solution is the general non-extremal charged rotating AdS₅ black hole of this theory.
- Four independent parameters:

$$(m, q, a, b)$$

- Limits: Kerr–AdS₅ for $q = 0$, RN–AdS₅ for $a = b = 0$
- Bulk $U(1)$: controlled finite-density deformation, not literal baryon number.

Horizon data

Temperature and chemical potential:

$$T = T(r_+, q, a, b)$$

$$\mu = -\Phi_H(r_+, q, a, b)$$

Rotations determine angular momenta and angular velocities:

$$a, b \longleftrightarrow J_a, J_b, \Omega_a, \Omega_b$$

The horizon generator is

$$\ell = \partial_t + \Omega_a \partial_\phi + \Omega_b \partial_\psi$$

Analytic problem

A stationary string in CCLP leads to coupled nonlinear **Nambu–Goto equations**; a fully general analytic solution is not known.

stationary string in CCLP $\xrightarrow{\text{NG}}$ coupled nonlinear ODEs \implies no generic closed form

Special exact sectors

- (1a) **Equal-spin CCLP** ($a = b$, charge allowed): constant strings and equilibrium.
- (1b) **Neutral Kerr–AdS₅** ($q = 0$): principal Killing string, arbitrary rotations, exact drag. The resulting drag has both thermal and kinematic origin.

Slow-rotation expansion

- generic charged CCLP;
- $(a, b, \omega_\phi, \omega_\psi) = O(\epsilon)$;
- bulk regularity fixes integration constants;
- finite transverse force $\mathcal{F}^{\text{ind}}(T, \mu)$.

Result I: equal-spin CCLP strings

Route (1a): exact equal-spin sector, charge allowed

For equal rotations $a = b$, the CCLP isometry is enhanced and one finds a one-parameter family of constant stationary embeddings:

$$\phi(t, r) = \phi_0 + \omega t, \quad \psi(t, r) = \psi_0 + \omega t, \quad \theta(t, r) = \theta_0$$

This family hides a small but important subtlety.

Zero flux

No momentum flow down the string for every ω :

$$\pi^r_M = 0$$

But

Zero drag alone does not determine the physical thermal saddle.

Result I: regularity selects co-rotation

Equilibrium is a regularity condition, not merely a zero-flux condition

The induced worldsheet metric contains

$$(g_{\text{ws}})_{tt} = G_{tt} + 2\omega(G_{t\phi} + G_{t\psi}) + \omega^2(G_{\phi\phi} + 2G_{\phi\psi} + G_{\psi\psi})$$

For a regular thermal saddle, the worldsheet horizon must coincide with the bulk Killing horizon:

$$(g_{\text{ws}})_{tt} = G(\ell, \ell), \quad \ell = \partial_t + \Omega_a(\partial_\phi + \partial_\psi)$$

Therefore

$$\boxed{\omega = \Omega_a}$$

Physical statement

In a rotating plasma, vanishing drag is not by itself sufficient for equilibrium. The dressed heavy quark must co-rotate with the thermal state selected by the horizon.

Result I: dragless stationary strings

Uniqueness within the stationary sector

Consider a general stationary embedding with asymptotically rigid rotation:

$$x^i(t, r) = x_0^i + \omega_i t + \tilde{x}^i(r), \quad x^i = (\theta, \phi, \psi)$$

If the radial momentum flux vanishes, then in physical gauge $(\sigma^0, \sigma^1) = (t, r)$ one obtains

$$\pi_i^r = 0 \quad \implies \quad \tilde{x}'^i(r) = 0 \quad (\text{away from the worldsheet horizon})$$

Thus a dragless stationary string must have a constant angular profile.

Equal-spin sector

The constant-profile family exists for $a = b$ and gives the candidates for equilibrium.

Generic rotations

For $a \neq b$, no analogous constant stationary solution exists within this class.

Result I: equilibrium one-body observable

A thermodynamic datum of the regular heavy probe

For the regular co-rotating solution, define the energy in the quark rest frame:

$$E_{\text{eff}} = -p \cdot \hat{\ell}, \quad \hat{\ell} = (g^2 - \Omega_a^2)^{-1/2} \ell$$

Using

$$\pi^0 \cdot \ell = 1$$

and subtracting the vacuum AdS string with the same boundary trajectory gives

$$\Delta E = \frac{r_+}{2\pi\alpha' \sqrt{g^2 - \Omega_a^2}}$$

Interpretation

This is the equilibrium insertion energy/free-energy shift of the regular heavy probe, complementary to the dissipative drag-force sector.

Result II: exact neutral branch

Route (1b): principal Killing string in neutral Kerr–AdS₅

In the neutral Kerr–AdS₅ limit, the principal Killing string gives an exact stationary solution for arbitrary rotations:

$$\theta = \theta_0 \quad \phi = ag^2t + \phi(r), \quad \psi = bg^2t + \psi(r)$$

Its hidden-symmetry origin is reflected in the simple induced worldsheet,

$$\det g_{\text{ws}} = -1$$

and in constant momentum fluxes:

$$\pi_{\phi}^r = \frac{a \sin^2 \theta_0}{\Xi_a} [1 + g^2 \cos^2 \theta_0 (b^2 - a^2)],$$
$$\pi_{\psi}^r = \frac{b \cos^2 \theta_0}{\Xi_b} [1 + g^2 \sin^2 \theta_0 (a^2 - b^2)]$$

Boos–Frolov 2018; Kerr–AdS hidden symmetries: Frolov–Krtouš–Kubizňák 2017

Result II: anisotropic tangential drag

Exact drag for arbitrary rotation parameters in Kerr–AdS₅

At the boundary,

$$\vec{v} = ag \partial_\phi + bg \partial_\psi$$

The drag force is

$$\vec{F}_{\text{drag}} = -\frac{1}{2\pi\alpha'} \left(\frac{\pi_\phi^r}{\sin^2 \theta_0} \partial_\phi + \frac{\pi_\psi^r}{\cos^2 \theta_0} \partial_\psi \right)$$

Unequal rotations

$$a \neq b \Rightarrow \vec{F}_{\text{drag}} \not\parallel \vec{v}$$

The force is tangential, but anisotropic.

Equal rotations

$$a = b \Rightarrow \vec{F}_{\text{drag}} = -\frac{g\Xi}{2\pi\alpha'} \vec{v}$$

The viscous form is restored.

Result III: charged CCLP at slow rotation

Route (2): perturbative charged solution for generic rotations

For the charged CCLP background, no exact analogue of the principal string is known for generic unequal rotations. We use

$$(a, b, \omega_\phi, \omega_\psi) = O(\epsilon)$$

and expand the stationary embedding:

$$\theta(r) = \theta_0 + \frac{\epsilon^2}{2}\theta_{(2)}(r) + O(\epsilon^4),$$

$$\phi(t, r) = \phi_0 + \epsilon\omega_\phi g^2 t + \epsilon\phi_{(1)}(r) + O(\epsilon^2),$$

$$\psi(t, r) = \psi_0 + \epsilon\omega_\psi g^2 t + \epsilon\psi_{(1)}(r) + O(\epsilon^2)$$

Role of the expansion

At zeroth order one has RN–AdS₅; the first angular profiles and the polar deformation encode the rotating finite-density response.

First-order profiles and regularity data

The angular profiles at first order are

$$\phi_{(1)}(r) = \mathfrak{p} \int_{r_+}^r \frac{\bar{r}^2 d\bar{r}}{h(\bar{r})}, \quad \psi_{(1)}(r) = \mathfrak{q} \int_{r_+}^r \frac{\bar{r}^2 d\bar{r}}{h(\bar{r})}$$

Here

$$h(r) = g^2 r^6 + r^4 - 2mr^2 + q^2$$

is the RN-AdS₅ blackening polynomial.

The constants $\mathfrak{p}, \mathfrak{q}$ determine the axial momentum fluxes.

Regularity

Finiteness of the Lorentzian worldsheet curvature at $r = r_+$ fixes

$$\mathfrak{p}, \mathfrak{q} = \mathfrak{p}, \mathfrak{q}(T, r_+, q, \omega_\phi, \omega_\psi)$$

Renormalised transverse drag force

The polar component contains a UV rest-mass contribution and a finite medium-induced term.

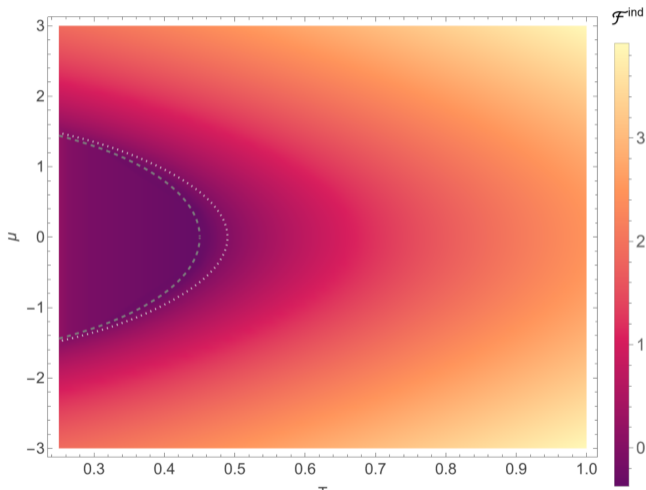
Asymptotic transverse force

$$\begin{aligned} \left. \frac{dp_\theta}{dt} \right|_{r \rightarrow \infty} &= -g^2(a^2 - b^2 + \omega_\psi^2 - \omega_\phi^2) \left(M_{\text{rest}} + \frac{r_+}{2\pi\alpha'} \right) \sin 2\theta_0 \\ &\quad - \frac{1}{4\alpha'} \frac{F(-r_0^2) \sin 2\theta_0}{r_0(3g^2r_0^4 - 2r_0^2 - 2m)} + O(\epsilon^3) \end{aligned}$$

For a static endpoint, define the finite plotted quantity

$$\mathcal{F}^{\text{ind}}(T, \mu) = -\frac{2\pi\alpha'}{g^2(a^2 - b^2) \sin 2\theta_0} \left[\frac{dp_\theta}{dt} + g^2(a^2 - b^2) \sin 2\theta_0 M_{\text{rest}} \right]$$

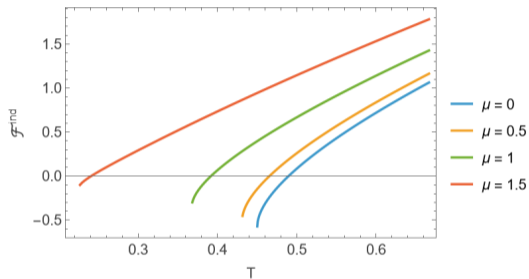
Finite-density dependence: $T-\mu$ plane



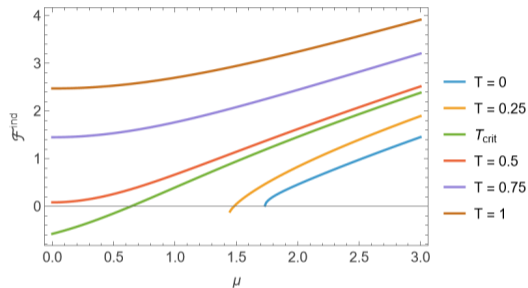
Data shown

- finite medium-induced transverse force $\mathcal{F}^{\text{ind}}(T, \mu)$;
- large-black-hole thermodynamic branch;
- dashed curve: minimal temperature for black-hole existence;
- dotted curve: zero of \mathcal{F}^{ind} .

Finite-density dependence: slices



\mathcal{F}^{ind} versus temperature at fixed μ



\mathcal{F}^{ind} versus chemical potential at fixed T

For the displayed large-black-hole branch, increasing $|\mu|$ enhances the medium-induced transverse drag; the temperature dependence is visibly non-linear.

Summary of results

Equilibrium

Only in the $a = b$ sector do stationary dragless strings exist; worldsheet regularity then selects the unique co-rotating thermal saddle:

$$\omega = \Omega_a$$

Neutral exact drag

The principal Killing string gives exact Kerr–AdS drag for arbitrary a, b :

$$\vec{F}_{\text{drag}} \not\parallel \vec{v} \quad (a \neq b)$$

Charged perturbative drag

For CCLP at slow rotation, regularity fixes the solution data and gives

$$\mathcal{F}^{\text{ind}} = \mathcal{F}^{\text{ind}}(T, \mu)$$

1. **Beyond slow rotation.**

Construct generic charged stationary strings numerically or analytically; determine whether the transverse force persists at finite rotation.

2. **Worksheet fluctuations.**

Compute fluctuation spectra, Langevin coefficients and momentum broadening around the regular co-rotating saddle.

3. **Other heavy-quark observables.**

Temporal Wilson loops, heavy-quark potentials, lightlike Wilson loops and jet-quenching observables in the same charged rotating background.

4. **Model dependence.**

Multi-charge gauged-supergravity backgrounds and bottom-up holographic-QCD models closer to phenomenology.

Thank you!

Questions?

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$$ds^2 = - \frac{\Delta_\theta [(1 + g^2 r^2) \rho^2 dt + 2q\nu] dt}{\Xi_a \Xi_b \rho^2} + \frac{2q\nu\omega}{\rho^2} + \frac{f}{\rho^4} \left(\frac{\Delta_\theta dt}{\Xi_a \Xi_b} - \omega \right)^2 + \frac{\rho^2 dr^2}{\Delta_r} + \frac{\rho^2 d\theta^2}{\Delta_\theta} \\ + \frac{r^2 + a^2}{\Xi_a} \sin^2 \theta d\phi^2 + \frac{r^2 + b^2}{\Xi_b} \cos^2 \theta d\psi^2,$$

$$A = \frac{\sqrt{3}q}{\rho^2} \left(\frac{\Delta_\theta dt}{\Xi_a \Xi_b} - \omega \right),$$

$$\nu = b \sin^2 \theta d\phi + a \cos^2 \theta d\psi, \quad \omega = \frac{a \sin^2 \theta}{\Xi_a} d\phi + \frac{b \cos^2 \theta}{\Xi_b} d\psi$$

$$\begin{aligned}\Delta_r &= r^{-2} [(r^2 + a^2)(r^2 + b^2)(1 + g^2 r^2) + q^2 + 2abq] - 2m, \\ \Delta_\theta &= \Xi_a \cos^2 \theta + \Xi_b \sin^2 \theta, \quad \Xi_a = 1 - a^2 g^2, \quad \Xi_b = 1 - b^2 g^2, \\ \rho^2 &= r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \\ f &= 2m\rho^2 - q^2 + 2abqg^2\rho^2\end{aligned}$$

- $q = 0$: Kerr-AdS₅;
- $a = b = 0$: RN-AdS₅;
- $m = q = 0$: AdS₅ in a rotating/asymptotic chart.

$$\ell = \partial_t + \Omega_a \partial_\phi + \Omega_b \partial_\psi$$

$$\Omega_a = \frac{a(r_+^2 + b^2)(1 + g^2 r_+^2) + bq}{(r_+^2 + a^2)(r_+^2 + b^2) + abq}, \quad \Omega_b = \frac{b(r_+^2 + a^2)(1 + g^2 r_+^2) + aq}{(r_+^2 + a^2)(r_+^2 + b^2) + abq}$$

$$T = \frac{1}{2\pi} \frac{r_+^4 [1 + g^2 (2r_+^2 + a^2 + b^2)] - (ab + q)^2}{r_+ [(r_+^2 + a^2)(r_+^2 + b^2) + abq]}$$

$$\mu = -\Phi_H, \quad \Phi_H = \frac{\sqrt{3}qr_+^2}{(r_+^2 + a^2)(r_+^2 + b^2) + abq}$$

Backup: regularity condition for p, q

At the horizon, finiteness of the worldsheet curvature gives two constraints. The useful solution is

$$\begin{aligned} p^2 - q^2 &= (a^2 - b^2)r_+(2\pi T + g^4 r_+^3) - 2g^2 r_+^2 (1 + g^2 r_+^2)(a\omega_\phi - b\omega_\psi) \\ &\quad + 2g^2 q(a\omega_\psi - b\omega_\phi) + g^4 r_+^4 (\omega_\phi^2 - \omega_\psi^2), \\ q^2 &= r_+^{-2} \left(\frac{aq}{r_+} + br_+(1 + g^2 r_+^2) - g^2 \omega_\psi r_+^3 \right)^2 \end{aligned}$$

Use in the main calculation

The integration constants entering the first-order profiles are therefore IR data fixed by worldsheet regularity.

Kerr–AdS admits a principal conformal Killing–Yano tensor. This provides the exact principal Killing string branch used in the neutral calculation.

For CCLP, the ordinary principal tensor is replaced by a generalized CCKY tensor with torsion,

$$T \propto \star F$$

Practical consequence

The neutral exact construction does not directly extend to generic charged CCLP.

charged CCLP \Rightarrow perturbative or numerical

Kubizňák–Kunduri–Yasui 2009; Boos–Frolov 2018

Backup: black-hole existence region in the plots

For the non-rotating charged branch,









$$T(r_+, \mu) = \frac{1}{2\pi} \left(2g^2 r_+ + \frac{1 - \mu^2/3}{r_+} \right)$$

Reality of r_+ gives the minimal-temperature curve

$$T > T_{\min}(\mu) = \frac{g}{\pi} \sqrt{2 \left(1 - \frac{\mu^2}{3} \right)}$$

This is the dashed curve in the heat map.

Backup: selected references

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