Rare four-leptonic decays of B^- mesons with three charged leptons in the final state

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Motivation

We study four-leptonic decays $B^- \to \ell^- \ell^+ \ell'^- \bar{\nu}_{\ell'}$ and $B^- \to \ell^- \ell^+ \ell^- \bar{\nu}_\ell$.

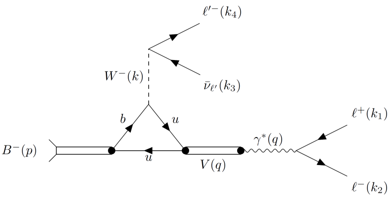
- Rare decays allow a precise test of Standard Model. Deviations from theoretical predictions may indicate physics Beyond the SM.
- The only experimental upper limit [1] on the partial width of the decay $B^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu$ is lower than all the previous theoretical predictions [2–9]:

$$\mathcal{B}r(B^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu) < 0.16 \times 10^{-7}.$$

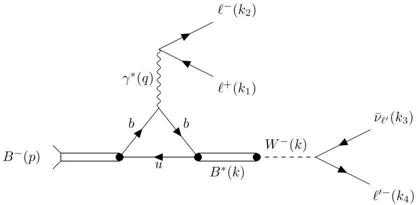
- The previous theoretical predictions have certain limitations:
  - Massless lepton approximation (simplifies bremsstrahlung contribution and kinematics).
  - Only light mesons resonances contribution.
  - Gauge non-invariant form of the amplitude.
  - Interference calculation in the decay $B^- \to \ell^- \ell^+ \ell^- \bar{\nu}_\ell$ is approximate.

  Our aim is to address these limitations to provide more accurate theoretical prediction.

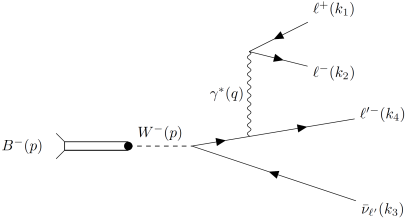
Feynman diagrams



u-quark radiation



b-quark radiation



bremsstrahlung

Amplitude of $B^- \to \ell^- \ell^+ \ell'^- \bar{\nu}_{\ell'}$

The amplitude for the decay $B^- \to \ell^- \ell^+ \ell'^- \bar{\nu}_{\ell'}$ (*NI* — Non-Identical):

$$\mathcal{M}^{(NI)}(k_1, k_2, k_3, k_4) = \mathcal{M}^{(u)} + \mathcal{M}^{(b)} + \mathcal{M}^{(brem)}.$$

Leptonic currents are defined as:

$$j^\nu(k_2, k_1) = \bar{u}(k_2)\gamma^\nu u(-k_1), \quad J^\mu(k_4, k_3) = \bar{u}(k_4)\gamma^\mu(1-\gamma^5)u(-k_3).$$

The bremsstrahlung amplitude:

$$\mathcal{M}^{(brem)} = \frac{\mathcal{A}}{q^2}(-\mathrm{i})\, f_{B_u}\, g_{\mu\nu}\, j^\nu(k_2, k_1)\, \tilde{J}^\mu(k_4, k_3),$$

$$\tilde{J}^\mu(k_4, k_3) = J^\mu(k_4, k_3) + \frac{m_{\ell'}}{(p-k_3)^2 - m^2{}_{\ell'}}(\bar{\ell'}(k_4)\gamma^\mu(\hat{p} + m_{\ell'})\,(1-\gamma_5)\,\nu_{\ell'}(-k_3)).$$

The b-quark radiation amplitude:

$$\mathcal{M}^{(b)} = \frac{1}{3}\frac{\mathcal{A}}{q^2}\frac{M_{B^*} f_{B^*}}{k^2 - M_{B^*}^2}\frac{2V_b(q^2)}{M_B + M_{B^*}}\varepsilon_{\mu\nu kq} j^\nu(k_2, k_1) J^\mu(k_4, k_3).$$

Hereinafter, $\mathcal{A} = -i\frac{G_F}{\sqrt{2}}4\pi\alpha_{\text{em}} V_{ub}$, $q = k_1 + k_2$, $k = k_3 + k_4$, $p = k + q$.

Vector meson resonances

All resonances in the u-quark radiation amplitude are described using the Breit-Wigner (VMD):

$$\begin{aligned}\mathcal{M}^{(u)} &= \frac{\mathcal{A}}{q^2} j^{\nu}(k_2,k_1) J^{\mu}(k_4,k_3) \times \left[\sum_V \frac{I_V\, M_V\, f_V}{q^2 - M_V^2 + \mathrm{i}\,\Gamma_V M_V}\, \mathcal{F}^{(V)}_{\mu\nu}(k^2)\right] = \\ &= \frac{\mathcal{A}}{q^2} j^{\nu}(k_2,k_1) J^{\mu}(k_4,k_3) \times \left[\frac{a'(x_{12},x_{34})}{M_B}\varepsilon_{\mu\nu k q} - \right. \\ & \left. -iM_B b'(x_{12},x_{34}) g_{\mu\nu} - 2i\frac{d'(x_{12},x_{34})k_\mu - c'(x_{12},x_{34})q_\mu}{M_B} k_\nu\right].\end{aligned}$$

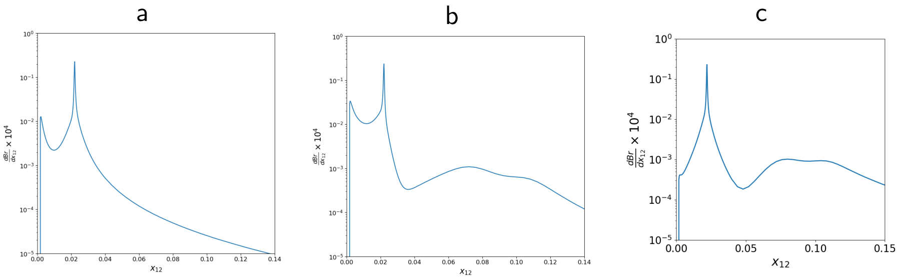
Dimensionless variables $x_{ij} = \frac{(k_i+k_j)^2}{M_B^2}$ are used. At $q^2 = 0$ gauge invariance constraints functions $a(x_{12},x_{34})$,...,$d(x_{12},x_{34})$. All form factors are modified via a subtraction procedure [10]:

$$\tilde{a}(q^2,k^2) = \tilde{a}(0,k^2) + \frac{q^2}{M_V{}^2} a'(q^2,k^2)$$

$$\tilde{a}(0,k^2) = 0, \quad \tilde{b}(0,k^2) = 0$$

$$\tilde{d}(0,k^2) = \frac{Q_B f_B}{M_1}\frac{1}{1-x_{34}}, \quad \tilde{c}(0,k^2) = -\frac{Q_B f_B}{M_1}\frac{1}{1-x_{34}} + \frac{b(0,k^2)}{1-x_{34}}.$$

Heavy meson contribution



Differential distributions $\frac{dBr(B^- \to \mu^- \mu^+ e^- \bar{\nu}_e)}{dx_{12}}$

a) $\rho\,(770)$ and $\omega\,(787)$ only,

b) $\rho\,(1450)$, $\omega\,(1420)$, $\rho\,(1700)$ and $\omega\,(1650)$ added,

c) form factors are taken in gauge invariant form.

Amplitude needs to be anti-symmetrized (I — Identical):

$$\mathcal{M}^{(I)}(k_1,k_2,k_3,k_4) = \frac{1}{\sqrt{2}}\left(\mathcal{M}^{(NI)}(k_1,k_2,k_3,k_4) - \mathcal{M}^{(NI)}(k_1,k_4,k_3,k_2)\right).$$

Massless lepton approximation is then applied. Amplitude of the decay $B^- \to \ell^-\ell^+\ell'^-\bar{\nu}_{\ell'}$ is as follows:

$$\mathcal{M}_0^{(NI)}(k_1,k_2,k_3,k_4) = \frac{\mathcal{A}}{q^2} j^\nu(k_2,k_1) J^\mu(k_4,k_3) \times \left[\frac{a(x_{12},x_{34})}{M_B}\varepsilon_{\mu\nu kq} - \right.$$

$$\left. - iM_B b(x_{12},x_{34}) g_{\mu\nu} + 2i\frac{c(x_{12},x_{34})}{M_B} q_\mu k_\nu\right] \equiv \mathrm{F}^{(0)}_{\mu\nu}(q,k) j^\nu(k_2,k_1) J^\mu(k_4,k_3).$$

Thus, amplitude of the decay $B^- \to \ell^-\ell^+\ell^-\bar{\nu}_\ell$ is:

$$\mathcal{M}_0^{(I)}(k_1,k_2,k_3,k_4) = \frac{1}{\sqrt{2}}\left(\mathrm{F}^{(0)}_{\mu\nu}(q,k) j^\nu(k_2,k_1) J^\mu(k_4,k_3) - \right.$$

$$\left. - \mathrm{F}^{(0)}_{\mu\nu}(\tilde{q},\tilde{k}) j^\nu(k_4,k_1) J^\mu(k_2,k_3)\right),$$

where $\tilde{q} = k_1 + k_4, \tilde{k} = k_2 + k_3$.

Method of orthogonal amplitudes I

We consider amplitudes of the type:

$$\mathcal{M} = \bar{u}(q)\,\hat{O}\,u(p).$$

One can view such amplitudes as vectors in a four-dimensional linear space [11–16]. If a scalar product operation is introduced, an orthogonal basis exists:

$$A_1 = \bar{u}(q)\,\hat{1}\,u(p), \quad A_2 = \bar{u}(q)\,\hat{U}\,u(p),$$
$$A_3 = \bar{u}(q)\,\hat{V}\,u(p), \quad A_4 = \bar{u}(q)\,\hat{U}\hat{V}\,u(p),$$

where U and V are auxiliary four-vectors, orthogonal to p, q and each other. Amplitude $\mathcal{M}$ and its square are then expressed in terms of this basis and the decomposition coeffitients:

$$\mathcal{M} = \sum_{i=1}^{4} \frac{1}{||A_i||^2}\omega_i A_i, \quad \omega_i = \sum_{s_p, s_q} \mathcal{M} A_i^\dagger, \quad \sum_{s_p, s_q} |\mathcal{M}|^2 = \sum_{i=1}^{4} \frac{1}{||A_i||^2} |\omega_i|^2.$$

Method of orthogonal amplitudes II

The method of orthogonal amplitudes can be applied to amplitudes with multiple spinor pairs. For the amplitude of the decay $B^- \to \ell^-\ell^+\ell^-\bar{\nu}_\ell$, the square takes the form:

$$\sum_{s_1,s_2,s_3,s_4} \left|\mathcal{M}_0^{(I)}(k_1,k_2,k_3,k_4)\right|^2 = \sum_{i=1}^{4}\sum_{j=1}^{4} \frac{1}{2||A_i||^2||\tilde{A}_j||^2} \left|\zeta_{ij}^{(0)} - \xi_{ij}^{(0)}\right|^2,$$

where

$$\zeta_{ij}^{(0)} = \sum_{s_1,s_2,s_3,s_4} \mathrm{F}_{\mu\nu}^{(0)}(q,k) j^\nu(k_2,k_1) J^\mu(k_4,k_3) A_i^\dagger \tilde{A}_j^\dagger,$$

$$\xi_{ij}^{(0)} = \sum_{s_1,s_2,s_3,s_4} \mathrm{F}_{\mu\nu}^{(0)}(\tilde{q},\tilde{k}) j^\nu(k_4,k_1) J^\mu(k_2,k_3) A_i^\dagger \tilde{A}_j^\dagger.$$

Basis A_i is constructed with k_1, k_2. Basis $\tilde{A}_j$ is constructed with k_3, k_4.

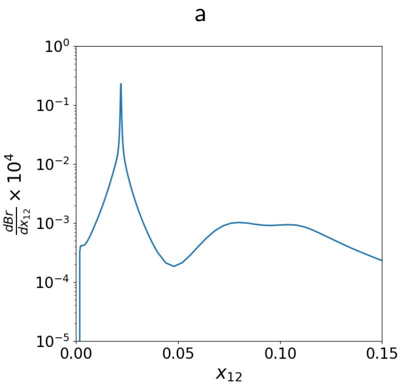
The decay $B^- \to \ell^-\ell^+\ell'^-\bar{\nu}_{\ell'}$ was processed similarly. Decomposition coefficients of amplitudes were calculated analytically using the Wolfram Mathematica environment and the FeynCalc [17–20] package. Monte-Carlo integration was performed using the Vegas [21] routine.

Numerical results I

Comparison of theoretical predictions obtained by different authors

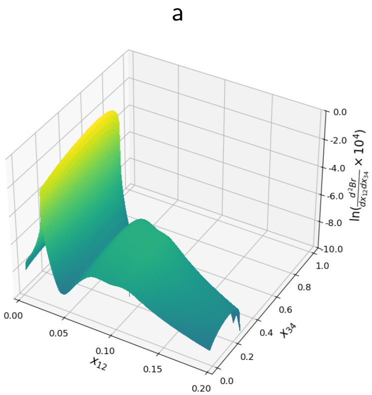
	$\mathrm{Br}(B^- \to \mu^-\mu^+e^-\bar{\nu}_e)$	$\mathrm{Br}(B^- \to e^-e^+\mu^-\bar{\nu}_\mu)$
[3]	0.6×10^{-7}	0.8×10^{-7}
[4]	1.78×10^{-8}	2.48×10^{-8}
[5]	5.19×10^{-8}	7.59×10^{-8}
[6]	1.23×10^{-9}	
[7]	3.19×10^{-8}	3.78×10^{-8}
[8]	3.01×10^{-8}	6.38×10^{-8}
	3.6×10^{-8}	3.7×10^{-8}
	$\mathrm{Br}(B^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu)$	$\mathrm{Br}(B^- \to e^-e^+e^-\bar{\nu}_e)$
[3]	0.7×10^{-7}	
[4]	1.77×10^{-8}	2.57×10^{-8}
[5]	6.02×10^{-8}	6.02×10^{-8}
[9]	3.02×10^{-8}	
	3.6×10^{-8}	3.8×10^{-8}

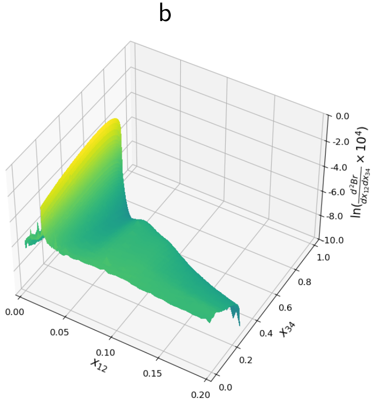
Numerical results II



Differential distributions $\frac{dBr(B^- \to \mu^- \mu^+ e^- \bar{\nu}_e)}{dx_{12}}$ (a); $\frac{dBr(B^- \to \mu^- \mu^+ e^- \bar{\nu}_e)}{dx_{34}}$ (b).

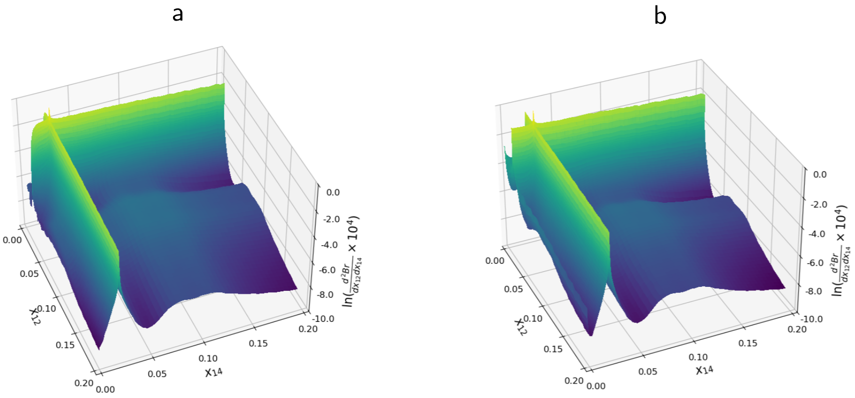
Numerical results III





Double differential distributions $\frac{d^2Br(B^- \to \mu^- \mu^+ e^- \bar{\nu}_e)}{dx_{12}dx_{34}}$ (a); $\frac{d^2Br(B^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu)}{dx_{12}dx_{34}}$ (b).

Numerical results IV



Double differential distributions $\frac{d^2Br(B^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu)}{dx_{12}dx_{14}}$ (a); $\frac{d^2Br(B^- \to e^- e^+ e^- \bar{\nu}_e)}{dx_{12}dx_{14}}$ (b).

Summary

1. Contributions from heavy vector meson resonances $\rho\,(1450)$, $\omega\,(1420)$, $\rho\,(1700)$, $\omega\,(1650)$ are included.
2. A subtraction procedure is applied to the amplitudes to maintain gauge invariance.
3. A precise bremsstrahlung amplitude is used.
4. For the decays $B^- \to \ell^-\ell^+\ell'^-\bar{\nu}_{\ell'}$, lepton masses are fully taken into account.
5. For the decays $B^- \to \ell^-\ell^+\ell^-\bar{\nu}_\ell$, the interference contribution is calculated with no additional approximations.

New branching ratios are:

$$
\begin{aligned}
\mathcal{B}r(B^- \to \mu^-\mu^+e^-\bar{\nu}_e) &= 3.6\times 10^{-8}\\
\mathcal{B}r(B^- \to e^-e^+\mu^-\bar{\nu}_\mu) &= 3.7\times 10^{-8}\\
\mathcal{B}r(B^- \to \mu^-\mu^+\mu^-\bar{\nu}_\mu) &= 3.6\times 10^{-8}\\
\mathcal{B}r(B^- \to e^-e^+e^-\bar{\nu}_e) &= 3.8\times 10^{-8}
\end{aligned}
$$

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Backup

Subtraction procedure

$$T_{\nu\mu}(q,k) = \varepsilon_{\nu\mu qk}\frac{e\,a(q^2,k^2)}{M_1} - \mathrm{i}\left(g_{\nu\mu} - \frac{q_\nu q_\mu}{q^2}\right) eM_1\, b(q^2,k^2)$$

$$- \quad \mathrm{i}e\left(k_\nu - \frac{(q\cdot k)}{q^2}q_\nu\right)\left(k_\mu\frac{2d(q^2,k^2)}{M_1} - q_\mu\frac{2c(q^2,k^2)}{M_1}\right) - \mathrm{i}Q_{B_u}ef_{B_u}\frac{q_\nu k_\mu}{q^2},$$

singularity at $q^2 \to 0$.

$$Q_{B_u}f_{B_u} - \frac{2(qk)}{M_1}d(q^2,k^2)|_{q^2\to 0} = 0 \quad \Rightarrow \quad d(0,k^2) = \frac{Q_{B_u}f_{B_u}}{M_1}\frac{1}{1-x_{34}}$$

$$d(q^2,k^2) = d(0,k^2) + \Delta(q^2,k^2)$$

$$\text{VMD:} \quad \Delta(q^2,k^2) = \frac{f(k^2)}{q^2-\delta}, \quad \delta = M_V{}^2 - i\Gamma_V M_V \approx M_V{}^2$$

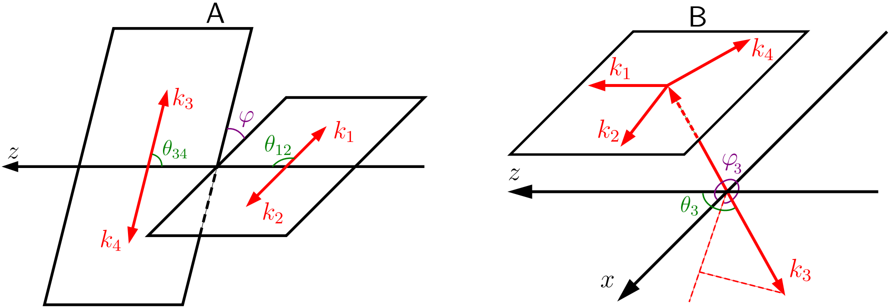
$$\Delta(0,k^2) = -f(k^2)/\delta \neq 0 \Rightarrow \text{subtraction procedure :}$$

$$\tilde{\Delta}(q^2,k^2) = \Delta(q^2,k^2) - \Delta(0,k^2) \approx \frac{q^2}{m_V{}^2}\Delta(q^2,k^2)$$

$$\tilde{\Delta}(q^2,k^2) = 0$$

$$d(q^2,k^2) = \frac{Q_{B_u}f_{B_u}}{M_1}\frac{1}{1-x_{34}} + \frac{q^2}{M_V{}^2}\Delta(q^2,k^2)$$

Four-particle kinematics



$$d\Phi_4^A = \frac{M_B^4}{2^{14}\pi^6}\lambda^{1/2}(1, x_{12}, x_{34})\sqrt{1-\frac{4\hat{m}_\ell{}^2}{x_{12}}}\left(1-\frac{\hat{m}_{\ell'}{}^2}{x_{34}}\right) dx_{12}\, dx_{34}\, dy_{12}\, dy_{34}\, d\varphi,$$

$$d\Phi_4^B = \frac{M_B^4}{2^{13}\pi^6}\frac{1-x_{124}}{x_{124}}\, dx_{124}\, dx_{12}\, dx_{14}\, dy_3\, d\varphi_3.$$