Relativistic effects in heavy mesons

based on the work

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Motivation

- Existing experimental data on spectra and radiative transitions in heavy mesons (contain at least one c or b quark) cannot be explained within the framework of existing approaches

- Lattice calculations provide answers, but do not provide insight into the phenomena

- Perturbative QCD cannot be used to describe the spectra of heavy mesons, since these systems have a “large“ size

Perturbative QCD

“Running“ coupling constant:

$$\alpha_s(q^2) = \frac{4\pi}{\beta_0 \ln(q^2/\Lambda_{QCD}^2)}\,, \quad \Lambda_{QCD} = 300\,\mathrm{MeV}\,.$$

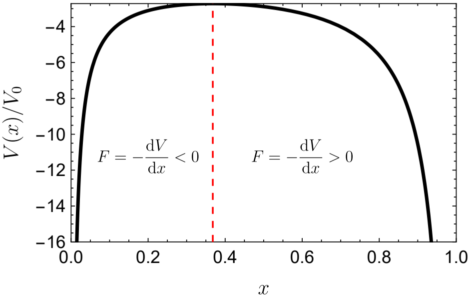
Interaction potential:

$$V_F(q^2) = -\frac{4}{3}\cdot\frac{4\pi\alpha_s(q^2)}{q^2}\,, \quad V(r) = \frac{4}{3}\cdot\frac{2\pi}{\beta_0 \ln(\Lambda r)}\,\frac{1}{r}\,,$$

$$\Lambda = e^C\,\Lambda_{QCD}\,, \quad C = 0.577\,, \quad \Lambda = 534\,\mathrm{MeV}\,.$$

We have

$$V(x) = \frac{V_0}{x\,\ln x}\,, \quad V_0 = \frac{8\pi\Lambda}{3\beta_0}\,, \quad x = \Lambda r\,.$$



Infrared pole: $x^* = 1$, $r^* = 0.374\,\text{fm}$.

Maximum: $x_{max} = 1/e$, $r_{max} = 0.138\,\text{fm}$.

When $r > r_{max}$ instead of attraction we have repulsion!

Perturbative QCD

The size a of the quarkonium wave function can be estimated using the lepton width Γ_{ee}

$$a \sim \left(\frac{4\alpha^2 e_q^2}{m_q^2 \, \Gamma_{ee}} \right)^{1/3} ,$$

$$a_{c\bar{c}} \sim 0.42 \, \mathrm{fm} \approx 3 \, r_{max} \, , \quad a_{b\bar{b}} \sim 0.18 \, \mathrm{fm} \gtrsim r_{max} \, .$$

Here e_q and m_q are the charge and mass of the quark, $\alpha = 1/137$.

To describe the spectra and widths of radiative transitions in heavy mesons, it is necessary to construct phenomenological models

Non-relativistic potential models

To describe the spectra of quarkonia $c\bar{c}$, $b\bar{b}$ in some approximation, nonrelativistic potential models with potentials of different types were used

$$V(r) = C_0 \, \ln \frac{r}{r_0},$$
$$V(r) = A + B\, r^{\alpha}, \qquad \alpha = 0.1\,,$$
$$V(r) = -\frac{a}{r} + br\,.$$

The radiation operators obtained in the non-relativistic approximation are used

$$H_{rad} = -e\frac{\boldsymbol{A}\cdot\boldsymbol{p}}{m} - 2\mu\boldsymbol{s}\cdot\boldsymbol{B}$$

Taking into account relativistic effects

The first important step to take into account relativistic effects in spectra was made in the work:
S. Godfrey and N. Isgur, Mesons in a Relativized Quark Model with Chromodynamics, Phys. Rev. D **32**, 189 (1985)

$$
\begin{aligned}
&H = H^{(0)} + \Delta H^{(S)}\,,\\
&H^{(0)} = h_1 + h_2 + U_g(r) + U_{conf}(r)\,,\\
&h_1 = \sqrt{m_1^2 + \boldsymbol{p}^2}, \quad h_2 = \sqrt{m_2^2 + \boldsymbol{p}^2},\\
&U_g(r) = -\frac{g}{r}\,, \quad U_{conf}(r) = br + \mathcal{C}\,.
\end{aligned}
$$

The potential $U_g(r)$ corresponds to the exchange of a 4-vector, and the potential $U_{conf}(r)$ corresponds to the exchange of a scalar

Taking into account relativistic effects

The term $\Delta H^{(S)}$ is the sum of the linear $\Delta H_1^{(S)}$ and quadratic $\Delta H_2^{(S)}$ over the spins of the contributions,

$$\Delta H_1^{(S)} = \frac{1}{4}\left(\frac{g}{r^3} - \frac{b}{r}\right)\left[(\boldsymbol{L}\cdot\boldsymbol{S}_{tot})\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)\right.$$
$$\left. + (\boldsymbol{L}\cdot\boldsymbol{\Sigma})\left(\frac{1}{m_1^2}-\frac{1}{m_2^2}\right)\right] + \frac{g}{m_1 m_2 r^3}(\boldsymbol{L}\cdot\boldsymbol{S}_{tot}),$$

$$\Delta H_2^{(S)} = \frac{g}{m_1 m_2}\left[\frac{8\pi}{3}\delta(\boldsymbol{r})(\boldsymbol{S}_1\cdot\boldsymbol{S}_2) + \frac{1}{2r^3}\left[3(\boldsymbol{S}_{tot}\cdot\boldsymbol{n})^2 - \boldsymbol{S}_{tot}^2\right]\right].$$

Here $\boldsymbol{S}_{tot} = \boldsymbol{S}_1 + \boldsymbol{S}_2$, $\boldsymbol{\Sigma} = \boldsymbol{S}_1 - \boldsymbol{S}_2$, $\boldsymbol{n} = \boldsymbol{r}/r$.

Disadvantages of the GI model: there is no spin-independent correction $\Delta H^{(0)}$, quark masses in the denominators in $\Delta H^{(S)}$, H_{rad} is used in the non-relativistic approximation

Guiding considerations for constructing the model

A.E. Bondar and A.I. Milstein, Relativistic effects in M1 radiative decays of heavy-light mesons, Phys. Rev. D, **112**, 054037 (2025)

Let's expand $\sqrt{(\boldsymbol{p}-e\boldsymbol{A})^2+m^2}$ in terms of $1/m$

$$\sqrt{(\boldsymbol{p}-e\boldsymbol{A})^2+m^2}=m+\frac{(\boldsymbol{p}-e\boldsymbol{A})^2}{2m}-\frac{((\boldsymbol{p}-e\boldsymbol{A})^2)^2}{8m^3}+\ldots$$

linear terms in $\boldsymbol{A}$ will be in all terms.

Now let's expand $\sqrt{(\boldsymbol{p}-e\boldsymbol{A})^2+m^2}$ in $1/\sqrt{\boldsymbol{p}^2+m^2}$ to terms linear in $\boldsymbol{A}$

$$\sqrt{(\boldsymbol{p}-e\boldsymbol{A})^2+m^2}=\sqrt{\boldsymbol{p}^2+m^2}-e\boldsymbol{A}\cdot\boldsymbol{v}\,,\quad \boldsymbol{v}=\frac{\boldsymbol{p}}{\sqrt{\boldsymbol{p}^2+m^2}}\,,$$

we have only one term!

Let us recall the formula from classical electrodynamics

$$dI_\omega=\frac{e^2\omega^2}{(2\pi)^2c^3}\left|\int dt\,[\boldsymbol{n}\times\boldsymbol{v}(t)]\,\exp\left[i\omega\Big(t-\boldsymbol{n}\cdot\boldsymbol{r}(t)/c\Big)\right]\right|^2 d\Omega_{\boldsymbol{n}}\,d\omega\,.$$

In our model, the Hamiltonian describing the spectrum has the form $H = H^{(0)} + \Delta H^{(0)} + \Delta H^{(S)}$, where $H^{(0)}$ coincides with GI.
For $\Delta H^{(0)} = \Delta H_g^{(0)} + \Delta H_b^{(0)}$ we have

$$\Delta H_g^{(0)} = -\frac{g}{4}\left[\frac{p^i}{h_2}\left(\frac{\delta^{ij}}{r} + \frac{r^i r^j}{r^3}\right)\frac{p^j}{h_1} + \frac{p^i}{h_1}\left(\frac{\delta^{ij}}{r} + \frac{r^i r^j}{r^3}\right)\frac{p^j}{h_2}\right] + \frac{\pi g}{2}\left[\frac{1}{h_1}\,\delta(\boldsymbol{r})\frac{1}{h_1} + \frac{1}{h_2}\,\delta(\boldsymbol{r})\frac{1}{h_2}\right],$$

$$\Delta H_b^{(0)} = \frac{b}{2}\left[\frac{1}{h_1}\left(\frac{1}{2r} - p^i r p^i\right)\frac{1}{h_1} + \frac{1}{h_2}\left(\frac{1}{2r} - p^i r p^i\right)\frac{1}{h_2}\right],$$

where $h_{1,2} = \sqrt{\boldsymbol{p}^2 + m_{1,2}^2}$. If we replace $h_{1,2} \to m_{1,2}$, we obtain the Breit Hamiltonian.

For the spin part of the Hamiltonian $\Delta H^{(S)}$ we have

$$\Delta H^{(S)} = \frac{1}{2h_1}\left(\frac{g}{r^3}-\frac{b}{r}\right)(\boldsymbol{L}\cdot\boldsymbol{S}_1)\frac{1}{h_1} + \frac{1}{2h_2}\left(\frac{g}{r^3}-\frac{b}{r}\right)(\boldsymbol{L}\cdot\boldsymbol{S}_2)\frac{1}{h_2}$$
$$+\frac{g}{2h_1}\frac{(\boldsymbol{L}\cdot\boldsymbol{S}_{tot})}{r^3}\frac{1}{h_2} + \frac{g}{2h_2}\frac{(\boldsymbol{L}\cdot\boldsymbol{S}_{tot})}{r^3}\frac{1}{h_1} + \frac{g}{2}\left[\frac{1}{h_1}\mathcal{P}\frac{1}{h_2} + \frac{1}{h_2}\mathcal{P}\frac{1}{h_1}\right],$$
$$\mathcal{P} = \frac{8\pi}{3}\delta(\boldsymbol{r})(\boldsymbol{S}_1\cdot\boldsymbol{S}_2) + \frac{1}{2r^3}\left[3(\boldsymbol{S}_{tot}\cdot\boldsymbol{n})^2 - \boldsymbol{S}_{tot}^2\right].$$

The lowest radial states with $L=0,\ J=0$ are considered; $L=0,\ J=1$; $L=1,\ J=2$; $L=1,\ J=0$ and two states with $L=1,\ J=1$.

Two states, Ψ_1 and Ψ_1', with $L = 1,\ J = 1$ are a superposition of the state ψ_1 with $S_{tot} = 0$ and ψ_2 with $S_{tot} = 1$:

$$\Psi_1(\boldsymbol{r}) = c_1\psi_1 + c_2\psi_2(\boldsymbol{r})\,, \quad \Psi_1' = c_2\psi_1(\boldsymbol{r}) - c_1\psi_2(\boldsymbol{r})\,.$$

The coefficients $c_{1,2}$ are found from the solution of the secular equation.

Important characteristic of the system: $\varphi = \arctan(c_1/c_2)$.

In the limit $m_2 \to \infty$, the Hamiltonian $\Delta H^{(S)}$ has the form

$$\Delta H^{(S)} = \frac{1}{2h_1}\left(\frac{g}{r^3} - \frac{b}{r}\right)\frac{1}{h_1}(\boldsymbol{L}\cdot\boldsymbol{S}_1)\,,$$

$$c_1 = -\sqrt{\frac{1}{3}}\,,\, c_2 = \sqrt{\frac{2}{3}}\,,\, \varphi^* = -\arctan(1/\sqrt{2}) = -35.3^\circ\,,\, \sigma = 1\,,$$

$$c_1 = \sqrt{\frac{2}{3}}\,,\, c_2 = \sqrt{\frac{1}{3}}\,,\, \varphi^* = \arctan(\sqrt{2}) = 54.7^\circ\,,\, \sigma = -1\,,$$

$$\sigma = \mathrm{sgn}\left\langle \psi_1 \left| \frac{1}{2h_1}\left(\frac{g}{r^3} - \frac{b}{r}\right)\frac{1}{h_1} \right| \psi_1 \right\rangle\,.$$

In the limit $m_2 \to \infty$, it is convenient to introduce the operator $\boldsymbol{j}_1 = \boldsymbol{L} + \boldsymbol{S}_1$. Then the probabilities $W_{1/2}$ and $W_{3/2}$ of finding $j_1 = 1/2$ and $j_1 = 3/2$ in the state corresponding to the wave function Ψ_1' are equal to

$$W_{1/2} = \frac{1}{3}(\sqrt{2}c_1 + c_2)^2 = \delta_{\sigma,-1}\,, \quad W_{3/2} = 1 - W_{1/2} = \delta_{\sigma,1}\,.$$

We received:

$W_{1/2} = 0.05$ for $b\bar{u}$, $b\bar{d}$, $b\bar{s}$, $b\bar{c}$

$W_{1/2} = 0.5$ for $c\bar{u}$, $c\bar{d}$ and $W_{1/2} = 0.23$ for $c\bar{s}$

Conclusion: The b quark can be considered heavy enough, but the c quark cannot

Radiative transitions in the relativistic potential model

Single-photon transitions are described by the Hamiltonian

$$H_{rad} = G^{(0)} + \frac{1}{2}\boldsymbol{S}_{tot}\cdot\boldsymbol{G}^{(S)} + \frac{1}{2}\boldsymbol{\Sigma}\cdot\boldsymbol{G}^{(\Sigma)} ,$$

$$G^{(0)} = -\frac{1}{2}\left\{\frac{1}{h_1}, e_1\boldsymbol{A}_1\cdot\boldsymbol{p}\right\} + \frac{1}{2}\left\{\frac{1}{h_2}, e_2\boldsymbol{A}_2\cdot\boldsymbol{p}\right\}$$

$$+\frac{g}{4}\Bigg[\frac{p^j}{h_2}\left(\frac{\delta^{ij}}{r}+\frac{r^i r^j}{r^3}\right)e_1 A_1^i\frac{1}{h_1} + \frac{1}{h_1}e_1A_1^i\left(\frac{\delta^{ij}}{r}+\frac{r^i r^j}{r^3}\right)\frac{p^j}{h_2}$$

$$-\left(h_1\leftrightarrow h_2,\, e_1\to e_2,\, A_1\to A_2\right)\Bigg]$$

$$+\frac{b}{2}\left[\frac{1}{h_1}\{r\,, e_1\boldsymbol{A}_1\cdot\boldsymbol{p}\}\frac{1}{h_1} - \frac{1}{h_2}\{r\,, e_2\boldsymbol{A}_2\cdot\boldsymbol{p}\}\frac{1}{h_2}\right].$$

Here $\{A,B\} = AB + BA$, $\boldsymbol{A}^*_W$ is the photon polarization vector, $\boldsymbol{A}^*_W\cdot\boldsymbol{k} = 0$,

$$\boldsymbol{A}_i = \boldsymbol{A}^*_W e^{-i\boldsymbol{k}\cdot\boldsymbol{r}_i}\,,\ \boldsymbol{r}_1 = \frac{m_2}{M}\boldsymbol{r}\,,\ \boldsymbol{r}_2 = -\frac{m_1}{M}\boldsymbol{r}\,,\ M = m_1 + m_2.$$

Comparison with the experiment

Model parameters: m_b, m_c, m_s, m_u, m_d, b, g, $\mathcal{C}_0$, $\mathcal{C}_1$

These parameters were found from the analysis of mass spectra:

- We examined the systems $b\bar{b}$, $c\bar{c}$. We found m_b, m_c, b.
- We considered the systems $b\bar{u}$, $b\bar{d}$. We found m_u, m_d, neglecting isotopy violation.
- We examined the system $b\bar{s}$. We found m_s.

Each system has its own g, $\mathcal{C}_0$, $\mathcal{C}_1$, since the sizes of the wave functions are different. The quark masses and the b parameter are the same in all systems.

Everything else (the remaining masses and widths of the radiative transitions) is predicted!

Comparison with the experiment

As a result, we obtained the following parameter values

Parameter $b = 0.1\,\mathrm{GeV}^2$

Quark masses (in GeV):
$m_b = 5$, $m_c = 1.6$, $m_s = 0.55$, $m_u = m_d = 0.25$

Parameter g and reduced mass μ (in GeV)

	$b\bar{b}$	$b\bar{c}$	$c\bar{c}$	$b\bar{s}$	$c\bar{s}$	$b\bar{u}$	$c\bar{u}$
g	0.57	0.75	0.78	0.82	0.93	0.94	0.95
μ	2.5	1.21	0.8	0.5	0.41	0.24	0.22

Comparison with the experiment

Mass spectrum in the $c\bar{c}$ and $b\bar{b}$ systems (in MeV)

	$M(\eta_c)$	$M(J/\psi)$	$M(\chi_{c0})$	$M(\chi_{c1})$	$M(h_c)$	$M(\chi_{c2})$
Predictions	2984	3093	3466	3513	3525	3544
Experiment	2984	3097	3415	3511	3525	3556
	$M(\eta_b)$	$M(\Upsilon)$	$M(\chi_{b0})$	$M(\chi_{b1})$	$M(h_b)$	$M(\chi_{b2})$
Predictions	9399	9459	9876	9894	9899	9907
Experiment	9399	9460	9859	9893	9899	9912

Mass spectrum in the $b\bar{u}$ and $b\bar{d}$ systems (in MeV)

	$M(B^{(-)})$	$M(B^{(-)*})$	$M(B_0^{(-)})$	$M(B_1^{(-)})$	$M(B_1^{(-)\prime})$	$M(B_2^{(-)})$
Predictions	5280	5382	5677	5711	5726	5740
Experiment	5279	5325	...	...	5726	5737
	$M(B^{(0)})$	$M(B^{(0)*})$	$M(B_0^{(0)})$	$M(B_1^{(0)})$	$M(B_1^{(0)\prime})$	$M(B_2^{(0)})$
Predictions	5280	5382	5677	5711	5726	5740
Experiment	5280	5325	...	...	5726	5740

Comparison with the experiment

Mass spectrum in the $b\bar{s}$ system (in MeV)

	$M(B_s)$	$M(B_s^*)$	$M(B_{s0})$	$M(B_{s1})$	$M(B'_{s1})$	$M(B_{s2})$
Predictions	5367	5443	5786	5815	5828	5840
Experiment	5367	5415	...	...	5829	5840

Mass spectrum in the $c\bar{u}$ and $c\bar{d}$ systems (in MeV)

	$M(D^{(0)})$	$M(D^{(0)*})$	$M(D_0^{(0)})$	$M(D_1^{(0)})$	$M(D_1^{(0)\prime})$	$M(D_2^{(0)})$
Predictions	1867	2020	2343	2413	2427	2452
Experiment	1865	2007	2343	2412	2422	2461
	$M(D^{(+)})$	$M(D^{(+)*})$	$M(D_0^{(+)})$	$M(D_1^{(+)})$	$M(D_1^{(+)\prime})$	$M(D_2^{(+)})$
Predictions	1867	2020	2343	2413	2427	2452
Experiment	1870	2010	2343	...	2426	2464

Mass spectrum in the $c\bar{s}$ system (in MeV)

	$M(D_s)$	$M(D_s^*)$	$M(D_{s0})$	$M(D_{s1})$	$M(D'_{s1})$	$M(D_{s2})$
Predictions	1968	2122	2455	2525	2542	2569
Experiment	1968	2112	2317	2460	2535	2569

Predictions for the widths of the radiative transitions $D_{sJ} \to D_s\gamma$ and $D_{sJ} \to D_s^*\gamma$ (in keV)

Paper	[?]	[?]	[?]	[?]	[?]	[?]	Our work
$\Gamma(D_{s0} \to D_s^*\gamma)$	1.9	14.5-24.9	5.46	4.92	2.06-2.07	...	4.51
$\Gamma(D_{s1} \to D_s\gamma)$	6.2	10.3-17.2	13.2	12.8	3.53-3.61	...	49.66
$\Gamma(D_{s1} \to D_s^*\gamma)$	5.5	14.0-25.1	17.4	15.5	4.74-4.79	...	1.95
$\Gamma(D'_{s1} \to D_s\gamma)$	15	25.2-31.1	61.2	54.5	18.18-18.85	1.6 ± 2.3	1.51
$\Gamma(D'_{s1} \to D_s^*\gamma)$	5.6	14.6-22.8	9.21	8.9	2.96-3.02	$10.4 \pm 1.$	16.35
$\Gamma(D_{s2} \to D_s^*\gamma)$	19	41.5-55.9	49.6	44.1	15.23-15.66	9.4 ± 2.0	3.75

Conclusion

- Within the framework of the relativistic potential model, a detailed analysis of the spectra and partial widths of radiative transitions in various systems of heavy mesons was carried out
- Qualitative agreement with all available experimental data was obtained
- Taking relativistic effects into account allows us to explain experimental data that cannot be explained by other models
- Predictions for meson masses and partial widths of radiative transitions remain finite even for zero light quark mass
- The results will be useful in planning experiments dedicated to the study of heavy meson physics