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COSMOLOGY AND
ASTROPARTICLE PHYSICS

«Pockets» of macroscopic extra dimensions in $f(R)$ -gravity

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Plan

- ❖ Motivation
- ❖ Quadratic $f(R)$ -gravity
- ❖ Domain walls in $f(R)$ -gravity
- ❖ Mechanism of inflationary generation
- ❖ Primordial Black Holes?
- ❖ Conclusion

Motivation: $f(R)$ -gravity

$$L_G = f(R) + c_1 R_{AB} R^{AB} + c_2 R_{ABCD} R^{ABCD} + \dots \quad (1)$$

- Higher order corrections for curvature should arise in effective gravitational action after quantization.
- The most successful model of cosmological inflation is based on $f(R)$ -gravity (the Starobinsky model):

$$S = \frac{1}{2\kappa} \int \left(R + \frac{R^2}{6M^2} \right) \sqrt{|g|} d^4x, \quad (2)$$

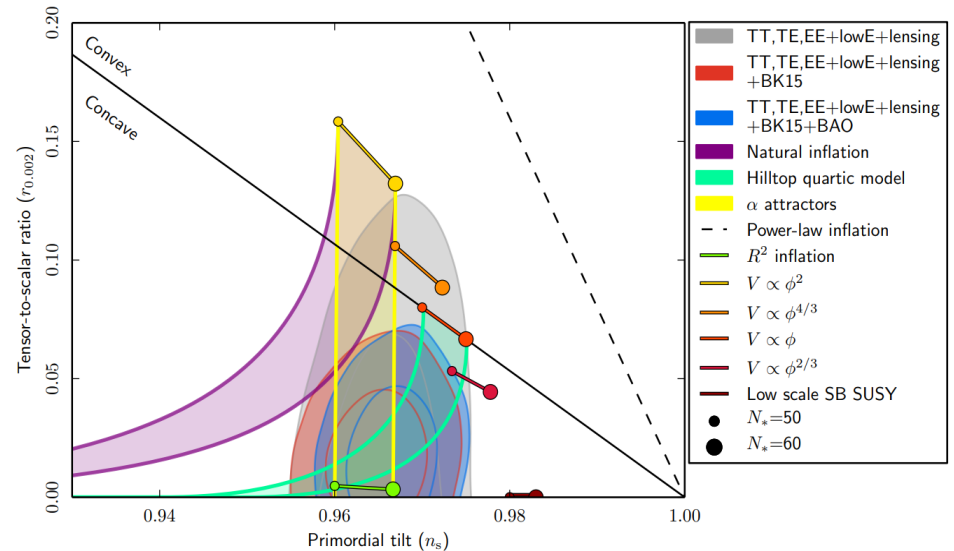
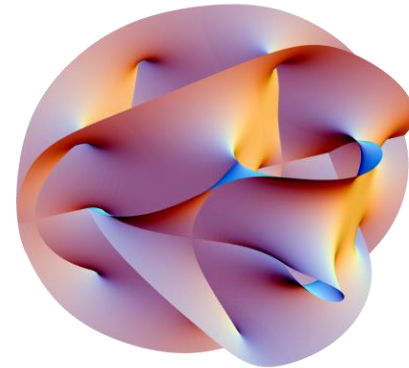


Fig. 3. Constraints on inflationary models according to [Planck 2018 results. X. Constraints on inflation]. The Starobinsky model lies at the very center of the constraints.

Motivation: Extra dimensions

$$S[g_{\mu\nu}] = \frac{m_D^{D-2}}{2} \int d^{4+n}x \sqrt{|g_D|} \dots \quad (3)$$

- Hopes for quantum gravity
(cancelling anomalies in string theory)
- Geometrization of gauge symmetry
in Kaluza-Klein style
- Solving gauge hierarchy problems
(for example in Randall-Sundrum type
models)
- Just “why 3+1?”



Quadratic $f(R)$ -gravity

$$S = \frac{m_D^{D-2}}{2} \int d^{4+n}x \sqrt{|g_D|} [f(R) + c_1 R_{AB} R^{AB} + c_2 R_{ABCD} R^{ABCD}] , \quad (4)$$

$$f(R) = a_2 R^2 + R - 2\Lambda_D ,$$

1. Subspace decomposition¹: $M = M_4 \times S_n$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\beta(x)} b_{ab} dx^a dx^b$$

$$R = R_4 + R_n + P_k , \quad P_k = 2n \partial^2 \beta + n(n+1)(\partial\beta)^2 , \quad (5)$$

2. Integrating out coordinates of the extra space :

$$S = \frac{1}{2} \mathcal{V}[n] m_D^2 \int \sqrt{^4g} d^4x e^{n\beta} [f(R) + \dots] \quad (6)$$
$$V_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})} .$$

¹ K. A. Bronnikov and S. G. Rubin, Phys. Rev. D 73 (2006) 124019 [[gr-qc/0510107](https://arxiv.org/abs/gr-qc/0510107)]

Effective 4-dim theory

3. Small extra dimensions and slow change approximation ($R_n \equiv \phi$):

$$R_4, P_k \ll R_n$$

$$S = \frac{1}{2} \mathcal{V} [n] m_D^2 \int \sqrt{4g} d^4x e^{n\beta} [f(R_n) + f'(R_n)(R_4 + P_k) + \dots] \quad (7)$$

4. Conformal transformation: $g_{\mu\nu} \mapsto \tilde{g}_{\mu\nu} = |e^{n\beta} f'(\phi)| g_{\mu\nu}$

$$S_{low} = \frac{1}{2} v_n \int d^4x \sqrt{g_4} \text{sign}(f') [R_4 + K(\phi)(\partial\phi)^2 - 2V(\phi)], \quad (8)$$

$$V(\phi) = -\frac{\text{sign}(f')}{2(f')^2} \left[\frac{|\phi|}{n(n-1)} \right]^{n/2} \left[f(\phi) + \frac{c_1 + 2c_2/(n-1)}{n} \phi^2 \right] \quad (9)$$

$$K(\phi) = \frac{1}{4\phi^2} \left[6\phi^2 \left(\frac{f''}{f'} \right)^2 - 2n\phi \left(\frac{f''}{f'} \right) + \frac{n(n+2)}{2} \right] + \frac{c_1 + c_2}{f'\phi}$$

Domain walls in $f(R)$

$$V(\phi) = -\frac{\text{sign}(f')}{2(f')^2} \left[\frac{|\phi|}{n(n-1)} \right]^{n/2} \left[f(\phi) + \frac{c_1 + 2c_2/(n-1)}{n} \phi^2 \right] \quad (10)$$

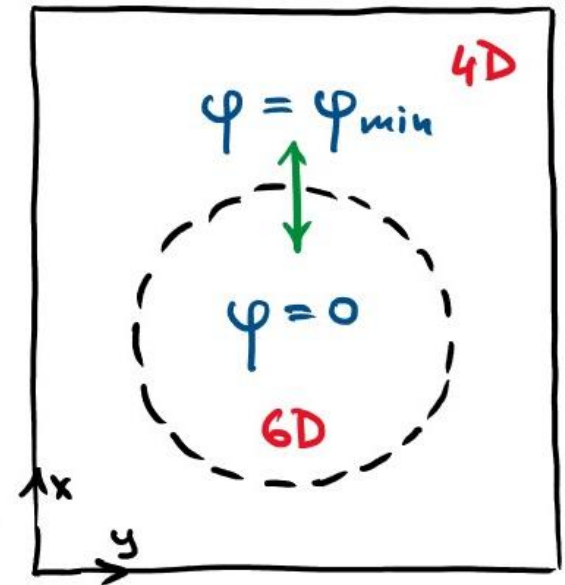
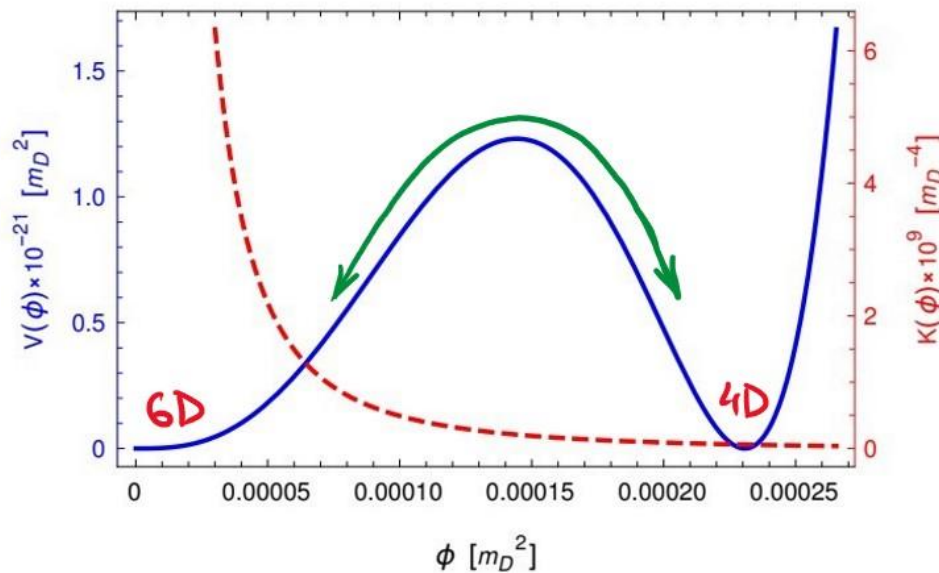


Fig. 4. Potential $V(\phi)$ and kinetic factor $K(\phi)$ of the scalar field for the parameters:
 $n = 6, c_1 = -8000, c_2 = -5000, a_2 = -500$.

Generation on Inflationary stage

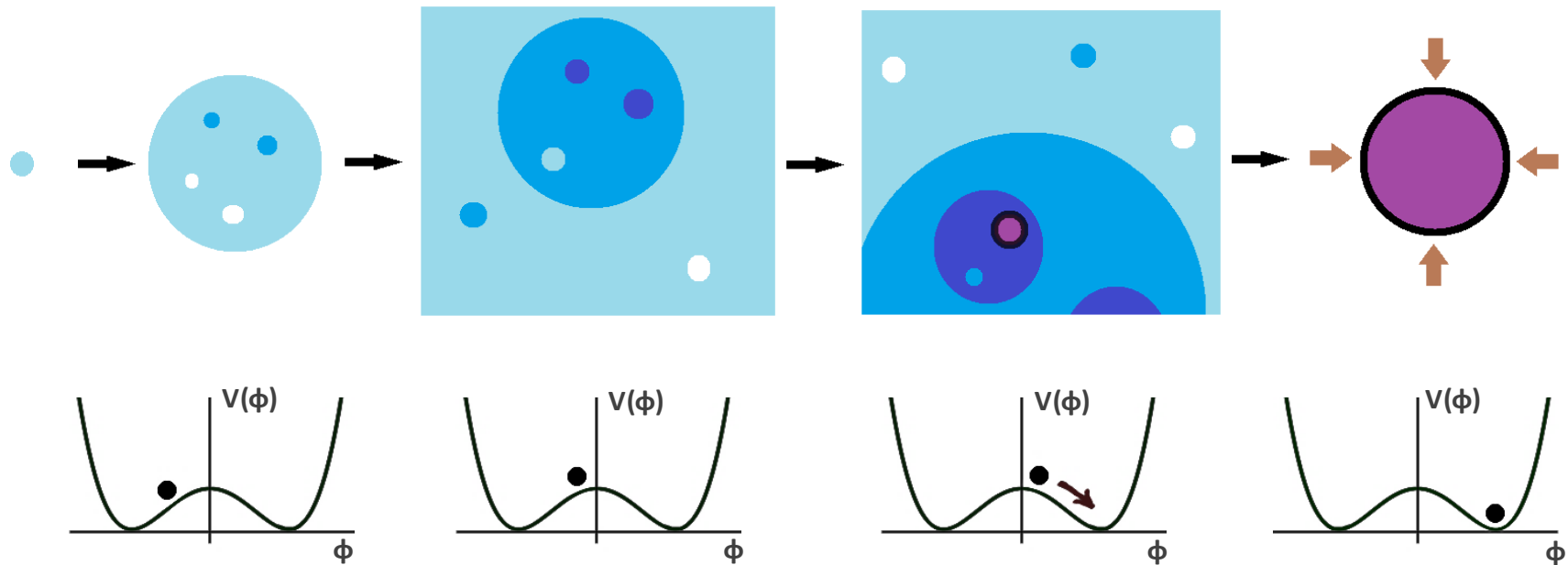


Fig.6. The formation² of a critical region, which will be further surrounded by a domain wall as a result of repeated fluctuations on a background of inflationary stage.

² S. G. Rubin, M. Yu. Khlopov and A. S. Sakharov, Grav. & Cosm. 6 (2000) [[hep-ph/0005271](https://arxiv.org/abs/hep-ph/0005271)]

Inflationary constraints

- Slow change approximation must continue to satisfy $\phi \equiv R_n \gg R_4$.
Curvature of 4-dim space during inflation: $R_4 \simeq 12 H^2$.
- The generation of domain walls by quantum fluctuations requires a slow rolling of ψ : $\sqrt{V''(\psi_{max})} \ll H$.
- The domain walls should not be too dense so as not to dominate the inflaton: $\varepsilon_\psi \ll \varepsilon_{inf} \sim H^2 m_4^2$.
- The field fluctuations during inflation should not be too large to prevent overproduction of the domain walls: $\delta\psi = \frac{H}{2\pi} \ll \psi$.

Black holes or not?

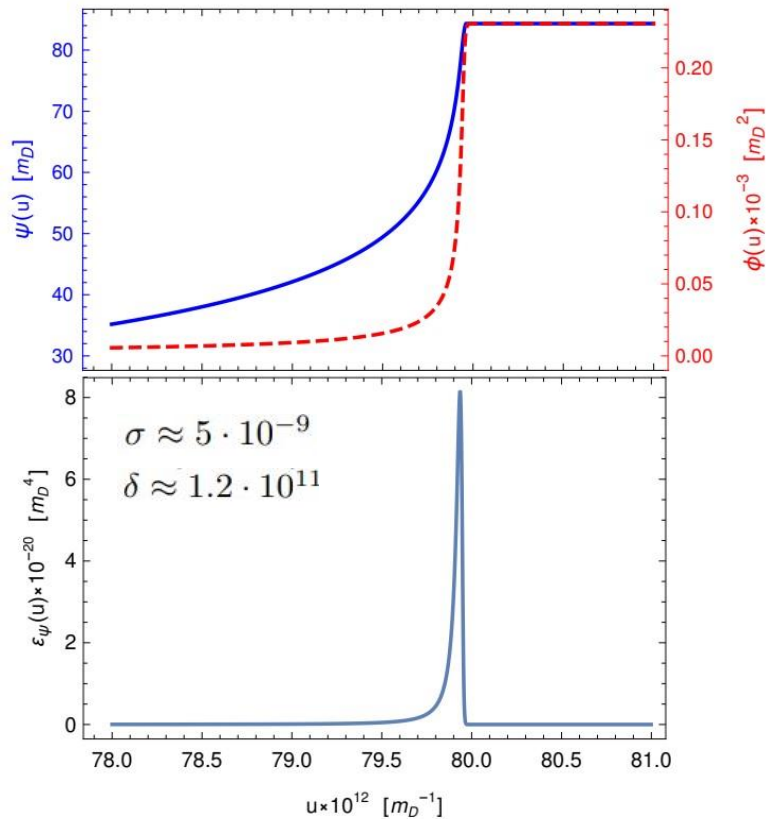


Fig. 5. Top - the value of the field when moving through the wall. Bottom - energy density when moving through the wall.

$$\psi = m_4 \int_{\phi_0}^{\phi} \sqrt{K(\phi')} d\phi', \quad (11)$$

$$\psi_{uu} + \frac{2\psi_u}{u} - \tilde{V}'(\psi) = 0, \quad (12)$$

- Black hole immediately forms if:

$$\frac{u_g}{u_w} = 8\pi G\sigma u_w > 1 \quad (13)$$

Conclusion

- In multidimensional $f(R)$ -gravity, the formation of bubbles of decompactified vacuum inside the compactified 4-dimensional Universe is possible during cosmological inflation.
- This do not disturb the inflaton dominance necessary for cosmological inflation, if the parameters of the $f(R)$ -model are chosen correctly.
- If the domain wall density is high enough, such bubbles immediately become submerged under the gravitational radius. It becomes the mechanism for production of primordial black hole (PBH).
- If there exists a region of initial $f(R)$ -parameters where the domain walls are light, then pockets of decompactified vacuum become observable.
- Do such “pockets” stabilize or collapse? What do they look like (gravitational lensing)? What happens if matter gets into the pocket?

[arXiv:2204.06360](https://arxiv.org/abs/2204.06360)

[10.3389/fspas.2022.927144](https://doi.org/10.3389/fspas.2022.927144)

Thank you for attention!

ANY QUESTIONS?

Фазовые переходы

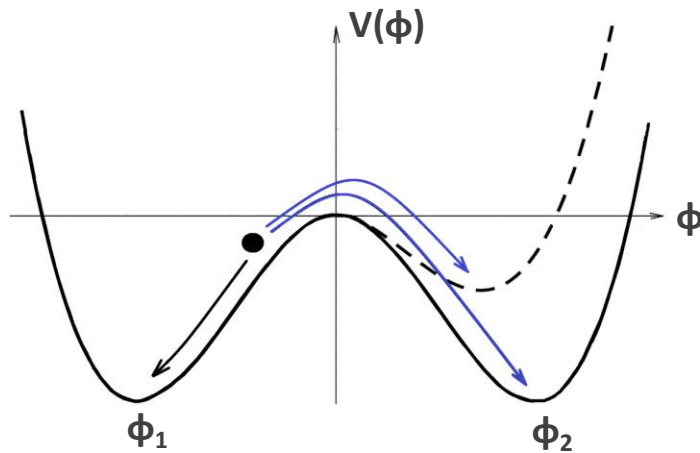


Рис. 1. Потенциалы содержащие несколько минимумов создают формирования полевых доменных стенок.

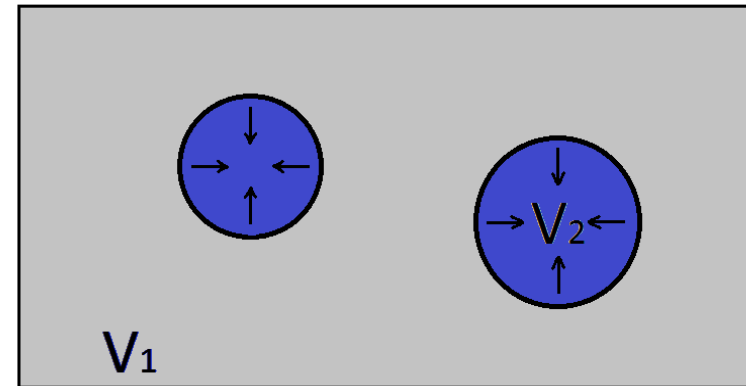


Рис. 1. Области с иным вакуумом оказываются окружены замкнутой полевой стенкой, которая начинает сжиматься под действием натяжения.