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Neutrino mass variables in 3 active and 2 sterile neutrino model

Rajeev Narasimhamurthy

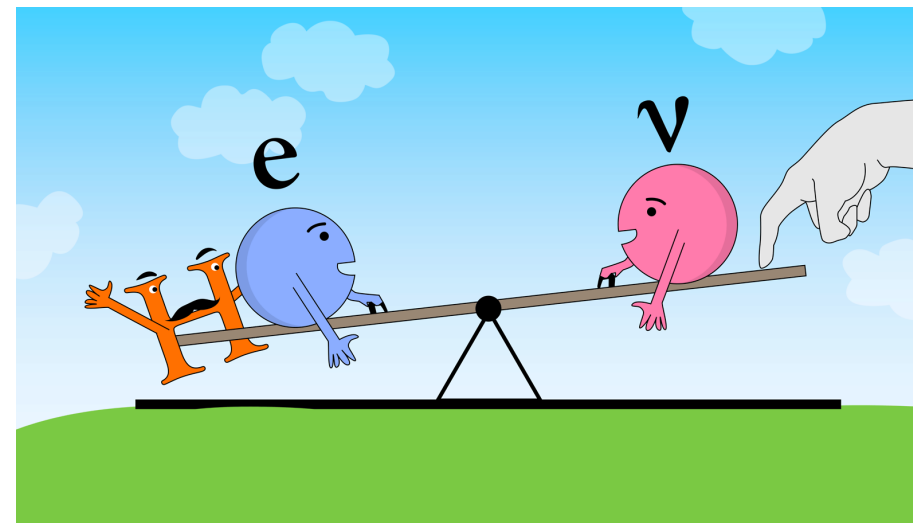
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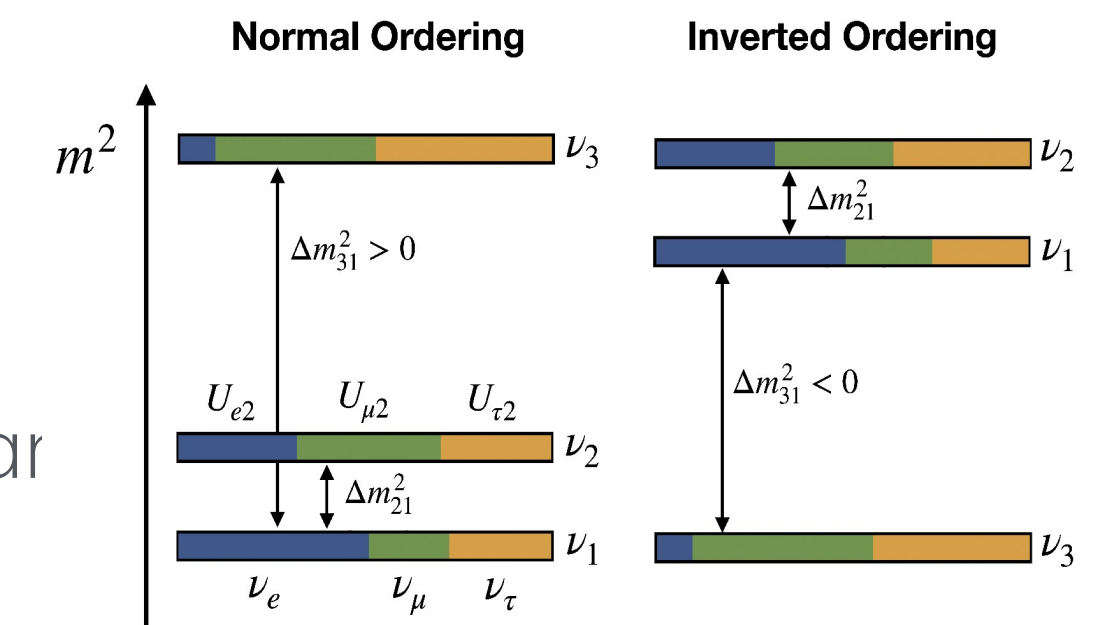
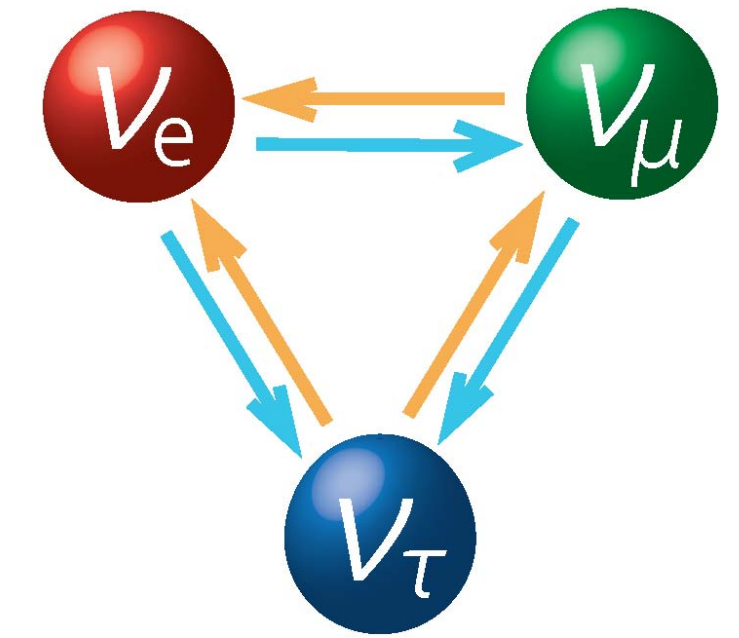


arXiv: 2603.17689 [hep-ph]

Neutrinos..!



- Within the **SM**, neutrinos were originally postulated as exactly **massless** particles, appearing in three copies (ν_e, ν_μ, ν_τ) corresponding to each of the charged leptons.
- **Lepton number** was assumed to be **conserved** independently for each lepton family.
- The discovery of **neutrino oscillations** demonstrated that neutrinos have tiny but **nonzero** masses and that lepton flavor is **not** a conserved quantity.
- The first evidence came from **atmospheric neutrinos** observed at Super-Kamiokande and solar neutrinos studied by SNO.
- Followed by confirmation from **reactor** (KamLAND, Daya Bay, RENO, Double Chooz) and **accelerator** experiments (K2K, MINOS, T2K, NOvA).
- These observations firmly establish that the three active neutrinos mix via the PMNS matrix, characterized by two mass-squared splittings ($\Delta m_{21}^2, \Delta m_{31}^2$), three mixing angles, and possibly CP-violating phases.
- While the three-flavor oscillation paradigm successfully explains the bulk of experimental data, several **anomalies** persist which cannot be accommodated within this framework



Known in Standard Picture

$$\Delta m_{21}^2 = (6.82 - 8.04) \times 10^{-5} eV^2$$

$$|\Delta m_{31}^2| = (2.42 - 2.59) \times 10^{-2} eV^2$$

$$\sin^2 \theta_{12} = (0.275 - 0.344)$$

$$\sin^2 \theta_{13} = (0.023 - 0.024)$$

$$\sin^2 \theta_{23} = (0.407 - 0.620)$$

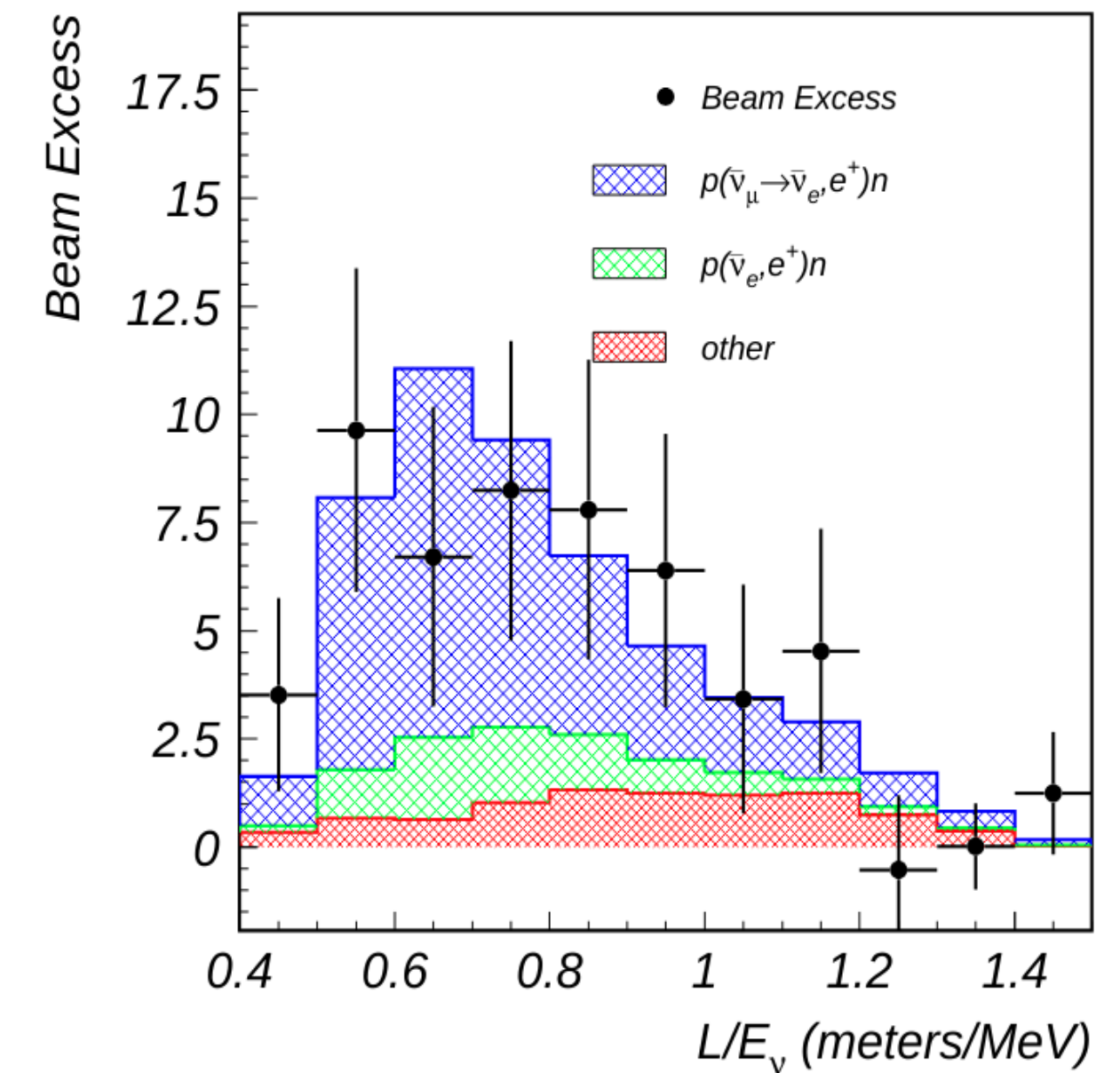
Unknown in Standard Picture

θ_{23} octant : $\theta_{23} > 45^\circ / \theta_{23} < 45^\circ$
 Mass ordering : $\Delta m_{31}^2 > 0 / \Delta m_{31}^2 < 0$
 Value of CP phase (δ_{CP}) = δ_{13}
 Absolute mass scale
 Dirac/Majorana

Neutrino Anomalies..!

LSND Anomaly

- LSND stands for Liquid Scintillator Neutrino Detector at Los Alamos National Laboratory, USA (1993–1998). This base has a length of ~30m and a neutrino energy of ~30 MeV.
- LSND used $\bar{\nu}_\mu$ produced from the muon decay at rest $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$
- Looked for $\bar{\nu}_e$ appearance via inverse beta decay $\bar{\nu}_e + p \rightarrow e^+ n$
- Found an **excess of $\bar{\nu}_e$** events over the expected background with statistical significance 3.8σ .
- This suggests oscillation $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ with $\Delta m^2 \sim 1 \text{ eV}^2$ - far larger than solar (7.4×10^{-5}) or atmospheric (2.5×10^{-3}) mass-squared splittings.
- Requires **an additional neutrino state**, since three neutrinos can only provide two independent Δm^2 .



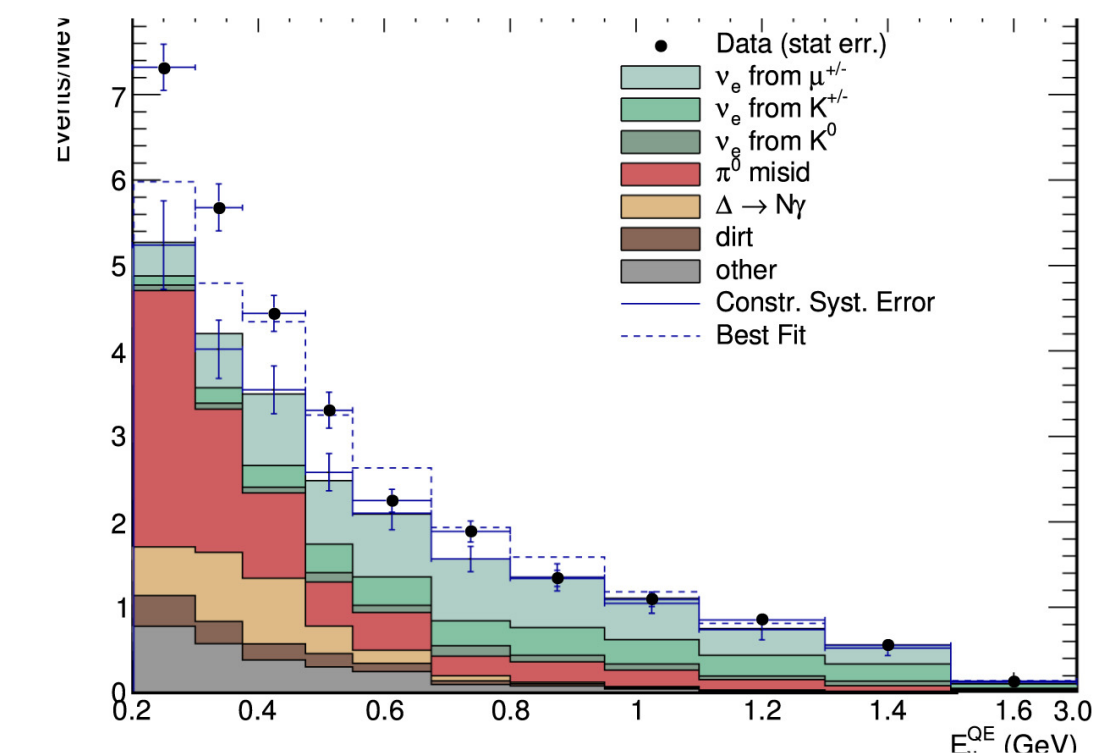
$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ at 3.8σ (C.

Athanassopoulos et al , PRL 1995)

MiniBooNE Anomaly

- MiniBooNE stands for Mini Booster Neutrino Experiment at Fermilab, USA (2002–2019): baseline: 541 m; Neutrino energy: 200–1250 MeV.
- **The goal was to** test LSND anomaly at higher energy and longer baseline independently.
- Setup: Neutrino beam from pion decay-in-flight, primarily ν_μ .
- MiniBooNE detected an excess of electron-like events in both ν and $\bar{\nu}$ modes.
- Combined excess significance: 4.8σ (2020).
- Consistent with **additional neutrino** oscillations $\nu_\mu \rightarrow \nu_e$ at $\Delta m^2 \sim 1 \text{ eV}^2$
- However, MiniBooNE can't distinguish electrons from single photons — so **photon-like backgrounds (e.g. π^0 misidentification)** might mimic a signal.
- **MicroBooNE** (2021, LArTPC detector) observed **no evidence for excess ν_e** events, casting doubt on the oscillation interpretation — though it doesn't fully explain MiniBooNE's anomaly.
- The puzzle remains unresolved.

MiniBooNE



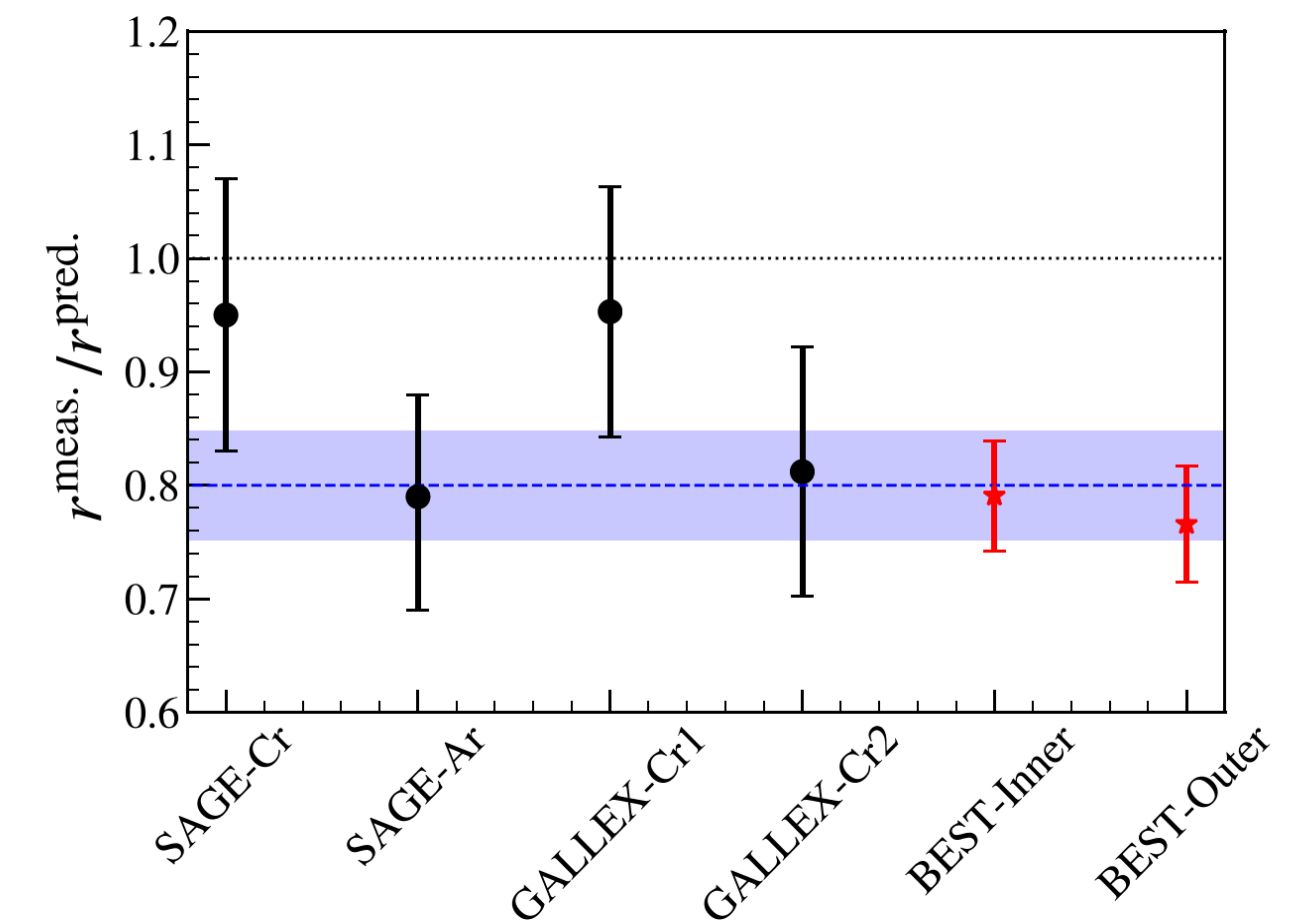
$\nu_\mu \rightarrow \nu_e$ at 4.8σ (Aguilar-Arevalo et al., PRL, 2009)

Reactor Antineutrino Anomaly

- Nuclear reactors produce huge fluxes of $\bar{\nu}_e$ from β -decays of fission fragments.
- Detected via inverse beta decay (IBD).
- Around **2011**, updated theoretical models (Huber and Mueller) predicted slightly higher $\bar{\nu}_e$ fluxes than older models.
- When compared to data from ~20 short-baseline reactor experiments ($L < 100$ m), a deficit of ~6% was observed. Leading to the statistical significance: 2.8σ .
- Could indicate oscillations into **an additional neutrino state** with $\Delta m^2 \sim 1 \text{ eV}^2$ and small mixing angle. However, recent data (DANSS, NEOS, STEREO, PROSPECT) suggest that the anomaly may instead arise from inaccurate reactor flux predictions, especially from U^{235} and Pu^{239} .
- Still debated. The flux-shape distortion (“5 MeV bump”) complicates the interpretation.

Gallium (Source) Anomaly

- **GALLEX** and **SAGE** — solar neutrino detectors calibrated using intense radioactive neutrino sources (eg. Cr^{51} and Ar^{37}) with baseline: ~1–2 m; Energy: ~1 MeV.
- Measured event rates were **10–20% lower** than predicted. Combined significance: **$\sim 2.8\sigma$** deficit.
- This may be due to the short-baseline oscillation $\nu_e \rightarrow \nu_s$ with $\Delta m^2 \sim 1 \text{ eV}^2$.
- Same mass range as LSND and MiniBooNE hints — potentially a common **additional neutrino** origin
- Recently, **BEST (Baksan Experiment on Sterile Transitions)** (2022) confirmed the deficit (~20%), strengthening the sterile neutrino interpretation.



Deficit in ν_e at GALLEX, SAGE, BEST ([Barinov et al., 2021](#))

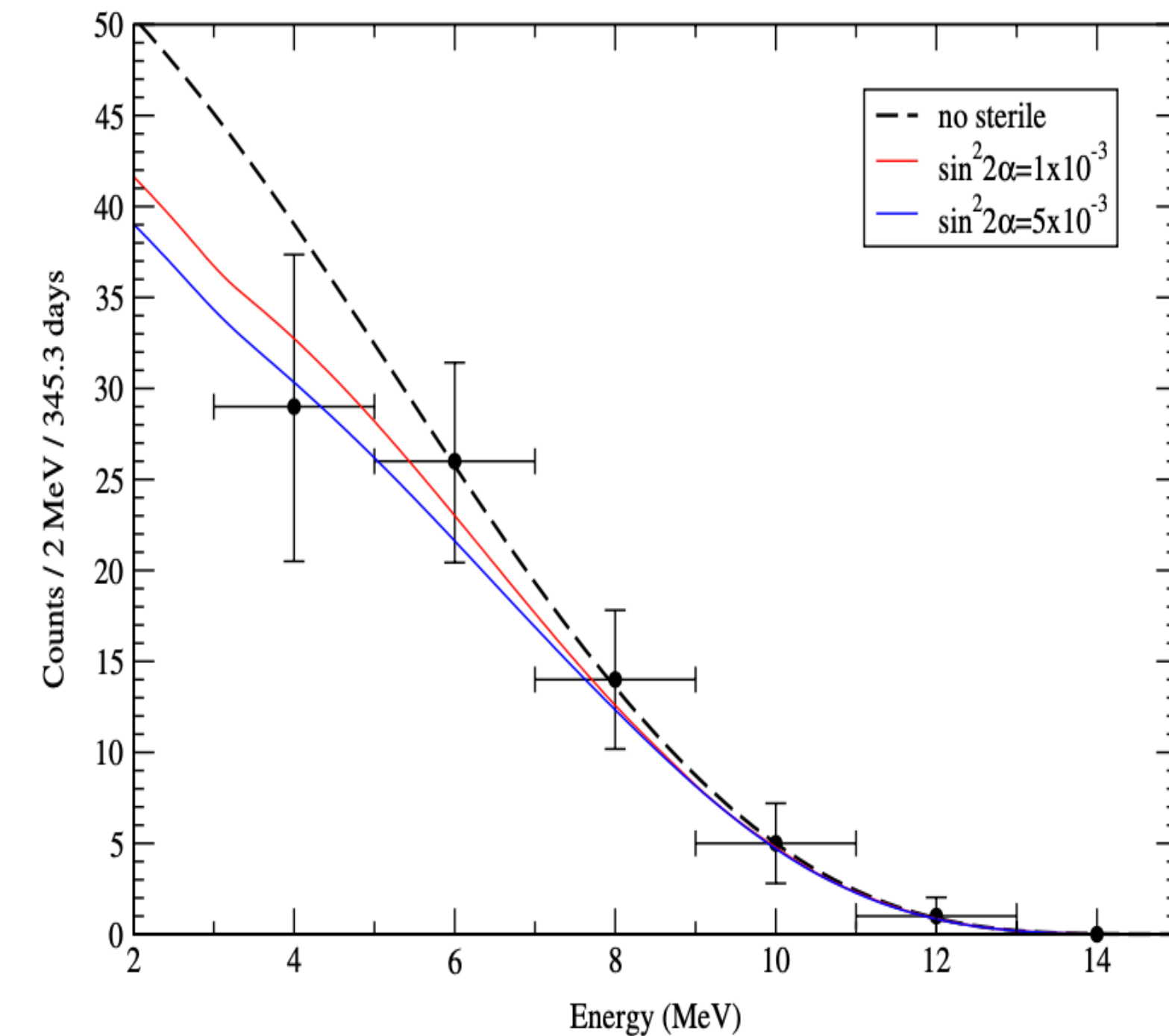
T2K and NOvA Tensions

- T2K (Japan) and NOvA (USA) are long-baseline accelerator neutrino experiments (study $\nu_\mu \rightarrow \nu_e$ appearance).
- **Goal:** Measure CP violation (δ_{CP}), mass ordering, and θ_{23} in the three-flavor framework.
- T2K favors large CP violation, NO and maximal $\theta_{23} \approx 45^\circ$: it observes more ν_e appearance events than NOvA.
- NOvA also prefers NO but δ_{CP} lies near 0 or π and a non-maximal θ_{23} : it observed ν_e appearance is **less** than T2K.
- When both datasets are combined under the **3-flavor framework**, there's a **$\sim 2-2.5\sigma$ tension** in the preferred values of δ_{CP} and θ_{23} .
- This may be interpreted as either statistical fluctuations or systematic uncertainty.
- If the δ_{CP} or θ_{23} tensions remain even after improved systematics, it could point to: NP - light **additional neutrino states** ($\Delta m_{n1}^2 \leq 10^{-2} \text{ eV}^2$)

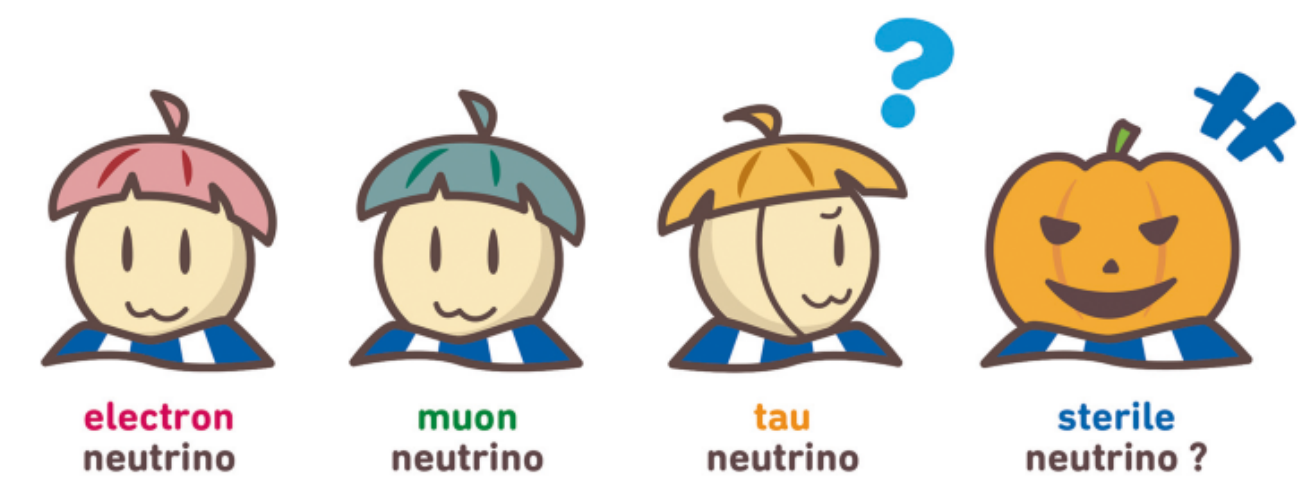
Solar upturn problem

- Solar MSW-LMA predicts a **rise (upturn)** in the electron-neutrino survival probability $P_{ee}(E)$ at low energies (few MeV \rightarrow sub-MeV), but current data (SNO, Super-K, Borexino) show a **weaker or absent upturn** than predicted.
- Introducing a very light **additional neutrino state** $\Delta m_{01}^2 \sim 10^{-5}$ adds an extra oscillation frequency with a long wavelength comparable to the Sun–Earth scale.
- This modifies the low-energy shape of $P_{ee}(E)$ and can suppress the expected upturn.

[PhysRevD. 83, 113011]



Sterile Neutrinos?



- Sterile neutrinos are hypothetical neutral fermions that **do not interact via the SM weak force** — only through **mixing with the active neutrinos** (ν_e, ν_μ, ν_τ).
- Why: Since the invisible width of the Z boson measured at LEP constrains the number of light active neutrinos to be three, any additional light states must be “sterile,” i.e., singlets under the SM gauge interactions.
- Additionally, these sterile states must be **non-thermal**: The early Universe’s expansion rate and structure formation depend on the **number of relativistic species** during recombination, quantified by N_{eff} .
- The Standard Model predicts $N_{eff} \approx 3.046$. A fully thermalized sterile neutrino would increase N_{eff} to ≈ 4 .
- Planck (CMB) + BAO data: $N_{eff} = 2.99 \pm 0.17$, disfavoring such extra species. Also, the total neutrino mass bound $\sum m_\nu < 0.12$ eV is inconsistent with an eV-scale sterile neutrino.
- This interprets: If sterile neutrinos exist, they must be **non-thermal, weakly coupled**, or **interacting with a dark sector**, preventing full thermalization.

Sterile Neutrinos?

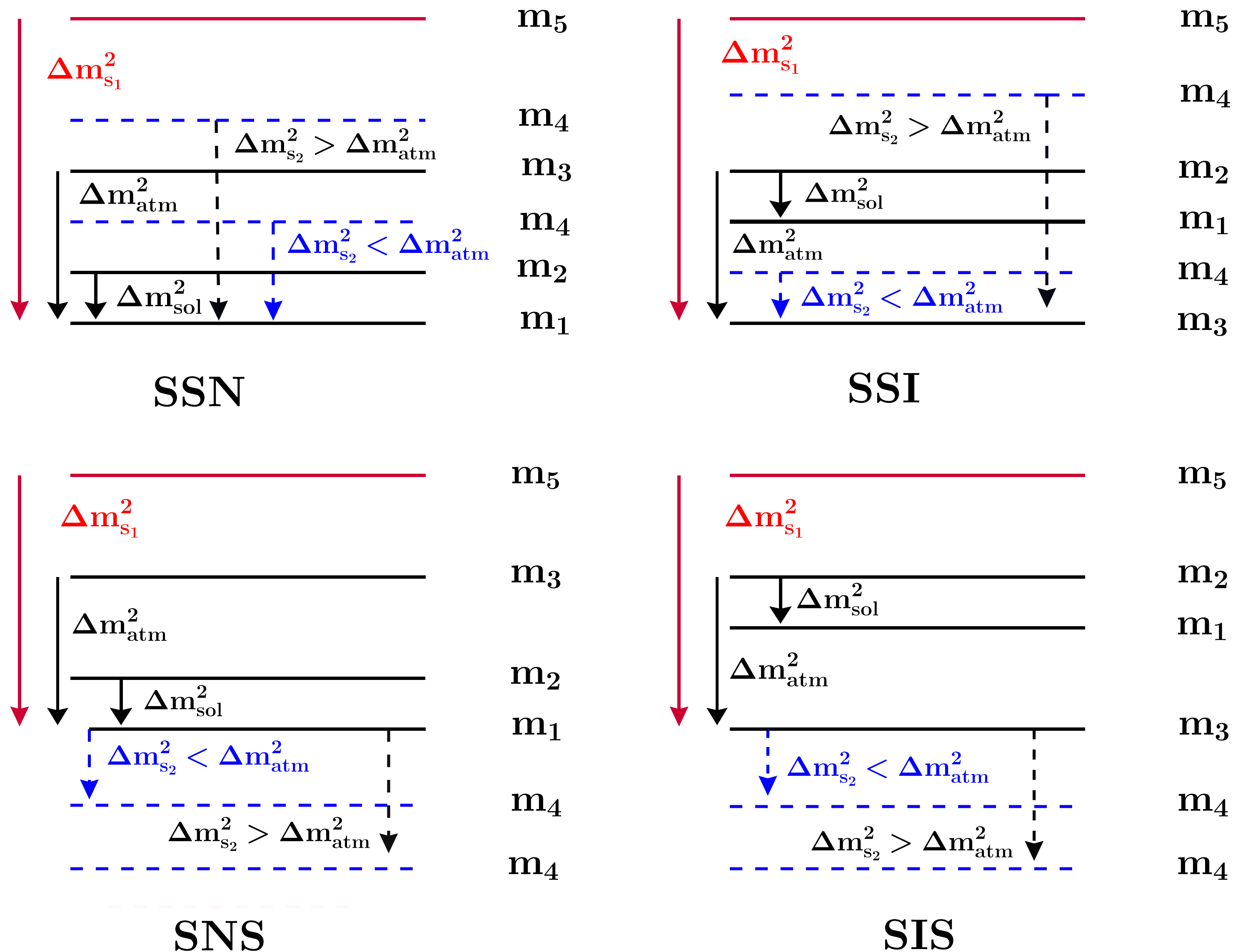


- Naturally appear in many extensions of the SM, e.g. **Type-I Seesaw models** (as right-handed neutrinos).
- Proposed to explain **neutrino anomalies** such as LSND, MiniBooNE, reactor and Gallium deficits. Also, help with tensions in long-baseline data (T2K–NOvA).
- They appear in Oscillations just by expanding the flavor states to include one (or more) sterile states:

$$\nu_{\alpha} = \sum_{i=1}^{3+N_s} U_{\alpha i} \nu_i, \quad \alpha = e, \mu, \tau, s$$

- This led to new mass-squared splittings and mixing angles.
- The extra oscillation frequency can cause **short-baseline appearance/disappearance** or **interference effects** in long-baseline experiments.
- Sterile neutrinos can exist in several mass scales: GeV–TeV (Seesaw), KeV (Warm dark matter), eV (LSND, MiniBooNE anomalies) and sub-eV (T2K–NOvA tension)

Classification of mass ordering in 3 + 2 sterile neutrino framework



Parameters	Case 1	Case 2
$\Delta m_{s_1}^2$	1.3 eV ²	1.3 eV ²
$\sin^2 \theta_{15}$	0.001 : 0.01	0.001 : 0.01
$\Delta m_{s_2}^2$	10 ⁻² eV ²	10 ⁻⁴ eV ²
$\sin^2 \theta_{14}$	5 × 10 ⁻⁴ : 5 × 10 ⁻³	0.1 : 0.2

Mass Constraints on 3+2 Sterile Neutrino model

Sum of neutrinos Σ

- Direct probes of the absolute neutrino mass scale arise from cosmological observations, which are sensitive to the sum of all neutrino mass eigenvalues,

$$\Sigma = \sum_i m_i.$$

- In the minimal three-neutrino framework, this quantity is determined by the lightest neutrino mass m_{lightest} together with the precisely measured mass-squared differences from oscillation experiments.

NO

$$m_1 = m_{\text{lightest}}, \quad m_2 = \sqrt{m_{\text{lightest}}^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_{\text{lightest}}^2 + \Delta m_{31}^2},$$

IO

$$m_3 = m_{\text{lightest}}, \quad m_2 = \sqrt{m_{\text{lightest}}^2 + |\Delta m_{32}|^2}, \quad m_1 = \sqrt{m_{\text{lightest}}^2 + |\Delta m_{32}|^2 - \Delta m_{21}^2}.$$

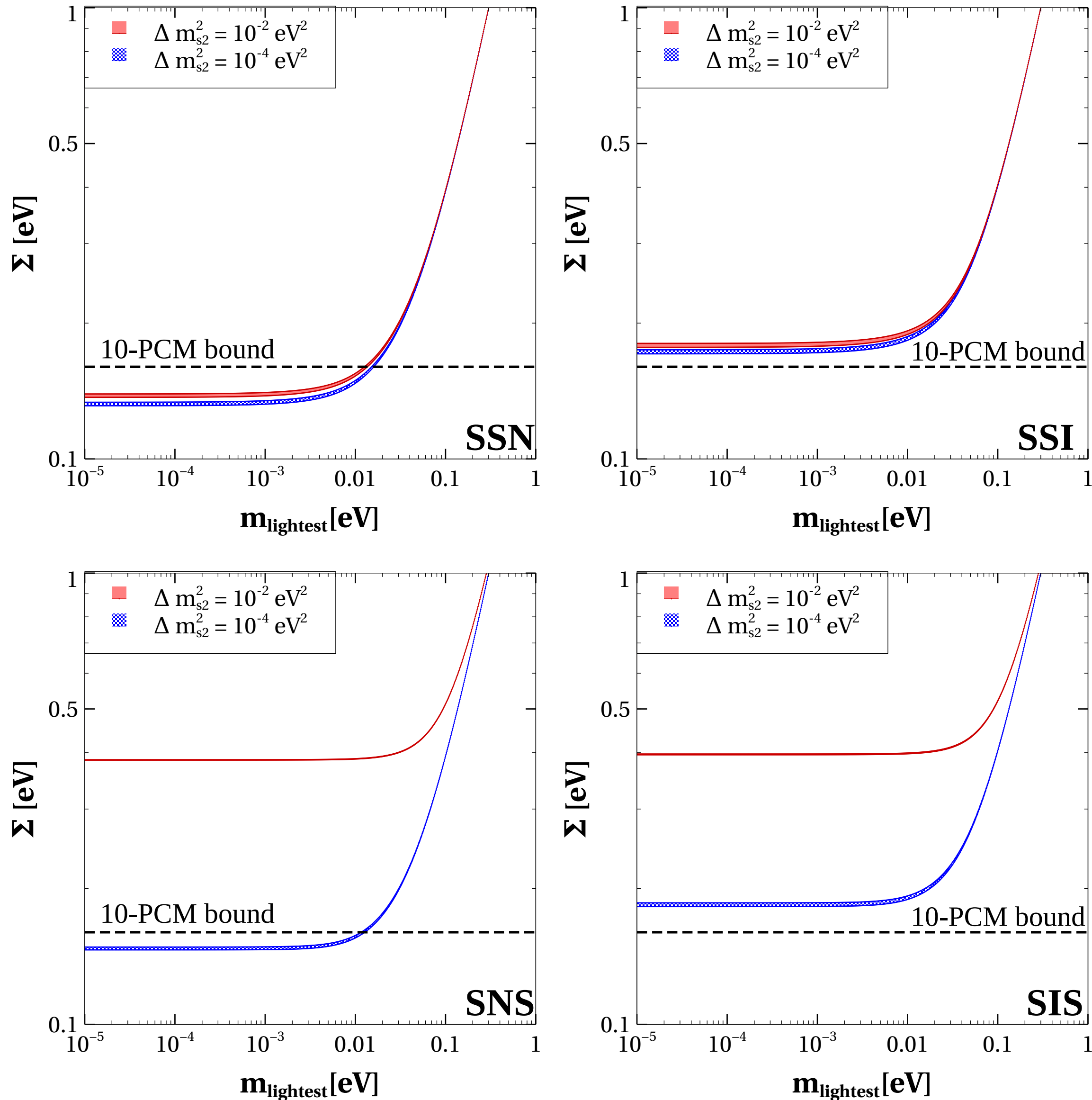
- The Planck 2018 data in combination with BAO measurements impose an upper bound $\Sigma \lesssim 0.12$ eV (95% C.L.)
- In the 3+2 framework, we have two additional mass eigenstates m_4 and m_5 which enter the sum.

Cosmological constraint on Σ

- If the active-sterile mixing is large, sterile neutrinos become fully thermalised in the early Universe, following the same Fermi-Dirac distribution as active flavors.
- As a result, they contribute equally to the effective number of relativistic degrees of freedom, N_{eff} , and to the total mass sum, which is enhanced relative to the three-flavor case.
- The precise measurement of CMB by the Planck collaboration constrains the $N_{\text{eff}} = 2.99^{+0.34}_{-0.33}$ (95 % *CL*) which ruled out the possibility of an extra sterile state.
- This tension can only be alleviated if the sterile neutrinos are not fully thermalized in the early Universe.
- In such cases, physical masses of the sterile neutrinos do not contribute to Σ .

$$\Sigma^{3+2} = m_1 + m_2 + m_3 + \Delta N_{\text{eff}} (m_4 + m_5).$$

Σ vs m_{lightest} in different mass schemes



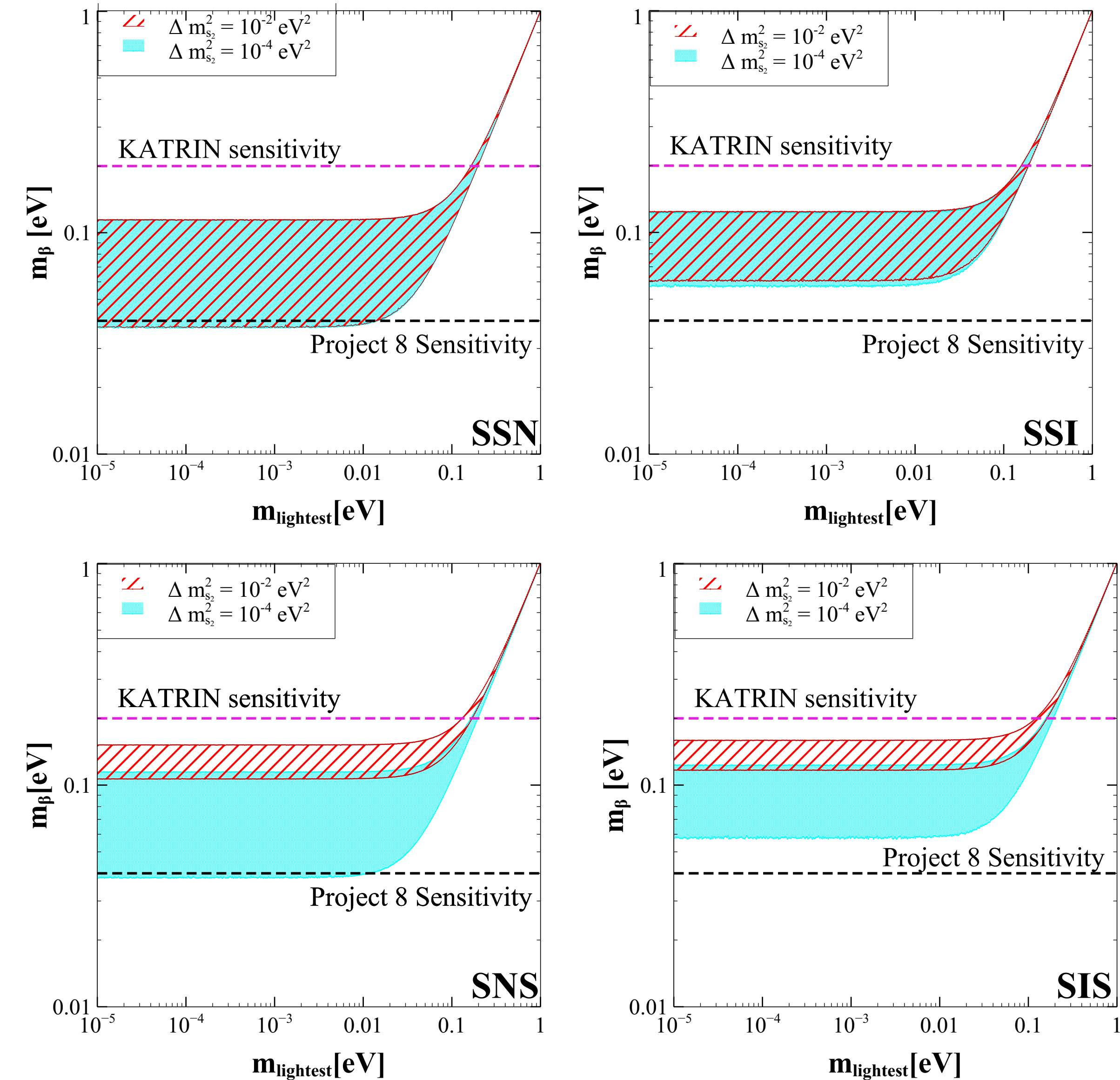
- SSN scenario is favored by 10-PCM bound up to $m_{\text{lightest}} \sim 0.015$ eV for both the mass squared differences, i.e $\Delta m_{s2}^2 = 0.01$ & 10^{-4} eV^2 .
- SSI scenario is disfavored by the 10-PCM bound for both the mass squared differences.
- SNS scenarios is disfavored for $\Delta m_{s2}^2 = 0.01 \text{ eV}^2$ however, it is allowed for $\Delta m_{s2}^2 = 10^{-4} \text{ eV}^2$ up to $m_{\text{lightest}} \sim 0.010$ eV as of 10-PCM bound is concerned.
- On the other hand, the SIS scenario is completely disfavor by the 10-PCM bound.

Mass ordering	$\Delta m_{s1}^2 = 1.3 \text{ eV}^2$	
	$\Delta m_{s2}^2 = 0.01 \text{ eV}^2$	$\Delta m_{s2}^2 = 10^{-4} \text{ eV}^2$
SSN	< 0.011	< 0.014
SSI	Disallowed	Disallowed
SNS	Disallowed	< 0.010
SIS	Disallowed	Disallowed

Effective Electron Neutrino Mass in Beta Decay

- The **effective electron-neutrino mass** measured in β -decay experiments is given by $m_\beta = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$,
- This quantity reflects the **incoherent sum** of neutrino mass contributions to the electron spectrum.
- In tritium β -decay: ${}^3\text{H} \rightarrow {}^3\text{He}^+ + e^- + \bar{\nu}_e$ the **endpoint energy** of the emitted electron depends on the neutrino mass.
- If neutrinos are massive, the **electron energy spectrum near the endpoint** is slightly distorted — the maximum kinetic energy of the electron is reduced by m_β .
- Thus, by precisely measuring this endpoint shape, one can put **a bound on the neutrino mass**, independent of any assumptions about whether neutrinos are Majorana or Dirac.
- KATRIN (2024) put the most stringent direct bound on the effective neutrino mass to date $m_\beta < 0.8$ eV (90% CL). Project 8 (atomic tritium): aims for $m_\beta \sim 40$ meV.
- When a sterile neutrino mixes with ν_e , the β -spectrum gains an **additional component**: it leaves a measurable imprint on the β -spectrum.

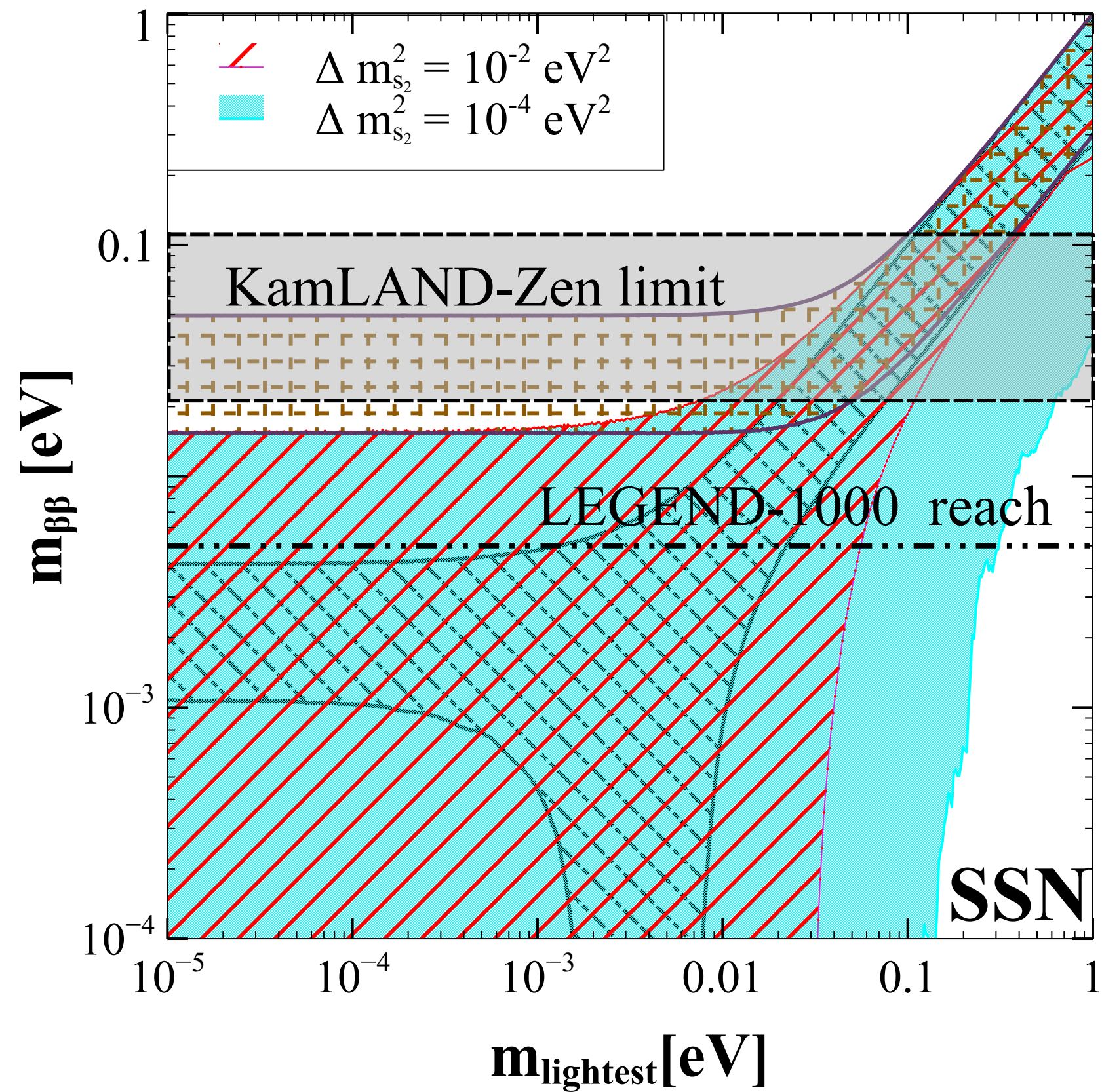
m_β vs m_{lightest} in different mass schemes



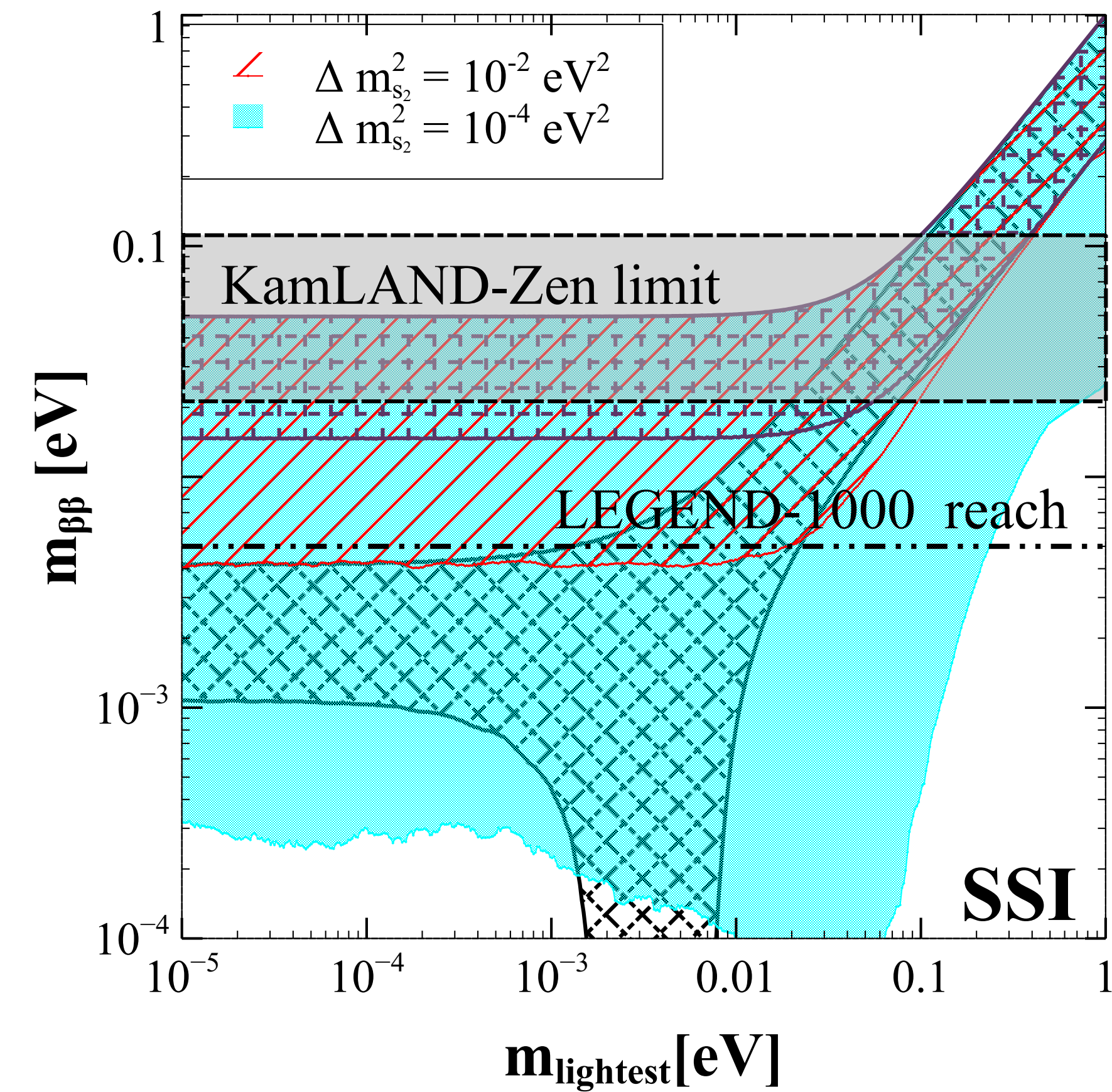
- There is an overlap of both contributions $\Delta m_{s_2}^2 = 0.01$ & 10^{-4} eV^2 in SSN and SSI. This is not the case with SNS and SIS.
- Although the current and projected KATRIN sensitivity ($m_\beta < 0.2$ eV) allows all the mass-ordering schemes considered, they are disfavored once the more stringent Project 8 sensitivity is taken into account.
- In the case of normal ordering, the SSN scenario is consistent with KATRIN up to $m_{\text{lightest}} \sim 0.015$ eV, while the SNS scheme is allowed up to $m_{\text{lightest}} \sim 0.01$ eV.
- $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$ contribution can meet Project 8 sensitivity in SNS upon considering the 3σ uncertainties.
- However, the 10-PCM bound on the lightest neutrino mass strongly excludes both inverted ordering schemes (SSI and SIS).

Effective Majorana Mass in Neutrinoless Double Beta Decay

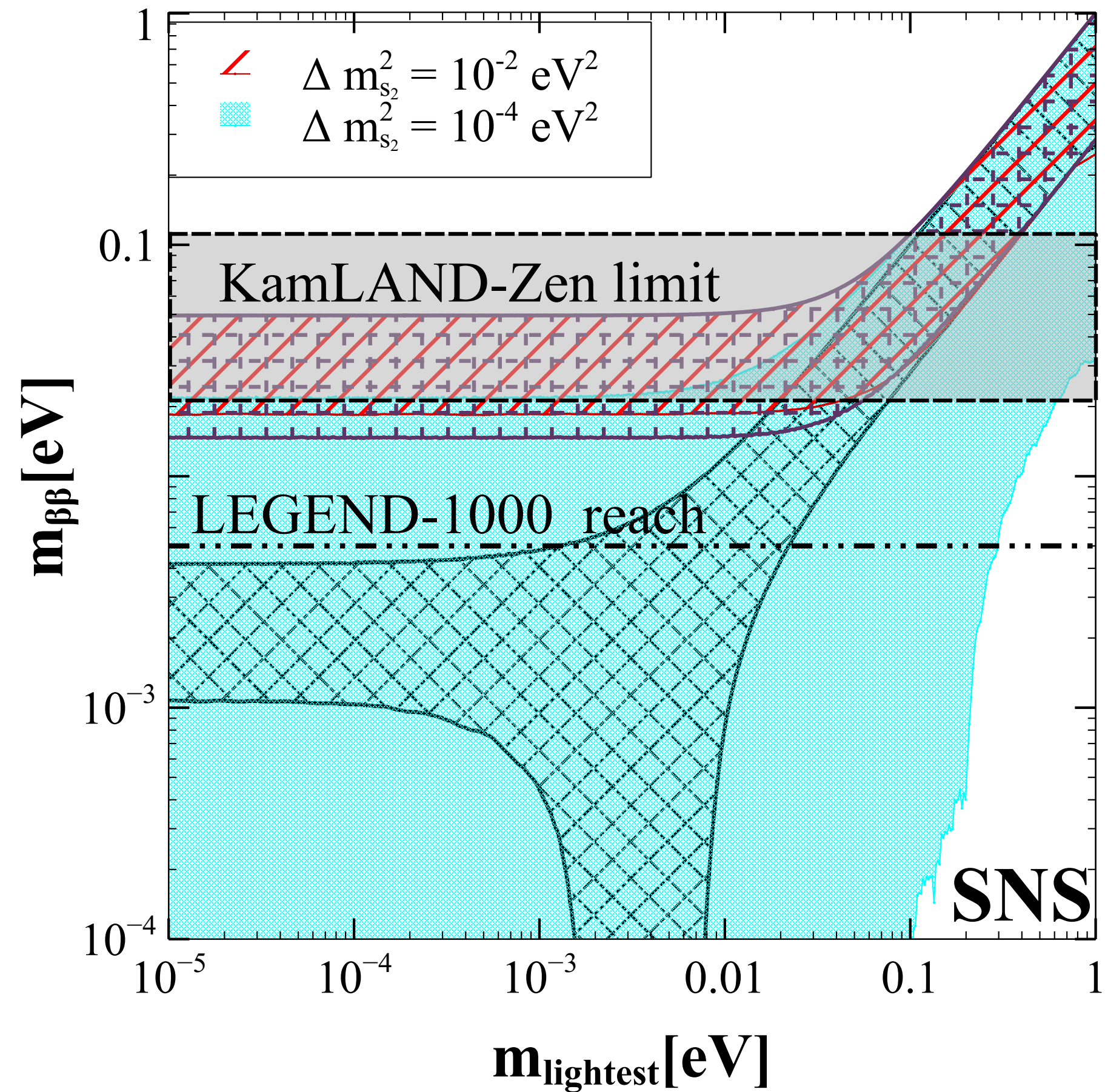
- The effective Majorana mass, $m_{\beta\beta} = \left| \sum_{i=1}^3 U_{ei}^2 m_i e^{i\alpha_i} \right|$,
- Probed in **neutrinoless double- β decay ($0\nu\beta\beta$)** — sensitive only if neutrinos are **Majorana**.
- Sensitive to phases and cancellations among mass terms: Because $m_{\beta\beta}$ involves coherent summation, the Majorana phases can lead to constructive or destructive interference
- Current limits (KamLAND-Zen, GERDA, CUORE): $m_{\beta\beta} \lesssim (0.028 - 0.122) \text{ eV}$.
- 3+2 sterile neutrinos **can significantly alter** both the magnitude and the interpretation of the $0\nu\beta\beta$ bounds.
- Next-generation detectors: **nEXO, LEGEND-1000, CUPID, KamLAND-Zen 800, NEXT** with target sensitivity $m_{\beta\beta} \sim 10 \text{ meV}$.
- We will next the plots for $m_{\beta\beta}$ vs m_{lightest} in different mass schemes



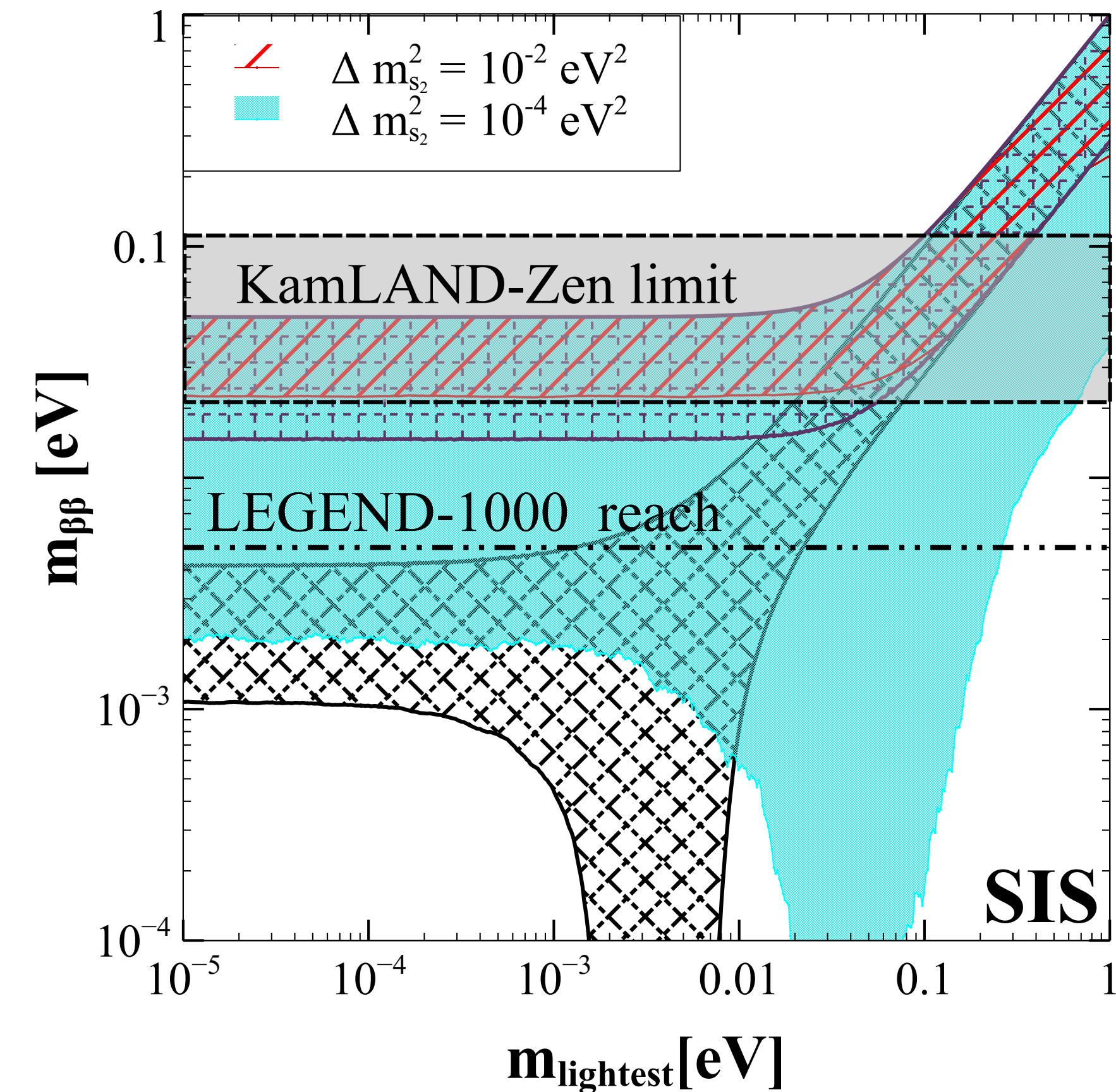
- **Light mass region:** Active contribution remains small $m_{\beta\beta}^{\text{NO}} \sim (1.2 - 4.1) \times 10^{-3}$ eV
- eV-scale sterile contribution dominates: $|t_{15}^2 m_5| \sim (1.14 - 11.52) \times 10^{-3}$ eV, while $|t_{14}^2 m_4|$ is subleading
- Complete cancellation possible for $\alpha_{51} \simeq \pi$; maximum value reaches $m_{\beta\beta}^{\text{SSN}} \lesssim 0.017$ eV.
- **Intermediate region:** Active contribution rises to $\mathcal{O}(10^{-2})$ eV and becomes comparable to sterile terms
- For $\Delta m_{s_2}^2 = 10^{-4}$ eV², $|t_{14}^2 m_4|$ becomes significant, allowing destructive interference: $m_{\beta\beta}^{\text{SSN}} \rightarrow \text{few} \times 10^{-4}$ eV
- **Quasi-degenerate region:** Active sector dominates: $m_{\beta\beta}^{\text{Std-NO}} \simeq (2.97 - 10.04) \times 10^{-2}$ eV
- Sterile contributions act as perturbations, though partial cancellation remains possible near $m_1 \sim 0.1$ eV
- Constructive phases $(\alpha_{41}, \alpha_{51}) = (0, 0)$ maximize $m_{\beta\beta}^{\text{SSN}}$, while $\alpha_{41}, \alpha_{51} \simeq \pi$ can induce strong cancellations.
- Sterile neutrinos significantly modify $m_{\beta\beta}$ through additional interference effects, especially in the low and intermediate mass regions



- **Light mass region:** Standard IO contribution dominates:
 $m_{\beta\beta}^{\text{Std-IO}} \simeq (1.53 - 4.92) \times 10^{-2} \text{ eV}$
- Sterile contributions remain smaller:
 $|t_{14}^2 m_4| \sim 10^{-3} (10^{-5}) \text{ eV}, \quad |t_{15}^2 m_5| \sim (0.11 - 1.15) \times 10^{-2} \text{ eV}$
- Complete cancellation is not possible, but destructive interference can suppress:
 $m_{\beta\beta}^{\text{SSI}} \sim 10^{-3} \text{ eV}$
- **Intermediate region:** Active contribution remains dominant:
 $m_{\beta\beta}^{\text{Std-IO}} \simeq (1.54 - 5.01) \times 10^{-2} \text{ eV}$
- For $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$, sterile effects increase and partial cancellation becomes stronger
- For $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$, active contribution dominates throughout and cancellation is absent
- For $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$, destructive interference between active and sterile terms can strongly suppress: $m_{\beta\beta}^{\text{SSI}} < 10^{-4} \text{ eV}$
- **Quasi-degenerate region:** Active neutrino contribution dominates for both sterile mass choices
- This regime may yield m_{lightest} values exceeding 0.1 eV , potentially in serious tension with the cosmological bounds ($\sum m_\nu < 0.16 \text{ eV}$).
- In the SSI scenario, sterile neutrinos mainly induce partial cancellations and broaden the allowed parameter space, with strongest effects appearing for $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$



- SNS mass ordering: $m_5 > m_3 > m_2 > m_1 > m_4$
- **Light mass region:** Light sterile contribution vanishes: $|t_{14}^2 m_4| \approx 0$
- $m_{\beta\beta}^{\text{SNS}}$ is controlled by the interplay between $m_{\beta\beta}^{\text{NO}}$ and $|t_{15}^2 m_5|$
- For $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$: complete cancellation becomes possible and extends up to $m_{\text{lightest}} \sim 0.1 \text{ eV}$
- For $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$: active contribution dominates and cancellation is absent
- For $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$: $m_{\beta\beta}^{\text{SNS}} \sim (1.79 - 11.20) \times 10^{-2} \text{ eV}$
- Part of the parameter space is already constrained by KamLAND-Zen, while future experiments like LEGEND-1000 can probe the remaining region
- **Large mass region:** Active neutrino contribution dominates over sterile terms
- Complete cancellation no longer possible for either sterile mass choice
- Both minimum and maximum values of $m_{\beta\beta}^{\text{SNS}}$ scale approximately with m_{lightest}
- SNS allows strong active--sterile interference and complete cancellation only for $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$, while the $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$ region remains highly testable in future $0\nu\beta\beta$ experiments



- **Light mass region:** Light sterile contribution is negligible: $|t_{14}^2 m_4| \simeq 0$
- $m_{\beta\beta}^{\text{SIS}}$ is determined by the interplay between the active IO contribution and the heavy sterile term $|t_{15}^2 m_5|$
- For $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$, partial cancellations are possible due to comparable active and sterile contributions
- $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$, active contribution strongly dominates and cancellation becomes ineffective
- **Intermediate region:** For $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$: $|t_{14}^2 m_4| \sim (0.56 - 1.25) \times 10^{-2} \text{ eV}$ becoming comparable to both heavy sterile and active contributions
- Combined sterile effects can destructively interfere with the active sector, strongly suppressing: $m_{\beta\beta}^{\text{SIS}}$
- For $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$, the light sterile contribution is highly suppressed and cancellation is absent
- **Large mass region:** Active neutrino contribution dominates: $m_{\beta\beta}^{\text{IO}} \sim (6 - 23) \times 10^{-2} \text{ eV}$
- Sterile terms remain subdominant, making interference effects ineffective
- $m_{\beta\beta}^{\text{SIS}}$ scales approximately linearly with m_{lightest}
- Strong sterile-induced suppression occurs mainly for $\Delta m_{s_2}^2 = 10^{-4} \text{ eV}^2$ while the $\Delta m_{s_2}^2 = 10^{-2} \text{ eV}^2$ scenario remains dominated by active neutrino contributions
- In the SIS scenario, significant active--sterile cancellations are possible only for small sterile mass splitting and moderate m_4 , whereas the quasi-degenerate regime is largely controlled by the active IO sector

Summary

- Although the three-flavor oscillation paradigm successfully explains several experimental data, anomalies persist which cannot be accommodated within this framework.
- LSND, MiniBooNE, etc, short-baseline-expt. suggest the need for eV-scale sterile neutrinos.
- T2K and NOvA (long-baseline-expt.) suggest the need for sub-eV scale sterile neutrinos.
- By combining oscillation data, constraints from neutrinoless double beta decay, cosmological considerations and tritium β decay, we aim to delineate the viable parameter space for the 3+2 model.
- 10-PCM bound allows SSN and SNS. KATRIN bound on m_β allows all 4 mass schemes, SSN and SNS show better for the future Project-8 sensitivity on m_β .
- For $m_{\beta\beta}$, eV-scale sterile is dominating, sub-eV sterile for 10^{-4} term becomes comparable to the active contribution, sub-eV sterile for 10^{-2} is subleading. Cancellation is possible depending on the phase.