Perturbations in scalar-tensor and scalar-vector-tensor theories.

partly based on papers with Volkova, Shtennikova, Valencia-Villegas

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Plan

- Scalar-tensor and scalar-vector-tensor theories, properties and perturbations
  - 2nd order equations (Horndeski type)
  - Degenerate theories (DHOST type)
  - Fully degenerate theories (MMG)
- Applications
  - early Universe cosmology
  - compact objects and other solutions
  - modern Universe cosmology

2nd order equations

Toy example:

$$\mathcal{L} = K(\pi, X)\Box\pi \qquad\qquad X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$$

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$$= ... + 2K_X \Box\pi\partial_\mu\pi\partial^\mu\delta\pi + \partial_\mu\partial^\mu K\delta\pi$$

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$$... - 2K_X\partial^\mu\partial_\nu\partial^\nu\pi\partial_\mu\pi\delta\pi + 2K_X\partial_\mu\partial^\mu\partial_\nu\pi\partial^\nu\pi\delta\pi$$

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$$... - 2K_X\partial^\mu\partial_\nu\partial^\nu\pi\partial_\mu\pi\delta\pi + 2K_X\partial_\mu\partial^\mu\partial_\nu\pi\partial^\nu\pi\delta\pi$$

$$= ...\text{only second derivatives}$$

Horndeski theory

- Most general scalar-tensor theory with 2nd order EoM

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

Properties

- Theory of a very general form under several assumptions:
- general covariance
- locality
- $S = \int d^4x \sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, \phi, \nabla\phi, \nabla\nabla\phi)$
  (1 additional degree of freedom)
- 2nd order equations of motion.

- Have sufficiently much freedom to modify gravity and scalar dynamics in different ways

  Encodes wide class of modified gravity models

Examples

- Minimally coupled scalar (k-essence)

- $F = F(\phi, X), \quad K = 0, \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$
- Canonical case: $G_2 = X - V(\phi)$

- Brans–Dicke theory

- $G_4 = \phi, \quad F = -\frac{\omega}{\phi} X - V(\phi), \quad K = G_5 = 0$
- $f(R)$ gravity equivalent to Brans–Dicke with $\omega = 0$

$$S = \int d^4x \sqrt{-g}\,[f'(\chi)(R - \chi) + f(\chi)]$$

- $G_4 = \frac{\phi}{2}, \quad F = -V(\phi), \quad K = G_5 = 0$

Examples

- Kinetic Gravity Braiding

  $F = F(\phi, X), \quad K = K(\phi, X), \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$

  Qubic Galileon: $F = X, \quad K = \frac{X}{\Lambda^3}, \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$

- Coupling to Gauss-Bonnet term
  $\xi(\phi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\right)$

$$F = 8\xi_{\phi\phi\phi\phi}X^2(3 - \ln X) \qquad K = 4\xi_{\phi\phi\phi}X(7 - 3\ln X)$$
$$G_4 = 4\xi_{\phi\phi}X(2 - \ln X) \qquad G_5 = -4\xi_{\phi}\ln X$$

Degenerate theories

Toy example:

Kinetic matrix

$\mathcal{L} = \frac{A}{2}\,\ddot{\phi}^2 + B\,\ddot{\phi}\dot{g} + \frac{C}{2}\,\dot{g}^2 + \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi^2 - \frac{1}{2}g^2 ...$

Equations of motion

$A\,\ddddot{\phi} + B\,\dddot{g} - \ddot{\phi} - \phi = 0$

$B\,\dddot{\phi} + C\,\ddot{g} + g = 0$

if $AC - B^2 = 0$ system is equivalent to

$\ddot{\phi} + \frac{B}{C}\,\dot{g} + \phi = 0$

$\left(1 - \frac{B^2}{C^2}\right)\,\ddot{g} - \frac{B}{C}\,\dot{\phi} + \frac{1}{C}\,g = 0$

Main idea:

Hessian

$\det\begin{pmatrix} \partial^2\mathcal{L}/\partial\ddot{\phi}^2 & \partial^2\mathcal{L}/\partial\ddot{\phi}\partial\dot{g} \\ \cdots & \partial^2\mathcal{L}/\partial\dot{g}^2 \end{pmatrix} = 0$

Primary constraint $\Rightarrow$ no extra DOF

beyond Horndeski

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}} \right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X) \Box \pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X) R + 2 G_{4X}(\pi, X) \left[(\Box \pi)^2 - \pi_{;\mu\nu} \pi^{;\mu\nu} \right],$$

$$\mathcal{L}_5 = G_5(\pi, X) G^{\mu\nu} \pi_{;\mu\nu} + \frac{1}{3} G_{5X} \left[(\Box \pi)^3 - 3 \Box \pi \pi_{;\mu\nu} \pi^{;\mu\nu} + 2 \pi_{;\mu\nu} \pi^{;\mu\rho} \pi_{;\rho}{}^{\nu} \right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X) \epsilon^{\mu\nu\rho}{}_{\sigma} \epsilon^{\mu'\nu'\rho'\sigma} \pi_{,\mu} \pi_{,\mu'} \pi_{;\nu\nu'} \pi_{;\rho\rho'} +$$

$$+ F_5(\pi, X) \epsilon^{\mu\nu\rho\sigma} \epsilon^{\mu'\nu'\rho'\sigma'} \pi_{,\mu} \pi_{,\mu'} \pi_{;\nu\nu'} \pi_{;\rho\rho'} \pi_{;\sigma\sigma'}$$

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$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

$$F_4 \; G_{5X}X = -3F_5 \left[G_4 - 2XG_{4X} + \frac{1}{2}G_{5\pi}X\right],$$

beyond Horndeski

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$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

$$F_4\ G_{5X}X = -3F_5\left[G_4 - 2XG_{4X} + \frac{1}{2}G_{5\pi}X\right],$$

$$\mathrm{H} \to \mathrm{BH} \qquad g_{\mu\nu} \to g_{\mu\nu} + \Gamma(\pi, X)\partial_\mu\pi\partial_\nu\pi.$$

DHOST theory

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

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$$\mathcal{L}_4 = F_2(\pi, X)R + \sum_{i=1}^{5} A_i(\pi, X)\, L_i^{(2)},$$

$$\mathcal{L}_5 = F_3(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \sum_{j=1}^{10} B_j(\pi, X)\, L_j^{(3)},$$

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$$L_1^{(2)} = (\pi_{\mu\nu})^2\,, \quad L_2^{(2)} = (\Box\pi)^2\,, \quad L_3^{(2)} = \Box\pi\,(\pi_{\mu\nu}\pi^{\mu}\pi^{\nu})\,,$$

$$L_4^{(2)} = (\pi_{\mu\rho}\pi^{\mu})^2\,,\ L_5^{(2)} = (\pi_{\mu\nu}\pi^{\mu}\pi^{\nu})^2\,,$$

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$$L_4^{(2)} = (\pi_{\mu\rho}\pi^\mu)^2 \,,\; L_5^{(2)} = (\pi_{\mu\nu}\pi^\mu\pi^\nu)^2 \,,$$

$$A_2 = -A_1 \,,$$

$$A_4 = \frac{1}{8(F_2 - XA_1)^2}\left[-16XA_1^3 + 4(3F_2 + 16XF_{2X})A_1^2 \right.$$
$$-(16X^2F_{2X} - 12XF_2)A_3A_1 - X^2F_2A_3^2$$
$$\left. -16F_{2X}(3F_2 + 4XF_{2X})A_1 + 8F_2(XF_{2X} - F_2)A_3 + 48F_2F_{2X}^2\right],$$

$$A_5 = \frac{(4F_{2X} - 2A_1 + XA_3)\left(-2A_1^2 - 3XA_1A_3 + 4F_{2X}A_1 + 4F_2A_3\right)}{8(F_2 - XA_1)^2} \,.$$

DHOST theory

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$$L_4^{(3)} = (\Box\pi)^2(\pi_{\mu\nu}\pi^\mu\pi^\nu)\,, \qquad L_5^{(3)} = \Box\pi\,(\pi_{\mu\nu}\pi^\mu)^2\,,$$

$$L_6^{(3)} = (\pi_{\rho\sigma})^2\,(\pi_{\mu\nu}\pi^\mu\pi^\nu)\,, \qquad L_7^{(3)} = \pi^{\mu\nu}\pi_{\nu\rho}\pi^{\rho\sigma}\pi_\mu\pi_\sigma\,,$$

$$L_8^{(3)} = (\pi^{\mu\nu}\pi_\mu)^2(\pi^{\rho\sigma}\pi_\rho\pi_\sigma)\,, \qquad L_9^{(3)} = \Box\pi(\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^2\,,$$

$$L_{10}^{(3)} = (\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^3$$

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$$L_{10}^{(3)} = (\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^3$$

+ Relations between F_3 and B_j

DHOST theory

$$
\begin{aligned}
S &= \int \mathrm{d}^4 x \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,, \\
\mathcal{L}_2 &= F(\pi, X), \\
\mathcal{L}_3 &= K(\pi, X)\Box\pi, \\
\mathcal{L}_4 &= F_2(\pi, X)R + \sum_{i=1}^{5} A_i(\pi, X)\, L_i^{(2)}, \\
\mathcal{L}_5 &= F_3(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \sum_{j=1}^{10} B_j(\pi, X)\, L_j^{(3)},
\end{aligned}
$$

+ Relations

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$$\mathcal{L}_4 = F_2(\pi, X)R + \sum_{i=1}^{5} A_i(\pi, X)\, L_i^{(2)},$$

$$\mathcal{L}_5 = F_3(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \sum_{j=1}^{10} B_j(\pi, X)\, L_j^{(3)},$$

+ Relations

H $\rightarrow$ DHOST $\qquad g_{\mu\nu} \rightarrow \Omega^2(\pi, X)\, g_{\mu\nu} + \Gamma(\pi, X)\partial_\mu\pi\partial_\nu\pi.$

Fully degenerate theories

- Additional constraint on the kinetic matrix $\Rightarrow$ only two DOF remains.

  Scalar is a non-dynamical auxiliary field

- Minimally Modified Gravity theories:
- Type-I MMG
- Have an Einstein frame
- Without matter are equivalent to GR
- "Veiled gravity"

- Type-II MMG

  = VCDM theory
- Does not have an Einstein frame
- "Cuscuton"

Perturbations above FLRW background: DHOST

$$S = \int \mathrm{d}t\mathrm{d}^3 x a^3 \left[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}^T_{ik} \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h^T_{kl} \right)^2 + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \right]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}. \end{aligned} \quad \Rightarrow \quad \begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a\left(\mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi} + \mathcal{F}_\mathcal{T}\Delta\right)\mathcal{G}_\mathcal{T}}{\Theta}. \end{aligned}$$

Perturbations above FLRW background: simplest MMG - generalized cuscuton

$$S=\int \mathrm{d}t\mathrm{d}^3x a^3\Big[\frac{\mathcal{G}_{\mathcal{T}}}{8}\left(\dot{h}_{ik}^T\right)^2-\frac{\mathcal{F}_{\mathcal{T}}}{8a^2}\left(\partial_i h_{kl}^T\right)^2+\mathcal{G}_{\mathcal{S}}\dot{\zeta}^2-\mathcal{F}_{\mathcal{S}}\frac{(\nabla\zeta)^2}{a^2}\Big]$$

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These coefficients are combinations of Lagrangian functions and have non-trivial relations

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Perturbations above spherically symmetric background

2 tensor modes ↙ ↘ 1 scalar mode ↙

1 odd mode 2 even modes

$$S^{(2)}_{odd} = \int \mathrm{d}t\, \mathrm{d}r \left[\mathcal{A}\dot{q}^2 - \mathcal{B}(q')^2 + \mathcal{C}\dot{q}(q') - \frac{l(l+1)}{J^2} \cdot \mathcal{H}q^2 - V(r)q^2 \right]$$

$$S^{(2)}_{even} = \int \mathrm{d}t\mathrm{d}r \left[\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + Q_{ij}\dot{v}^i v^{j\prime} + \text{...less derivative terms...} \right]$$

$i,\ j = 1..2$

Scalar-Vector-Tensor theories

- Scalar-tensor theory + Vector interactions

  Basis for U(1) gauge vector-ST vertexes:

$$F_{\alpha\beta}F^{\alpha\beta}\,, \quad F_{\alpha\beta}F^{\alpha\beta}(\nabla_\gamma\nabla^\gamma\pi)\,, \quad F_{\alpha\beta}F_{\gamma\delta}(\nabla_\gamma\nabla^\gamma\pi)\nabla^\beta\pi\,\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\pi\,,$$
$$F^\alpha{}_\epsilon\nabla_\nu\pi\,\nabla^\nu\nabla_\mu\pi\,\nabla_\alpha F^{\epsilon\mu}\,, \quad F_{\delta\epsilon}\nabla_\nu\pi\,\nabla^\nu\nabla^\alpha\pi\,\nabla_\alpha F^{\delta\epsilon}\,,$$
$$F_{\alpha\beta}F_{\gamma\delta}\nabla_\nu\nabla^\alpha\pi\,\nabla^\nu\nabla^\gamma\pi\,\nabla^\beta\pi\,\nabla^\delta\pi\,, \quad F_{\alpha\beta}F^\alpha{}_\delta\nabla^\beta\pi\,\nabla^\delta\pi\,, \quad F_{\alpha\beta}F^\alpha{}_\delta\nabla^\delta\nabla^\beta\pi\,,$$
$$F_{\alpha\beta}F^{\alpha\beta}\nabla_\nu\nabla^\mu\pi\,\nabla^\nu\pi\,\nabla_\mu\pi\,, \quad F_{\alpha\beta}F^{\alpha\beta}R\,, \quad F_{\alpha\beta}F^{\alpha\beta}(\nabla_\gamma\nabla^\gamma\pi)^2\,,$$
$$F_{\alpha\beta}F^\alpha{}_\delta R\nabla^\beta\pi\,\nabla^\delta\pi\,, \quad F_{\alpha\beta}F^{\alpha\beta}\nabla_\eta\nabla_\mu\pi\,\nabla^\eta\nabla^\mu\pi\,, \quad F_{\alpha\beta}F^\alpha{}_\delta\nabla_\nu\nabla^\beta\pi\,\nabla^\nu\pi\,\nabla^\delta\pi\,,$$
$$F_{\alpha\beta}F^\alpha{}_\delta(\nabla_\gamma\nabla^\gamma\pi)\nabla^\delta\nabla^\beta\pi\,, \quad F_{\delta\epsilon}(\nabla_\gamma\nabla^\gamma\pi)\nabla_\alpha F^{\delta\alpha}\nabla^\epsilon\pi\,,$$
$$F_{\alpha\beta}F^\alpha{}_\delta\nabla_\nu\nabla^\beta\pi\,\nabla^\nu\nabla^\delta\pi\,, \quad F_{\alpha\beta}F_{\gamma\delta}R^{\alpha\gamma}\nabla^\beta\pi\,\nabla^\delta\pi\,, \quad F_{\delta\epsilon}\nabla_\alpha F^{\delta\alpha}\nabla^\epsilon\pi\,,$$
$$F_{\delta\epsilon}\nabla_\alpha F^{\alpha\mu}\nabla^\delta\pi\,\nabla^\epsilon\nabla_\mu\pi\,, \quad F_{\alpha\beta}F_{\gamma\delta}\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\nabla^\beta\pi\,, \quad F_{\delta\epsilon}\nabla_\alpha F^{\delta\eta}\nabla^\epsilon\pi\,\nabla_\eta\nabla^\alpha\pi\,,$$
$$F_{\delta\epsilon}\nabla_\nu\pi\,\nabla^\nu\nabla^\epsilon\pi\,\nabla_\alpha F^{\delta\alpha}\,, \quad F_{\alpha\beta}F^\alpha{}_\delta(\nabla_\gamma\nabla^\gamma\pi)\nabla^\beta\pi\,\nabla^\delta\pi\,,$$
$$F_{\alpha\beta}F_{\gamma\delta}\nabla^\beta\pi\,\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\pi\,, \quad F_{\alpha\beta}F^\alpha{}_\delta R^{\beta\delta}\,, \quad F_{\alpha\beta}F_{\gamma\delta}R^{\alpha\gamma\beta\delta}\,,$$
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  (talk on Wednesday by M.Valencia-Villegas)

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  $F_{\alpha\beta}F^\alpha{}_\delta R\nabla^\beta\pi\nabla^\delta\pi$, $F_{\alpha\beta}F^{\alpha\beta}\nabla_\eta\nabla_\mu\pi\nabla^\eta\nabla^\mu\pi$, $F_{\alpha\beta}F^\alpha{}_\delta\nabla_\nu\nabla^\beta\pi\nabla^\nu\pi\nabla^\delta\pi$,
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  full degeneracy, additional constraint $\Rightarrow$???

Scalar-Vector-Tensor theories: another approach

KK reduction of Horndeski/DHOST/MMG theory

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} - \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & -\phi^2 \end{pmatrix}$$

- $$R^5 \longrightarrow R^4 \times S^1$$

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- Metric + scalar $\longrightarrow$ Metric + vector + scalar + scalar
  [U(1) gauge]

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Degenerate SVT: Perturbations above FLRW background

$$S = \int \mathrm{d}t\mathrm{d}^3 x a^3 \Big[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}^T_{ik}\right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h^T_{kl}\right)^2 +$$

$$+ \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla \zeta)^2}{a^2} + \mathcal{G}_\mathcal{V} \dot{A}_i^2 - \mathcal{F}_\mathcal{V} \frac{(\partial_j A_i)^2}{a^2} \Big]$$

The speeds of sound for tensor, scalar and vector perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}, \qquad c^2 = c_\mathcal{V}^2 = \frac{\mathcal{F}_\mathcal{V}}{\mathcal{G}_\mathcal{V}}$$

Perturbations above spherically symmetric background

2 tensor modes   2 vector modes   1 scalar

$\swarrow \searrow$ $\swarrow \searrow$ $\Downarrow$

odd   even   odd   even   even

$\searrow$ $\swarrow$

Odd sector of the theory (graviton+photon)

Applications of ST and SVT theories

Motivation to study them, solutions

- early Universe cosmology
- compact objects and other solutions
- modern Universe cosmology

Early Universe (pre-Big Bang)

$\pm$ Horndeski

$+$ beyond Horndeski and DHOST

$+$ MMG

\- SVT

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
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  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum
  - amplitude of tensor spectrum (r-ratio)
  - anisotropies and non-gaussianities

Early Universe: first challenge

Null Energy Condition $\Longleftrightarrow$ **Null Convergence Condition**

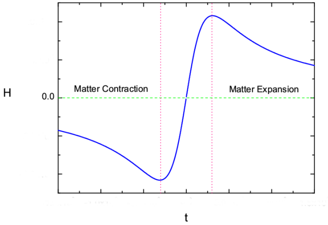
$T_{\mu\nu} k^\mu k^\nu \geq 0$ $\qquad$ $R_{\mu\nu} k^\mu k^\nu \geq 0$

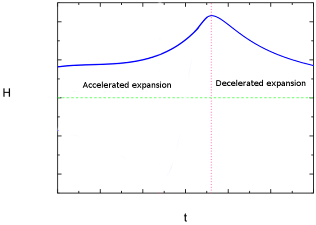
Friedmann equations

$\dot{H} = -4\pi G(p + \rho) \quad \leq 0$

Penrose theorem

no singularity $\Rightarrow$ NEC-violation





Cosmological bounce and genesis solutions require NEC-violation

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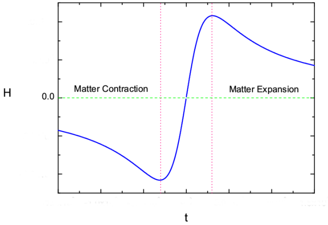

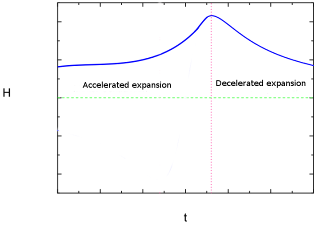

Cosmological bounce and genesis solutions require NEC-violation

As well as wormhole-like solutions

NEC-violation

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stability requirement: $\mathcal{G}_\mathcal{S}, \mathcal{G}_\mathcal{T} > 0 \,, \quad \mathcal{F}_\mathcal{S}, \mathcal{F}_\mathcal{T} > 0 \,.$

(sub)luminality requirement: $\mathcal{F}_\mathcal{T} \leq \mathcal{G}_\mathcal{T} \,, \ \ \mathcal{F}_\mathcal{S} \leq \mathcal{G}_\mathcal{S}$

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- **Horndeski theory**

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- **Horndeski theory** and beyond

Early Universe: second challenge

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

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  If one tries to construct globally stable globally non-singular solution

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  - geodesically incompliteness
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  - different geometry or formalism (torsion, Palatini)
  - **consider beyond Horndeski or DHOST theory**

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- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable globally non-singular solution

$$\mathcal{G}_\mathcal{S} = \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \quad \mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \quad \xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}. \qquad \Rightarrow \qquad \mathcal{G}_\mathcal{S} = 0, \quad \mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \quad \xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}.$$

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$
  - different geometry or formalism (torsion, Palatini)
  - **consider beyond Horndeski or DHOST theory**
  - **consider cuscuton or generalized cuscuton**

    (talk on Wednesday by V.Volkova)

Compact objects and other solutions

+ Horndeski

+ beyond Horndeski and DHOST

+ MMG

± SVT

Compact objects and other solutions

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- No-go for wormhole solutions in Horndeski theory
- "almost" resolved in beyond Hondeski
- Many different "successful" black hole solutions (with or without hair)

Modern Universe cosmology

\+ Horndeski

\+ beyond Horndeski and DHOST

\+ MMG

++ SVT

Modern Universe cosmology

- Dark matter
- "mimetic"dark matter
- heavy vector or scalar dark matter
- or even light scalar dark matter

- Dark energy
- quintessence and other models
- phantom crossing $\Leftarrow$ NEC
- selftuning models

- Modifications to hot stage

- Usually the scalar-tensor part works as background theory
- isotropy
- simplicity

- Why Vectors then?
- vector particles
- (electro) magnetic fields
- modified photon

Modern Universe challenges

- There is are additional phenomenological restrictions, when we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$
$$|\frac{c_{\mathcal{T}}}{c} - 1| < 10^{-15}$$

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- (SVT) Electromagnetic waves do not decay too

$$\mathcal{L} \supset \quad \zeta\zeta A_i \ \ \text{vertex is suppressed}$$

Horndeski cosmology Speed of graviton

$$S = \int \mathrm{d}^4 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} - \frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{S}_T = \int \mathrm{d}\eta\, \mathrm{d}^3 x\, a^4 \left[\frac{1}{2\, a^2}\left(\mathcal{G}_\tau \left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau (\partial_k\, h_{ij})^2\right)\right]$$

$$\mathcal{G}_T = 2\left[G_4 - 2XG_{4,X} - X\left(H\dot{\pi}\, G_{5X} - G_{5\phi}\right)\right]$$

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Viable ST models after GW 170817

- General relativity
- quintessence/k-essence
- Brans-Dicke/f(R)
- Kinetic Gravity Braiding
- Derivative Conformal
- Disformal Tuning
- quartic Galileons
- Fab Four
- quadratic DHOST
- quintic Galileons
- cubic DHOST
- $f(\phi)$ Gauss-Bonnet

Viable SVT models after GW 170817

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Subclass of Fab Four allows self tuning with simultaneous Vainshtein screening of Dark Energy

No decay for gravitational waves

$$
\begin{aligned}
\mathcal{L}_{ssh} \;=\; & a^3 \Big[\frac{c_1}{a^2} h_{ij}\zeta_{,i}\zeta_{,j} + \frac{c_2}{a^2}\dot{h}_{ij}\zeta_{,i}\zeta_{,j} + c_3 \dot{h}_{ij}\zeta_{,i}\psi_{,j} + \frac{c_4}{a^2}\partial^2 h_{ij}\zeta_{,i}\psi_{,j} \\
& + \frac{c_5}{a^4}\partial^2 h_{ij}\zeta_{,i}\zeta_{,j} + c_6 \partial^2 h_{ij}\psi_{,i}\psi_{,j} + \frac{c_7}{a^2}\dot{h}_{ij}\dot{\zeta}_{,i}\zeta_{,j} + c_8 \dot{h}_{ij}\dot{\zeta}_{,i}\psi_{,j} \Big] ,
\end{aligned}
$$

where $\partial^2 h_{ij} \equiv h_{ij,kk}$, $\psi \equiv \partial^{-2}\dot{\zeta}$ and the coefficients read

$$
\begin{aligned}
c_1 &= \mathcal{F}_S, & c_5 &= \frac{\mathcal{G}_T}{4\tilde{\Theta}^2}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1)^2, \\
c_2 &= -\frac{\mathcal{G}_T}{2\tilde{\Theta}}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1), & c_6 &= \frac{\mathcal{G}_S^2}{4\mathcal{G}_T}, \\
c_3 &= \frac{3}{2}\mathcal{G}_S, & c_7 &= \frac{\mathcal{G}_T(\Gamma - 3\mathcal{G}_T\Delta_1)}{2\tilde{\Theta}^2}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1), \\
c_4 &= -\frac{\mathcal{G}_S}{2\tilde{\Theta}}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1), & c_8 &= -\frac{\mathcal{G}_S(\Gamma - 3\mathcal{G}_T\Delta_1)}{2\tilde{\Theta}}.
\end{aligned}
$$

No decay constraint solved in SVT trivial gravity in ST

Conclusion and outlook

- Hot models:
  - Minimally modified gravities
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