Thermal false vacuum decay around black holes is aspherical

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Based on: Dmitry Gorbunov, Dmitry Levkov, VM, arXiv:soon

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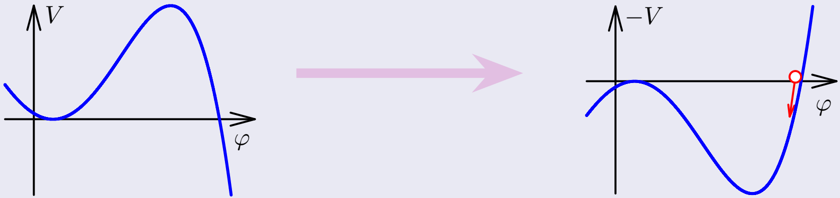
Introduction: false vacuum decay

$T = 0$

False vacuum

Euclidean time: $\tau = it$



Decay rate: $\Gamma \propto e^{-S_E[\varphi_b]}$,

φ_b — **bounce** solution

bounce with the lowest S_E **dominates**

- Example: Higgs field in Standard model

$$\lambda(\mu) < 0 \quad \text{at} \quad \mu \gtrsim 10^9\, GeV$$

Bednyakov, Fedoruk, Kazakov '25

- **Black holes** may catalyze the decay!

Hiscock '87; Tetradis '08

$T_H = (4\pi r_s)^{-1}$

Hawking '74

False vacuum decay at $T > 0$

- Euclidean time becomes periodic: $0 \leqslant \tau \leqslant \beta \equiv T^{-1}$
- Potential barrier E_{sph} = the energy of the critical bubble φ_{sph}

2 mechanisms of thermal decay

- Quantum tunneling under the barrier:

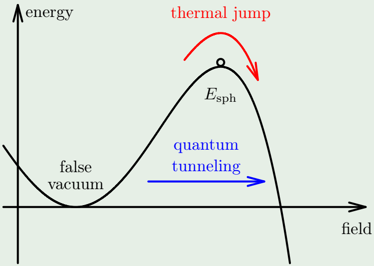
$$E < E_{sph} \implies P \propto e^{-S_E[\varphi_i]}$$

periodic instanton $\varphi_i(\tau, \boldsymbol{x})$

- Thermal jump *over* the barrier

$$E > E_{sph} \implies P \propto e^{-E_{sph}\beta}$$

critical bubble $\varphi_{sph}(\boldsymbol{x})$



Common assumption: least-action φ_i,φ_{sph} are $O(3)$–symmetric around **BH**

Are they?

Example model in flat space

$$V(\varphi) = \frac{m^2\varphi^2}{2} - \frac{g_4\varphi^4}{4}$$

Peculiarities:

- $T = 0,\ m = 0$:

  scale invariance

  $$\varphi(x_E) \to \gamma\varphi(\gamma x_E)$$

  family of instantons:

  $$\varphi_{FL} \propto \frac{a}{x_E^2 + a^2}$$

  $$S_E = \frac{8\pi^2}{3g_4}$$

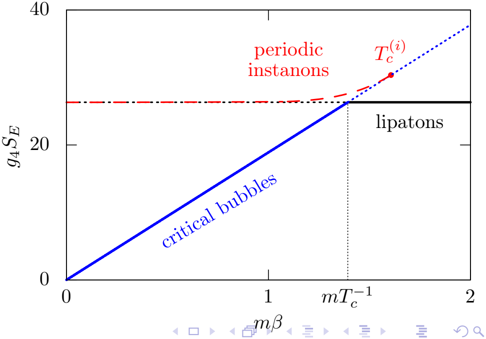
  Fubini '76
  Lipatov '77

- $T = 0,\ m \neq 0$:

  invariance is broken

  $a \to 0$ — "lipaton"

- $T \neq 0$:
  - lipatons → lipatonic chains ✓
  - finite–sized periodic instantons ✓
    but always suppressed *Kuznetsov, Tinyakov '97*

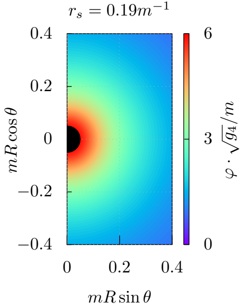


Spherical critical bubbles around the black hole

- Schwarzschild black hole (isotropic coordinates)

$$ds_E^2 = \left(\frac{1-R_s/R}{1+R_s/R}\right)^2 d\tau^2 + (1+R_s/R)^4\left[dR^2 + R^2 d\Omega\right]$$

$$r(R) = R\,(1+R_s/R)^2, \quad R_s = r_s/4$$



- Hartle-Hawking vacuum:

  BH in thermal equilibrium $\beta = T_H^{-1} = 4\pi r_s$

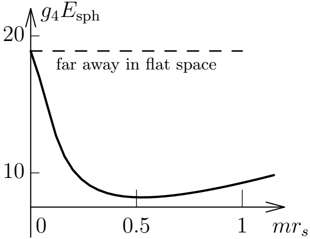
- $r_s \ll m^{-1} \iff$ very hot black hole

  thermal jumps dominate

- Spherical critical bubble:

$$\partial_R^2\varphi_s + \frac{2R}{R^2-R_s^2}\,\partial_R\varphi_s = \left(1+\frac{R_s}{R}\right)^4 V'(\varphi_s)$$

boundary condition: $\partial_R\varphi(R_s) = 0$



Spherical bubbles lose their dominance

Dominant bounce solution should have only one negative mode

Callan, Coleman '77

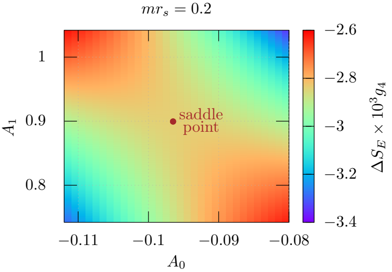
- Eigenmodes $\xi = \xi_\ell(R) P_\ell(\cos\theta)$ over spherical bubbles satisfy

$$\left[-\partial_R^2 - \frac{2R}{R^2 - R_s^2}\partial_R + \frac{\ell(\ell+1)}{R^2} + \left(1 + \frac{R_s}{R}\right)^4 V''(\varphi_s)\right]\xi_\ell = \mu_\ell \xi_\ell \qquad \boxed{R_s = \frac{r_s}{4}}$$

- For $r_s \lesssim 0.194 m^{-1}$: only $\ell = 0$ is negative
- For $r_s \gtrsim 0.194 m^{-1}$: $\ell = 1$ i.e. dipole mode becomes negative
- Search for a new saddle point as

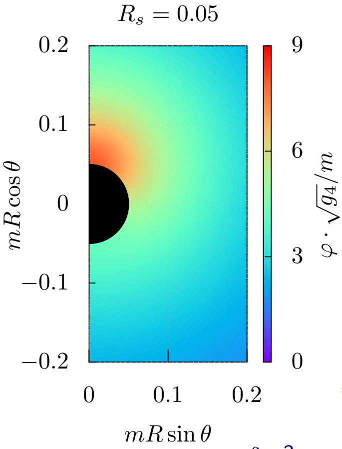
$$\varphi_s(R) + A_0 \xi_0(R) + A_1 \xi_1(R) \cos\theta$$

...and find it!



Aspherical critical bubbles

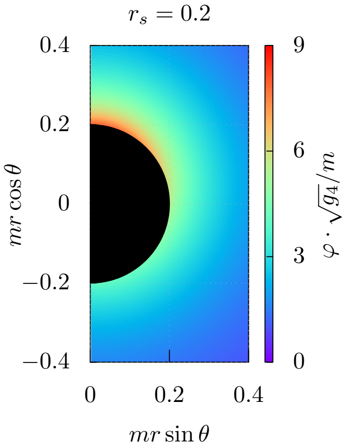
Isotropic coordinates:



$\longleftrightarrow$

$r = R\left(1 + \frac{R_s}{R}\right)^2$

Schwarzschild coordinates:



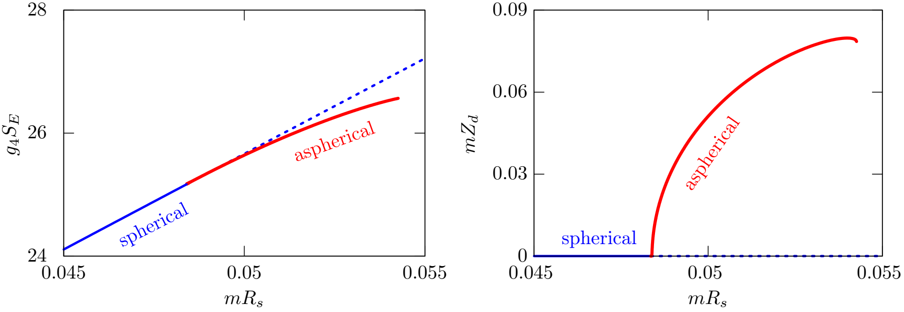
isotropic: $z = R\cos\theta$

Asphericity: $Z_d = \dfrac{\int d^3\mathbf{x}\sqrt{g_E} \cdot z \cdot \mathcal{P}[\varphi]}{\int d^3\mathbf{x}\sqrt{g_E} \cdot \mathcal{P}[\varphi]}$ — "center of mass", $\mathcal{P}[\varphi] > 0$

e.g. $\mathcal{P}[\varphi] = \varphi^4$

Aspherical critical bubbles for larger, cooler black holes

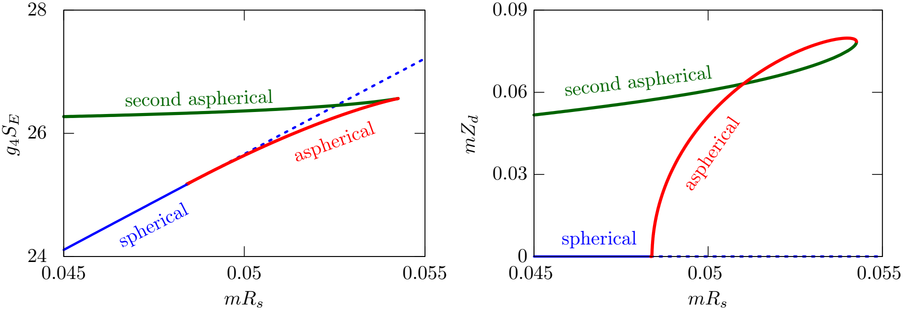
"Center of mass": $Z_d = \dfrac{\int d^3\mathbf{x}\sqrt{g_E}\cdot z\cdot\mathcal{P}[\varphi]}{\int d^3\mathbf{x}\sqrt{g_E}\cdot\mathcal{P}[\varphi]}$, $\mathcal{P}[\varphi] = \varphi^4$



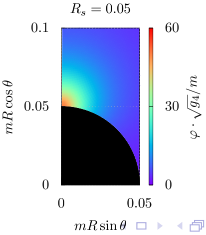
- Why do aspherical bubbles disappear at large r_s?

Aspherical critical bubbles for larger, cooler black holes

"Center of mass": $Z_d = \dfrac{\int d^3\mathbf{x}\sqrt{g_E}\cdot z\cdot \mathcal{P}[\varphi]}{\int d^3\mathbf{x}\sqrt{g_E}\cdot \mathcal{P}[\varphi]}$, $\mathcal{P}[\varphi] = \varphi^4$



- Why do aspherical bubbles disappear at large r_s?
- They **bifurcate** with the second, suppressed branch of solutions!



Branches of critical bubbles

Find saddle points with **fixed** Z_d:

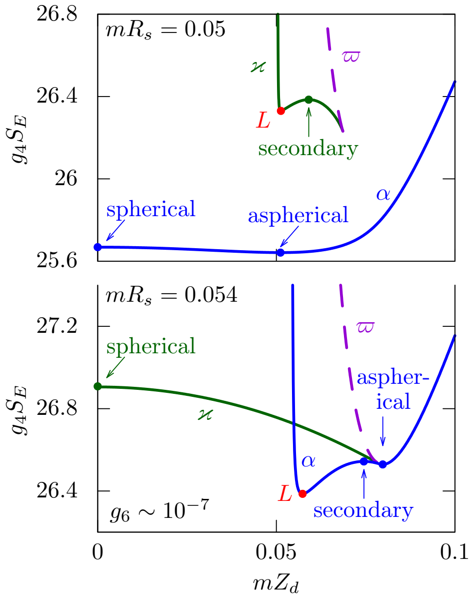
$$S \to S - \lambda\,\beta\,\mathcal{C}[\varphi]$$

$$\mathcal{C}[\varphi] = \int d^3\mathbf{x}\sqrt{g_E}\,(z - Z_d)\,\mathcal{P}[\varphi]$$

Takeaway:

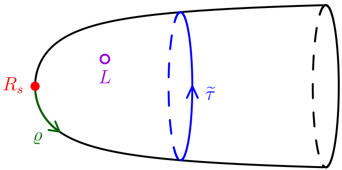
- Two separate branches
- Another minimum connected to secondary bubble, **welcome L**
- L beats asymmetric bubble at $r_s \approx 0.207 m^{-1}$
- Crossover between branches at $r_s \approx 0.215 m^{-1}$

What is L? A lipaton!



Lipaton on the horizon is also a critical bubble

- Euclidean space is regular on the event horizon!
- Regular coordinates:



$$\varrho = (R - R_s)\sqrt{R_s/R}$$

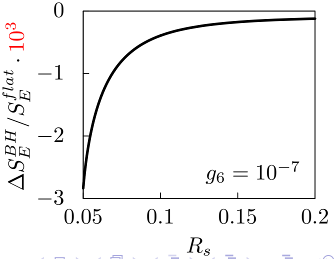
also: $\eta = \theta R_s, \quad \tilde{\tau} = \tau/8R_s$

$$\Downarrow$$

$$\frac{ds_E^2}{16} = \frac{\varrho^2 d\tilde{\tau}^2}{\nu_\varrho} + \nu_\varrho \left[d\varrho^2 + \nu_\varrho \left(d\eta^2 + R_s^2 \sin^2 \frac{\eta^2}{R_s^2} d\phi^2 \right) \right], \quad \nu_\varrho = 1 + \frac{\varrho^2}{4R_s^2}$$

- τ becomes angle on the horizon $\Longrightarrow$ lipaton does not depend on it
- Plain theory: $S_E = 8\pi^2/3g_4$
  lipatons exist everywhere, size $a \to 0$.
- Regularized theory $+g_6\varphi^6$:
  - lipaton acquires finite size $a \sim g_6^{1/4} > 0$
  - dominant contribution — on the horizon:

$$S_E^{BH} < S_E^{flat}, \quad \Delta S_E^{BH} \overset{\text{(num)}}{\propto} \left(\frac{a}{R_s}\right)^4$$



Results

- Spherical critical bubbles:

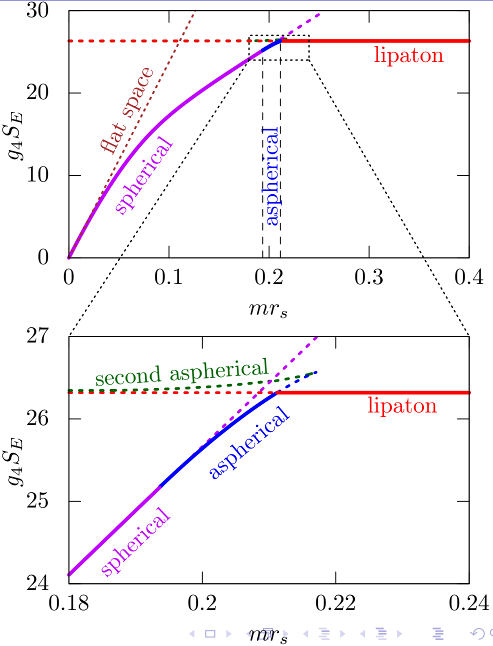
$$r_s \lesssim 0.194 m^{-1}$$
$$\Updownarrow$$
$$T_H \gtrsim 0.41 m,$$

- Aspherical critical bubbles:

$$0.194 m^{-1} \lesssim r_s \lesssim 0.207 m^{-1}$$
$$\Updownarrow$$
$$0.384 m \lesssim T_H \lesssim 0.41 m$$

- Lipatons:

$$r_s \gtrsim 0.207 m^{-1}$$
$$\Updownarrow$$
$$T_H \lesssim 0.384 m$$



Conclusions and perspectives

- False vacuum decay in the presence of a black hole may proceed aspherically
- In scalar model with $\frac{m^2\varphi^2}{2} - \frac{g_4\varphi^4}{4}$ aspherical decay dominates at $r_s \gtrsim 0.194 m^{-1}$
- Perspectives: application to electroweak sector of Standard model
  - The scale invariance of massless $-\lambda\varphi^4$ theory is also broken!..
  - ..but by running coupling constant $\lambda(\varphi)$, not mass term!
  - RG equations were solved numerically $\Longrightarrow$ fitting?
  - Thermal corrections to the Higgs should be accounted for

THANK YOU FOR YOUR ATTENTION

Backup: Negative modes quantity for large black holes

$$R \gg m^{-1} \implies R \approx R_s \text{ on the solution, } \rho \equiv R - R_s \ll R_s$$

$$\partial_R^2 \varphi_s + \frac{2R \cdot \partial_R \varphi_s}{R^2 - R_s^2} = \left(1 + \frac{R_s}{R}\right)^4 V'(\varphi_s) \implies \underbrace{\partial_\rho^2 \varphi + \frac{\partial_\rho \varphi}{\rho} = 16 V'(\varphi)}_{\text{critical bubble in 2D flat space}}$$

$$\left[-\partial_R^2 - \frac{2R}{R^2 - R_s^2}\partial_R + \frac{\ell(\ell+1)}{R^2} + \left(1 + \frac{R_s}{R}\right)^4 V''(\varphi_s)\right]\xi_\ell = \mu_\ell \xi_\ell$$

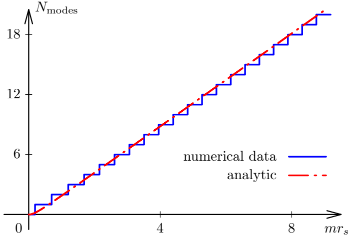
$$\Downarrow$$

$$-\partial_\rho^2 \xi_\ell - \frac{\partial_\rho \xi_\ell}{\rho} + 16 V''(\varphi_s)\xi_\ell = \underbrace{\left[\mu_\ell - \frac{\ell(\ell+1)}{R_s^2}\right]}_{\mu_{2D} < 0} \xi_\ell$$

Number of negative modes:

$$\mu_\ell = \mu_{2D} + \frac{\ell(\ell+1)}{R_s^2} < 0$$

$$N_{\text{modes}} = \sqrt{|\mu_{2D}| R_s^2 + \frac{1}{4}} - \frac{1}{2}$$



$$\delta V_{\text{reg}} = \frac{g_6 \varphi^6}{6}, \quad \text{with} \quad g_6 = \gamma_6 \cdot (g_4/m)^2$$

- Correction to the action in flat spacetime:

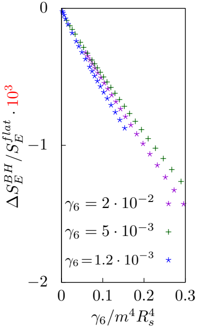
$$\delta S = \frac{8\pi^2}{g_4}(ma)^2 \left[\ln \frac{2}{ma} - \frac{1}{2} - \gamma_E\right] + \frac{64\pi^2}{15} \frac{g_6}{g_4^3} \frac{1}{a^2}$$

- Lipaton size a minimizes δS:

$$(ma)^4 \left[\ln \frac{2}{ma} - 1 - \gamma_E\right] = \frac{8}{15}\gamma_6$$

- Schwarschild BH, event horizon:

$$\Delta S_E^{BH} \propto (a/R_s)^4$$



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