Presymplectic BV-AKSZ for supersymmetric theories

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Based on *«M. Grigoriev, A. Mamekin, Presymplectic BV-AKSZ formulation of N = 1 D = 4 Supergravity», arXiv:2503.04559 [hep-th]*;
«M. Grigoriev, A. Mamekin, Presymplectic BV-AKSZ for supersymmetric theories», in progress

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Motivation

- In superspace supersymmetry is manifest
- Component formulation from the superspace one is obtained by integration over odd coordinates and elimination of auxiliary fields.
- Is it possible to systematically derive superspace formulation from the space-time one?

Results

At the level of equations of motion for arbitrary SUSY theory:

- “Presymplectic BV-AKSZ for N = 1, D = 4 Supergravity, M. Grigoriev, A. Mamekin, 2025”

At lagrangian level:

- $N = 1$ $D = 1$ toy model: “Presymplectic BV-AKSZ for N = 1, D = 4 Supergravity, M. Grigoriev, A. Mamekin, 2025”.
- $N = (1,1)$ $D = 2$, $N = 1$ $D = 3$, $N = 1$ $D = 4$: ”Presymplectic BV-AKSZ for supersymmetric theories, M. Grigoriev, A.Mamekin, in preparation”

For somewhat related approaches see [Ponomarev, Vasiliev, 2012], [Misuna, Vasiliev, 2014], [Misuna, 2026].

Basic notions

Definition: Q-manifold (M,Q) is $\mathbb{Z}$-graded manifold M equipped with odd nilpotent vector field Q, i.e.

$$Q^2 = 0, \quad gh(Q) = 1.$$

Example: Odd tangent bundle $(T[1]X,d_X)$. Local description:

$$(x^\mu, \eta^A, \theta^\mu, \zeta^A), \quad d_X = \theta^\mu \frac{\partial}{\partial x^\mu} + \zeta^A \frac{\partial}{\partial \eta^A}$$

$$gh(x^\mu) = gh(\eta^A) = 0, \quad \epsilon_F(x^\mu) = 0, \quad \epsilon_F(\eta^A) = 1$$

$$gh(\theta^\mu) = gh(\zeta^A) = 1, \quad \epsilon_F(\theta^\mu) = 0, \quad \epsilon_F(\zeta^A) = 1$$

Definition: Q-bundle $(E,Q) \stackrel{\pi_B}{\to} (B,\gamma)$ is a locally trivial bundle $E \stackrel{\pi_B}{\to} B$ such that $\pi_B^* \circ \gamma = Q$.

Presymplectic BV-AKSZ with background fields

[Alexandrov, Kontsevich, Schwarz, Zaboronsky, 1995], [Alkalaev, Grigoriev, 2013], [Dneprov, Grigoriev, 2025].

Ingredients:

- Target: Q-bundle $(E,Q) \overset{\pi_B}{\to} (B,\gamma)$ with compatible presymplectic form $\omega \in \Omega^2(E)$ of $gh(\omega) = n-1$:

$$\omega = d\chi, \quad i_Q\omega + dH \in \mathcal{I}_B,$$

where $\mathcal{I}_B$ is generated by $\pi_B^*(\alpha)$ for $\alpha \in \Omega^1(B)$.

- Base: Q-manifold (M,δ) with trace $Tr : C^\infty(M) \to \mathbb{R}$:

$$gh(Tr) = -n \quad Tr(\delta f) = 0 \, \forall\, f \in C^\infty(M).$$

For $M = T[1]X$, where X is ordinary real manifold of dimension n:

$$Tr(f) = \int_{T[1]X} d^n x d^n \theta f(x,\theta).$$

Presymplectic BV-AKSZ with background fields

Action and equations of motion:

- Fix background $\sigma_b : M \to B$ such that $\sigma_b^* \circ \gamma = \delta \circ \sigma_b^*$ and consider $\mathbb{Z}$-degree preserving supermaps $\sigma : M \to E$, $\sigma^* \circ \pi_B^* = \sigma_b^*$.
- Equations of motion:

$$\sigma^*(\omega_{AB})(\delta \circ \sigma^*(\psi^B) - \sigma^* \circ (Q\psi^B)) = 0,$$

  where ψ^A is fiber coordinate.
- AKSZ action reads as

$$S_{AKSZ}[\sigma] = Tr\left[\sigma^*(\chi)(\delta) + \sigma^*(H)\right],$$

- Passing to generic supermaps $\hat{\sigma} : M \to E$ presymplectic form ω_{BV} and BV-AKSZ action reads as

$$\omega_{BV} = Tr\left[\hat{\sigma}^*(\omega)\right], \qquad S_{BV-AKSZ}[\hat{\sigma}] = Tr\left[\hat{\sigma}^*(\chi)(\delta) + \hat{\sigma}^*(H)\right].$$

  Taking symplectic quotient one obtains odd symplectic form provided ω_{BV} is regular.

Presymplectic BV-AKSZ for N=1 D=3 scalar supermultiplet

Target:

- $B = \mathfrak{g}[1]$, where $\mathfrak{g} = \mathfrak{siso}(1,2)/\mathfrak{so}(1,2)$. Local coordinates and their nonzero degrees:
$$(\xi^{AB}, \psi^A), \qquad gh(\xi^{AB}) = gh(\psi^A) = 1, \qquad \epsilon(\psi^A) = 1.$$
- Total space $E = B \times F$, where F is locally described by complex coordinates $(\phi, \phi_{AB}, \lambda^A)$ with all degrees be 0, but $\epsilon(\lambda^A) = 1$
- Q-structure:
$$Q\phi = \xi^{AB}\phi_{AB} + \psi^A\lambda_A, \qquad Q\lambda^A = \psi_B\phi^{AB}, \qquad Q\phi_{AB} = 0,$$
$$Q\xi^{AB} = \psi^A\psi^B, \qquad Q\psi^A = 0.$$
- Compatible presymplectic structure
$$\chi^E = -\tilde{\xi}_{AB}\phi^{AB}d\bar{\phi} + \tilde{\xi}_{AB}\lambda^A d\bar{\lambda}^B + \xi_{AB}\psi^A\phi d\bar{\lambda}^B + c.c,$$
$$H = -2\tilde{\xi}\phi^{AB}\bar{\phi}_{AB} + \frac{1}{2}\xi_{AB}\psi^A\psi^C\lambda_C\bar{\lambda}^B + \tilde{\xi}_{AB}\phi^{AB}\psi^C\bar{\lambda}_C + c.c.$$

Presymplectic BV-AKSZ for N=1 D=3 scalar supermultiplet

Minkowski space-time formulation

- Take as base manifold $M = T[1]\mathbb{R}^3$ with local coordinates (x^{AB}, θ^{AB}).
- Fix background $\sigma_b : M \to B$:

$$\sigma_b^*(\xi^{AB}) = \theta^{AB}, \qquad \sigma_b^*(\psi^A) = 0.$$

- Consider $\mathbb{Z}$-degree preserving supermaps $\sigma : M \to E$

$$\sigma^*(\phi) = \phi(x), \qquad \sigma^*(\phi_{AB}) = \phi_{AB}(x), \qquad \sigma^*(\lambda^A) = \lambda^A(x).$$

- AKSZ action:

$$S_{AKSZ}[\sigma] = \int_{\mathbb{R}^3} d^3x \left(-4\phi^{AB}\partial_{AB}\bar{\phi} - 4\lambda^A\partial_{AB}\bar{\lambda}^B + 2\phi^{AB}\bar{\phi}_{AB} + c.c\right).$$

Eliminating ϕ_{AB}:

$$S[\phi,\lambda^A] = -4\int_{\mathbb{R}^3} d^3x \left(\frac{1}{2}\partial^{AB}\phi\partial_{AB}\bar{\phi} + \lambda^A\partial_{AB}\bar{\lambda}^B + c.c\right).$$

Presymplectic BV-AKSZ for N=1 D=3 scalar supermultiplet

Superspace formulation

- Take as base manifold $M = T[1]\mathbb{R}^{3|2}$ with local coordinates $(x^{AB}, \eta^A, \theta^{AB}, \zeta^A)$. For convenience we perform change coordinates $\theta^{AB} \to \theta^{AB} + \zeta^{(A}\eta^{B)}$.
- Introduce $Tr : C^\infty(M) \to \mathbb{R}$ as

$$Tr(f) = \int_{T[1]\mathbb{R}^{3|2}} d^3x d^2\eta d^3\theta d^2\zeta f(x,\eta,\theta,\zeta)\mathbb{Y},$$

where $\mathbb{Y} = \tilde{\theta}^{AB} \frac{\partial^2}{\partial\zeta^A\partial\zeta^B}\delta(\zeta^1)\delta(\zeta^2)$.
- Fix background $\sigma_b : M \to B$:

$$\sigma_b^*(\xi^{AB}) = \theta^{AB}, \qquad \sigma_b^*(\psi^A) = \zeta^A.$$

Presymplectic BV-AKSZ for N=1 D=3 scalar supermultiplet

- Consider $\mathbb{Z}$-degree preserving supermaps $\sigma : M \to E$

$$\sigma^*(\phi) = \Phi(x,\eta), \quad \sigma^*(\phi_{AB}) = \Phi_{AB}(x,\eta), \quad \sigma^*(\lambda^A) = \Lambda^A(x,\eta).$$

- Equations of motion read as

$$\Phi_{AB} = \partial_{AB}\Phi, \quad \Lambda^A = \mathcal{D}^A\Phi, \quad \mathcal{D}^2\Phi = 0,$$

where $\mathcal{D}_A = \frac{\partial}{\partial\eta^A} - \eta^B\frac{\partial}{\partial x^{AB}}$.

- AKSZ action reads as

$$S[\Phi,\Phi_{AB},\Lambda^A] = 6\int d^3xd^2\eta\left(\Phi\mathcal{D}_A\bar{\Lambda}^A + \bar{\Phi}\mathcal{D}_A\Lambda^A + \Lambda_A\bar{\Lambda}^A\right),$$

- Eliminating Φ_{AB} and Λ^A one gets

$$S[\Phi] = -6\int_{\mathbb{R}^{3|2}} d^3xd^2\eta\mathcal{D}_A\Phi\mathcal{D}^A\bar{\Phi}.$$

- Passing to generic supermaps $\hat{\sigma} : M \to E$ it can be shown, that induced presymplectic form is regular and thus one gets full scale BV.

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

Target:

- $B = \mathfrak{g}[1]$, where $\mathfrak{g} = \mathfrak{siso}(1,3)/\mathfrak{so}(1,3)$. Local coordinates and their nonzero degrees:

$$(\xi^{AA'}, \psi^{A}), \qquad gh(\xi^{AA'}) = gh(\psi^{A}) = 1, \qquad \epsilon(\psi^{A}) = 1.$$

- Total space $E = B \times F$, where F is locally described by complex coordinates $(\phi, \phi_{AA'}, \lambda^{A})$ with all degrees be 0, but $\epsilon(\lambda^{A}) = 1$

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

- Q-structure:

$$Q\phi = \xi^{AA'}\phi_{AA'} + \psi^A\lambda_A, \quad Q\lambda_A = -2\bar{\psi}^{A'}\phi_{AA'}, \quad Q\phi_{AA'} = 0.,$$
$$Q\xi^{AA'} = 2\psi^A\bar{\psi}^{A'}, \qquad Q\psi^A = 0.$$

- Compatible presymplectic structure

$$\chi^E = -\tilde{\xi}_{AA'}\left(\phi^{AA'}d\bar{\phi} - \frac{1}{2}\bar{\lambda}^{A'}d\lambda^A\right) - $$
$$- i\xi_{AB}\bar{\phi}\psi^A d\lambda^B - \frac{i}{2}\xi_{AA'}\psi^A\bar{\psi}^{A'}\bar{\phi}d\phi + c.c,$$

$$H = -\tilde{\xi}\phi^{AA'}\bar{\phi}_{AA'} + \tilde{\xi}_{AA'}\bar{\phi}^{AA'}\psi^B\lambda_B +$$
$$+ \frac{i}{2}\xi_{AB}\psi^B\bar{\psi}^{A'}\lambda^A\bar{\lambda}_{A'} + i\xi_{AA'}\psi^A\bar{\psi}^{A'}\bar{\phi}\psi^B\lambda_B + c.c,$$

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

Minkowski space-time formulation

- Take as base manifold $M = T[1]\mathbb{R}^4$ with local coordinates $(x^{AA'}, \theta^{AA'})$.
- Fix background $\sigma_b : M \to B$:

$$\sigma_b^*(\xi^{AA'}) = \theta^{AA'}, \qquad \sigma_b^*(\psi^A) = 0.$$

- Consider $\mathbb{Z}$-degree preserving supermaps $\sigma : M \to E$

$$\sigma^*(\phi) = \phi(x), \qquad \sigma^*(\phi_{AA'}) = \phi_{AA'}(x), \qquad \sigma^*(\lambda^A) = \lambda^A(x).$$

- AKSZ action:

$$S_{AKSZ}[\sigma] = \int \left(2\bar{\phi}^{AA'}\partial_{AA'}\phi - \phi^{AA'}\bar{\phi}_{AA'} + \bar{\lambda}^{A'}\partial_{AA'}\lambda^A + c.c.\right) d^4x.$$

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

Superspace formulation

- Take as base manifold $M = T[1]\mathbb{R}^{4|4}$ with local coordinates $(x^{AA'}, \eta^A, \theta^{AA'}, \zeta^A)$. For convenience we perform change coordinates $\theta^{AA'} \to \theta^{AA'} + \zeta^A \bar{\eta}^{A'} + \bar{\zeta}^{A'} \eta^A$.
- Introduce $Tr : C^\infty(M) \to \mathbb{R}$ as

$$Tr(f) = \int_{T[1]\mathbb{R}^{4|4}} d^4x d^2\eta d^2\bar{\eta} d^4\theta d^2\zeta d^2\bar{\zeta} f(x,\eta,\theta,\zeta)\mathbb{Y},$$

where

$$\mathbb{Y} = \Bigg[\eta^2\theta^{AB}\frac{\partial^2}{\partial\zeta^A\partial\zeta^B} + \bar{\eta}^2\bar{\theta}^{A'B'}\frac{\partial^2}{\partial\bar{\zeta}^{A'}\partial\bar{\zeta}^{B'}} - \\ - \left(\eta^A\bar{\eta}_{B'}\bar{\theta}^{A'B'} + \bar{\eta}^{A'}\eta_B\theta^{AB}\right)\frac{\partial^2}{\partial\bar{\zeta}^{A'}\partial\zeta^A}\Bigg]\delta(\zeta^1)\delta(\zeta^2)\delta(\bar{\zeta}^{1'})\delta(\bar{\zeta}^{2'}).$$

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

- Fix background $\sigma_b : M \to B$:

$$\sigma_b^*(\xi^{AA'}) = \theta^{AA'}, \qquad \sigma_b^*(\psi^A) = \zeta^A.$$

- Consider $\mathbb{Z}$-degree preserving supermaps $\sigma : M \to E$

$$\sigma^*(\phi) = \Phi(x,\eta,\bar\eta),$$
$$\sigma^*(\phi_{AB}) = \Phi_{AA'}(x,\eta,\bar\eta), \quad \sigma^*(\lambda^A) = \Lambda^A(x,\eta,\bar\eta).$$

- Equations of motion read as

$$\Phi_{AA'} = \partial_{AA'}\Phi, \qquad \Lambda_A = \mathcal{D}_A\Phi, \qquad \overline{\mathcal{D}}_{A'}\Phi = 0, \qquad \mathcal{D}^2\Phi = 0,$$

where $\mathcal{D}_A = \partial_A - \bar\eta^{A'}\partial_{AA'}$. The first three are algebraic.
- AKSZ action reads as

$$S[\Phi,\Lambda^A] = \int d^4x d^2\eta d^2\bar\eta \Big(12\overline{\Phi}\mathcal{D}_A\Lambda^A\eta^2 +$$
$$+3\bar\eta^{A'}\eta^A(2\overline{\Phi}\overline{\mathcal{D}}_{A'}\Lambda_A - \Lambda_A\Lambda_{A'} + 2\overline{\Phi}\partial_{AA'}\Phi) + c.c\Big).$$

Presymplectic BV-AKSZ for N=1 D=4 scalar supermultiplet

To obtain explicitely SUSY invariant action we impose algebraic equations of motion:

$$S[\Phi] = 72 \int d^4x d^2\eta d^2\bar{\eta} \Phi \overline{\Phi},$$

where $\overline{\mathcal{D}}_{A'} \Phi = 0$. This is well known Wess-Zumino action.

Conclusion

- Superfield formulation at the level of EOM for arbibtrary SUSY theory can be derived systematically within Presymplectic BV-AKSZ framework.
- Lagrangian superspace formulations for $N = 1\, D = 3$, $N = (1{,}1)\, D = 2$ and $N = 1\, D = 4$ supersymmetric theories were recovered via presymplectic BV-AKSZ construction.

Thank you for attention!