

# New results on dynamical flattening of halo density cusps by Q-ball dark matter

Alexander Libanov, Sergey Troitsky

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# Motivation

- ? Numerical simulations of collisionless dark matter halos predict a steep central peak in the dark matter mass density distribution (Navarro-Frenk-White Profile) in the central regions of halos within the  $\Lambda$ CDM framework (cusp) and are in poor agreement with galaxy observations (flat dark matter density profile in the center of halo, core). There is no clear transition from cuspy to cored profiles (cusp-to-core transition) [1].

$$\rho_{NFW}(r) = \frac{\rho_N}{r/r_N(r/r_N + 1)^2}, \quad \text{-> cusp} \quad \rho_{Burkert} = \frac{\rho_B}{(r/r_B + 1)(r^2/r_B^2 + 1)}, \quad \text{-> core}$$

- ? Adding baryon feedback may be ineffective in galaxies with low baryon abundances [2].
- ? The cross section for self-interacting dark matter (SIDM) is tightly constrained [3].
- ✓ Dark matter Q-balls are capable of merging efficiently in the central regions of halos [4].

[1] Salucci, P., (2019), *The Astron. Astrophys. Rev.*, 27(1), 2.

[2] De Naray, R. K. et al, (2008), *The Astrophys. J.*, 676(2), 920-943.

[3] Andrade, K. E. et al, (2022), *MNRAS*, 510(1), 54-81.

[4] Libanov, A., (2025), *Phys. Rev. D*, 111(6), 063540.

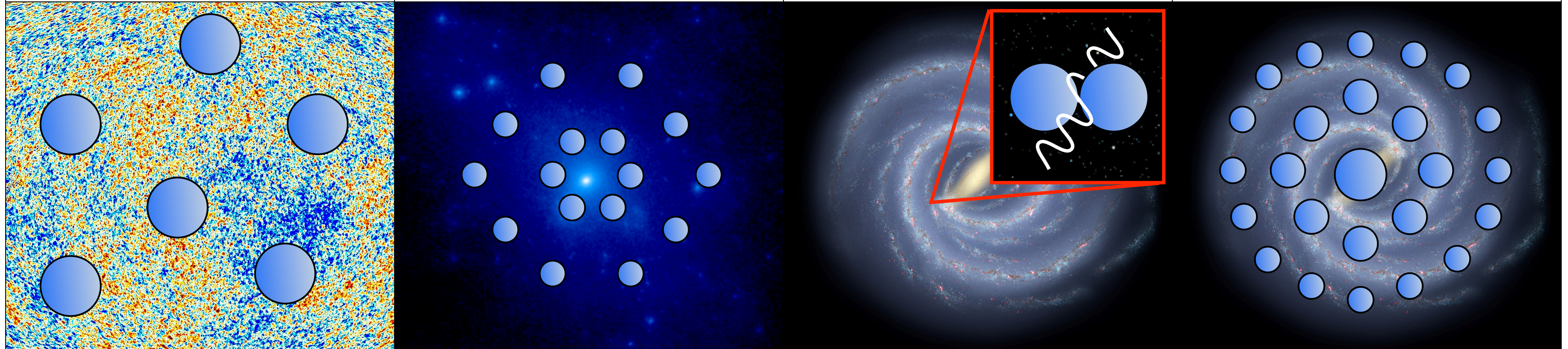
# Scenario

1. Q-balls are born in the early Universe with a characteristic charge  $Q_0$  and are the only component of dark matter.

2. Q-balls form galaxies with the Navarro-Frank-White profile.

3. Q-balls merge with each other and emit gravitational waves.

4. The emitted gravitational waves carry away some of the dark matter mass from the halo.



Time  $t$

Redshift  $z$



# Q-balls Lagrangian

Friedberg-Lee-Sirlin Lagrangian[5]:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - U(\varphi) + (\partial_\mu\chi)^*\partial_\mu\chi - k^2\varphi^2\chi^*\chi, \quad (1)$$

$$U(\varphi) = (\varphi^2 - v^2)^2.$$

Q-ball parameters:

$$R_Q = \left(\frac{Q}{4}\right)^{1/4} \frac{1}{v}, \quad (2)$$

$$M_Q = \frac{4\sqrt{2}\pi}{3}vQ^{3/4}. \quad (3)$$

Limitations on Q-ball charge:

$$Q_{min} = \frac{M_Q}{m_\chi}, \quad (4)$$

$$Q_{max} = \frac{9}{256\pi^2G^2} \frac{1}{v^4}. \quad (5)$$

[5] Friedberg, R., Lee, T. D., Sirlin, A., (1976), Phys. Rev. D, 13(10), 2739.

# Model assumptions

- Q-balls are born in the early Universe without specifying a specific birth mechanism (*phase transition? condensation? something else?*) with some characteristic charge  $Q_0$ , which is considered as a free parameter.
- In addition to the charge  $Q_0$ , the potential parameter  $\nu$  is also considered as a free parameter.
- Before the formation of structures, Q-balls do not interact with each other.
- Such Q-balls are the only component of dark matter and form halos with the Navarro-Frank-White dark matter halo density profile, without specifying the specific mechanism of structure formation.
- After the formation of the structure at  $z_{cusp}$  its halo is isolated.
- Q-balls interact between each other in the dark matter halo and Q-balls relative velocities are equal to the velocity dispersion of stars in the galaxy. To estimate this dispersion, the Jeans approximation for weakly collisional dark matter is used.
- Q-balls have no channel of interaction with the visible sector, except for gravitational.
- We do not take baryonic feedback into account and the two closest Q-balls are most likely to merge.
- As a result of the interaction, the two Q-balls merge with known probability, and part of their mass is emitted in the form of gravitational waves during this process.

# Q-balls merging mechanism

A change in the charge of a Q-ball due to merging with another identical Q-ball [4]:

$$\begin{cases} \frac{\partial Q(t, r)}{\partial t} = Q(t, r) \sigma_r(r) \sigma_Q(Q) n_Q(Q), & t \in [0; 13] \text{ Gyr}, \\ Q(0, r) = Q_0, \end{cases} \quad (6)$$

where

$$\sigma_r^2(r) = \frac{1}{\rho_{QBDM}(r)} \int_r^\infty \rho_{QBDM}(r) \frac{GM_h(< r)}{r^2} dr, \quad (7)$$

$$\sigma_Q = \frac{\pi R_Q^2}{2}, \quad (8)$$

$$n_Q(Q) = \frac{\rho_{QBDM}}{M_Q(Q)}. \quad (9)$$

[4] Libanov, A., (2025), Phys. Rev. D, 111(6), 063540.

# Q-balls merging mechanism

Charge conservation law:

$$Q_0 N_0(r) = Q(t, r) N(t, r) \Rightarrow Q_0 \frac{\rho_{NFW}(r)}{M_Q(Q_0)} dV_h = Q(t, r) \frac{\rho_{QBDM}(t, r)}{M_Q(Q)} dV_h \Rightarrow \quad (10)$$

$$\frac{\rho_{QBDM}(t, r)}{\rho_{NFW}(r)} = \left( \frac{Q(t, r)}{Q_0} \right)^{-1/4}. \quad (11)$$

Energy conservation law:

$$M_h(0, r) = M_h(t, r) + E_{GW}(t, r) \Rightarrow \rho_{NFW}(r) dV_h = \rho_{QBDM}(t, r) dV_h + \rho_{GW}(t, r) dV_h \Rightarrow \quad (12)$$

$$\rho_{NFW}(r) = \rho_{QBDM}(t, r) + \rho_{GW}(t, r). \quad (13)$$

# Q-balls merging mechanism

The final equation of the Q-ball merger:

$$\left\{ \begin{array}{l} \frac{\partial Q(t, r)}{\partial t} = \frac{3}{16\sqrt{2}v^3} \sigma_r(t, r) \rho_{QBDM}(t, r) Q^{3/4}(t, r), \quad t \in [0; t(z)] \text{ Gyr}, \\ \frac{\rho_{QBDM}(t, r)}{\rho_{NFW}(r)} = \left( \frac{Q(t, r)}{Q_0} \right)^{-1/4} \\ \rho_{NFW}(r) = \rho_{QBDM}(t, r) + \rho_{GW}(t, r), \\ Q(0, r) = Q_0, \end{array} \right. \quad (14)$$

where

$$t(z) = \frac{1}{H_0} \int_z^{z_{cusp}} \frac{dz}{(1+z) \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}}, \quad t_0 \equiv t(0) \quad (15) \quad \text{— the equivalent time associated with the redshift of the halo.}$$

# Density profile equation

We explicitly substitute the relation of density with charge into the equation of Q-balls merger and obtain the equation for density:

$$\left\{ \begin{array}{l} \frac{\partial \rho_{QBDM}(t, r)}{\partial t} = - \frac{3}{64\sqrt{2}} \frac{1}{v^3 Q_0^{1/4}} \frac{1}{\rho_{NFW}(r)} \sigma_r(t, r) \rho_{QBDM}^3(t, r), \quad t \in [0; t(z)] \text{ Gyr}, \\ \rho_{NFW}(r) = \rho_{QBDM}(t, r) + \rho_{GW}(t, r), \\ \rho(0, r) = \rho_{NFW}(r), \\ \rho_{NFW}(r) = \frac{\rho_N}{\frac{r}{r_N} (1 + \frac{r}{r_N})^2}, \end{array} \right. \quad (16)$$

or, moving on to dimensionless variables  $\theta = t/t_0$ ,  $\xi = \rho_{QBDM}/\rho_{NFW}$ ,  $x = r/r_N$ ,

$$\left\{ \begin{array}{l} \frac{\partial \xi}{\partial \theta} = - C \frac{\xi^{5/2}}{x^{1/2}(1+x)} \left( \int_x^\infty \frac{\xi \int_0^x x(x+1)^{-2} \xi dx}{x^3(1+x)^2} dx \right)^{1/2}, \quad \theta \in [0; \theta_{max}], \\ \xi(0, x) = 1, \\ C = \frac{3}{32} \sqrt{\frac{\pi}{2}} \frac{t_0 r_N \rho_N^{3/2} \sqrt{G}}{v^3 Q_0^{1/4}}. \end{array} \right. \quad (17)$$

# Parameter space

## I. Limitation on the cross section:

Since the probability of inelastic interaction of Q-balls is  $\sim 50\%$  [6], then  $\bar{\sigma}_Q^{\text{elastic}} = \bar{\sigma}_Q^{\text{inelastic}} = \bar{\sigma}_Q$ .

The mean cross sections is

$$\bar{\sigma}(Q) = \frac{\pi R_Q^2}{2M_Q}.$$

Q-balls parameters are

$$R_Q = \left(\frac{Q}{4}\right)^{1/4} \frac{1}{v},$$

$$M_Q = \frac{4\sqrt{2}\pi}{3} v Q^{3/4}.$$

$$\bar{\sigma}(Q) = \frac{3}{16\sqrt{2}} v^{-3} Q^{-1/4}.$$

The greater the charge, the smaller the cross-section.

[6] Multamäki, T., & Vilja, I, (2000), *Phys. Lett. B*, 484(3-4), 283-288.

# Parameter space

The limitation on elastic interaction cross section is different for different structures.

The most conservative limitation is  $\lesssim 1 \text{ cm}^2/\text{g}$  [7]:

$$\bar{\sigma}(Q_0) = \frac{3}{16\sqrt{2}} v^{-3} Q_0^{-1/4} \lesssim 1 \text{ cm}^2/\text{g}.$$

Limitation from Bullet Cluster [7]:

$$C = \frac{3}{32} \sqrt{\frac{\pi}{2}} \frac{t_0 r_N \rho_N^{3/2} \sqrt{G}}{v^3 Q_0^{1/4}} \rightarrow \bar{\sigma}(M_{BC}, r \approx 50 \text{ kpc}, z_{BC} = 0.3) \lesssim 0.13 \text{ cm}^2/\text{g} \rightarrow v Q_0^{1/12} \geq 19 \text{ MeV}. \quad (18)$$

[7] Adhikari, S. et al, (2025), *Reviews of Modern Physics*, 97(4), 045004.

# Parameter space

## II. Q-balls must not collapse into black holes:

$$R_Q > R_{BH} \Rightarrow v^4 Q_0 < \frac{9}{256\pi^2 G^2}. \quad (19)$$

## III. Are Q-balls MACHO?

The restriction on MACHO microlensing requires no more than 10% DM with masses  $M_{MACHO} \sim 10^{-7} M_\odot$  [8]. Since the main part of the Q-balls does not change, it is sufficient to limit the mass of the initial Q-balls:

$$M_Q(v, Q_0) \lesssim M_{MACHO} \iff v Q_0^{3/4} \lesssim \frac{3}{4\sqrt{2}} \pi M_{MACHO}. \quad (20)$$

[8] Tisserand, P. et al, (2007), *Astron. & Astroph.*, 469(2), 387-404.

# Parameter space

## IV. Gravitational waves background limitation.

Gravitational wave frequency near Q-ball during merger in Newtonian approximation:

$$f_0 = \frac{1}{\pi} \sqrt{\frac{2GM_Q}{(2R_Q)^3}} = 2\sqrt{\frac{G}{3\pi}} v^2. \quad (21)$$

The frequency does not depend on Q-balls macroscopic parameters  $\Rightarrow$  [halo is monochromatic source!](#)

Universe expands  $\Rightarrow$  gravitational wave "redshifts":

$$f_d = \frac{f_s}{1+z}. \quad (22)$$

# Parameter space

1. Characteristic amplitude  $h_c$  from Q-balls  $< h_c$  of detectors (a weaker restriction).
2. Observed frequency of GW from Q-balls is not in range of active detectors (a stronger constraint is used in this work).

Since the LIGO range is not realized within the framework of this model with any physically meaningful set of parameters and LISA is at the design stage, it is sufficient to satisfy NANOGrav observations.

NanoGrav frequency is in  $10^{-9} - 10^{-8}$  Hz. When taking into account the redshift of the GW from the Q-balls, it can be seen that it is enough to make two estimates:

1. Frequency near source  $f_0 < f_{NanoGrav}^{min} = 10^{-9}$  Hz.
2. Frequency observed on detector  $f_d = f_0 / (1 + z_{cusp}) > f_{NanoGrav}^{max} = 10^{-8}$  Hz.

# Parameter space

Then, taking into account the Newtonian approximation and the redshift,

$$v^2 \geq \frac{1}{2} \sqrt{\frac{3\pi}{G}} (1 + z_{cusp}) f_{NanoGrav}^{max}, \quad (23)$$

$$v^2 \leq \frac{1}{2} \sqrt{\frac{3\pi}{G}} f_{NanoGrav}^{min}. \quad (24)$$

It is easy to see that the final constraint **does not contain**  $Q_0$ .

# Parameter space

## V. Cusp flattening and cusp-to-core transition.

Let us introduce enclosed dark matter surface density:

$$\Sigma(< r) = \frac{M(< r)}{\pi R^2}, \quad (25)$$

and concentration-max velocity relation for NFW-profile [9]:

$$c_{V,N}(V_{max}) = c_0^N \left( 1 + \sum_{i=1}^3 \left[ a_i^N \log_{10} \left( \frac{V_{max}}{\text{km/s}} \right) \right]^i \right), \quad (26)$$

$$c_0^N = 7.21 \times 10^4, \quad a_1^N = -0.81, \quad a_2^N = -0.47, \quad a_3^N = -0.27, \quad V_{max}/(\text{km s}^{-1}) \in [7; 1500].$$

[9] Kaneda, Y. et al, (2024), *Publications of the Astronomical Society of Japan*, 76(5), 1026-1040.

# Parameter space

Using this relations it can be obtained theoretical relation between enclosed in  $0.01r_{max}$  dark matter surface density and max velocity of rotation curve for different halo masses  $M_{200}$  for NFW (cuspy) profile:

$$\Sigma_N(< 0.01r_{max}, V_{max}) = \frac{H_0 V_{max} f_N(0.01x_N) c_{V,N}^{1/2}}{0.01^2 \sqrt{2\pi} G f_N(x_N)}, \quad (27)$$

where

$$f_N(x) = \ln(1 + x) - \frac{x}{1 + x} \text{ and } x_N = 2.162$$

are standard relations for NFW-profile and  $r_{max}$  are radius at  $V_{max}$  [10],[11].

[10] Moliné, A. et al, (2023), *MNRAS*, 518(1), 157-173.

[11] Hayashi, K. et al, (2026), *Publications of the Astronomical Society of Japan*, 78(2), 745-763.

# Parameter space

The same relations can be obtained for Burkert (cored) profile:

$$c_{V,B} = c_0^B \left[ 1 + \sum_{i=1}^3 a_i^b \left( \log_{10} \left( \frac{V_{max}}{\text{km/s}} \right) \right)^i \right], \quad (28)$$

$$c_0^B = 4.925 \times 10^4, \quad a_1^B = -0.8505, \quad a_2^B = 0.2434, \quad a_3^B = -0.02314, \quad V_{max}/(\text{km s}^{-1}) \in [7; 1500].$$

$$\Sigma_B(< 0.01 r_{max}, V_{max}) = \frac{H_0 V_{max} f_B(0.01 x_B) c_{V,B}^{1/2}}{0.01^2 \sqrt{2} \pi G f_B(x_B)}, \quad (29)$$

where

$$f_B = \frac{1}{4} \ln(1 + x^2) + \frac{1}{2} \ln(1 + x) - \frac{1}{2} \arctan(x) \text{ and } x_B = 3.24$$

are standard relations for Burkert-profile [9-11].

[9] Kaneda, Y. et al, (2024), *Publications of the Astronomical Society of Japan*, 76(5), 1026-1040.

[10] Moliné, A. et al, (2023), *MNRAS*, 518(1), 157-173.

[11] Hayashi, K. et al, (2026), *Publications of the Astronomical Society of Japan*, 78(2), 745-763.

# Parameter space

Also in one hand we can find enclosed dark matter surface density and max velocity which corresponds to our density profile:

$$M(< r) = \int_0^r 4\pi r^2 \rho_{QBDM}(z=0) dr, \quad (30)$$

$$V^2 = \frac{GM(< r)}{r^2}, \quad (31)$$

$$\frac{dV}{dr} = 0 \rightarrow (V_{max}, r_{max}), \quad (32)$$

$$\Sigma_{QBDM}(< 0.01r_{max}) = \frac{M(< 0.01r_{max})}{\pi(0.01r_{max})^2}. \quad (33)$$

# Parameter space

On the other hand we assume that at  $z_{cusp} = 13$  our density profile is equal to Navarro-Frenk-White profile at  $z_{cusp} = 13$  and its parameters define constant  $C$  of our density equation.

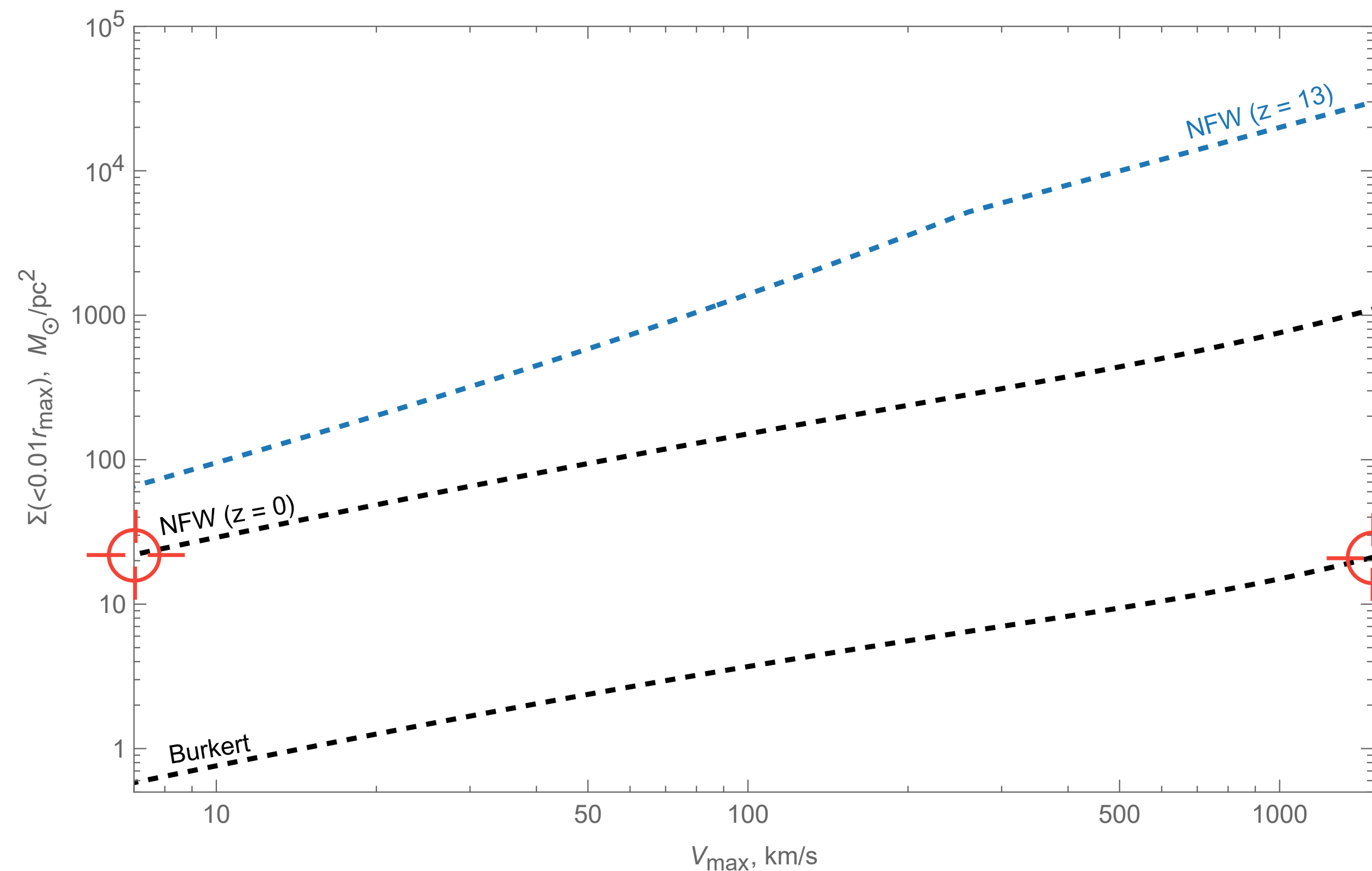
$$C = \frac{3}{32} \sqrt{\frac{\pi}{2}} \frac{t_0 r_N \rho_N^{3/2} \sqrt{G}}{v^3 Q_0^{1/4}}$$

To find Navarro-Frenk-White profile parameters at  $z_{cusp} = 13$  for different halo masses  $M_{200}/M_\odot \in [1.0 \times 10^4; 1.0 \times 10^{17}]$  we use the Uchuu cosmological simulations [12].

[12] Ishiyama, T. et al, (2021), *MNRAS*, 506(3), 4210-4231.

# Parameter space

On the one hand, it can be seen from the form of the density equation that the lower the constant, the worse the initial profile is flattened. On the other hand, the larger the halo, the greater the constant.



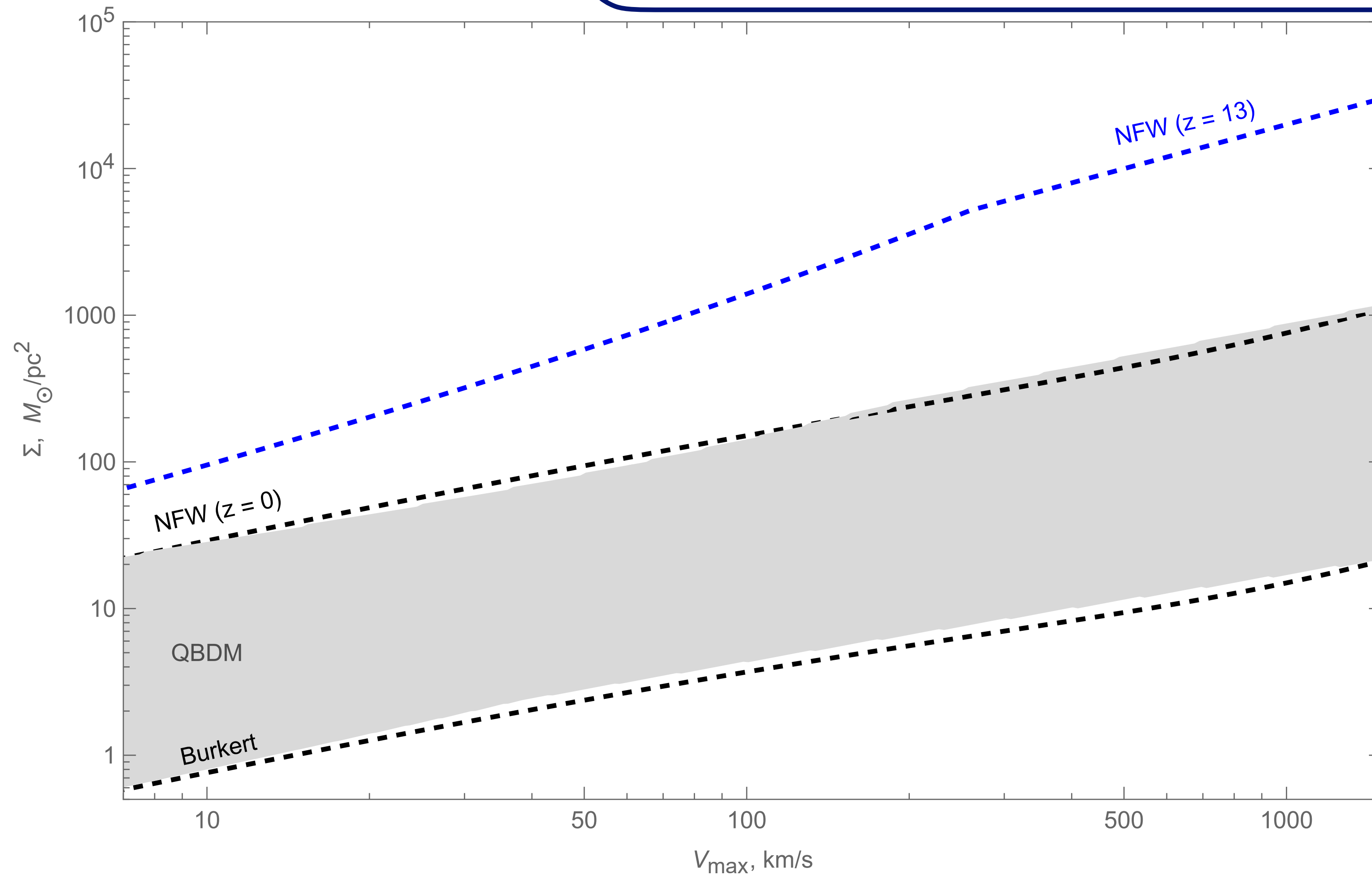
On the other hand, the larger the halo, the greater the constant. It follows that it is sufficient to shoot the constant for the small max velocity (and, accordingly, the small halo mass) according to condition (34) and shoot the constant for the large max velocity (and, accordingly, the large halo mass) according to condition (35).

$$\Sigma_{QBDM}(< 0.01r_{max}) \lesssim \Sigma_N(< 0.01r_{max}) | V_{max} \approx 7 \text{ km/s}, \quad (34)$$

$$\Sigma_{QBDM}(< 0.01r_{max}) \gtrsim \Sigma_B(< 0.01r_{max}) | V_{max} \approx 1500 \text{ km/s}, \quad (35)$$

# Parameter space

$$C_{min} |_{M_{200}=4.3 \times 10^6 M_{\odot}} \approx 0.5 \leq C \leq C_{max} |_{M_{200}=6.3 \times 10^{16} M_{\odot}} \approx 1.5 \times 10^9. \quad (36)$$



$$\Sigma_{QBDM}(< 0.01 r_{max}) |_{C_{min}} \approx 21.6 M_{\odot}/\text{pc}^2,$$

$$V_{max} |_{C_{min}} \approx 7 \text{ km/s}.$$

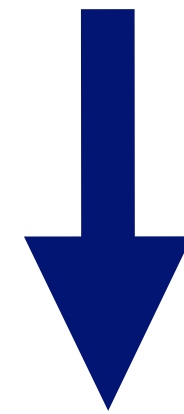
$$\Sigma_{QBDM}(< 0.01 r_{max}) |_{C_{max}} \approx 22.5 M_{\odot}/\text{pc}^2,$$

$$V_{max} |_{C_{max}} \approx 1500 \text{ km/s}.$$

# Parameter space

$$C_{min} |_{M_{200}=4.3 \times 10^6 M_{\odot}} \approx 0.5 \leq C \leq C_{max} |_{M_{200}=6.3 \times 10^{16} M_{\odot}} \approx 1.5 \times 10^9,$$

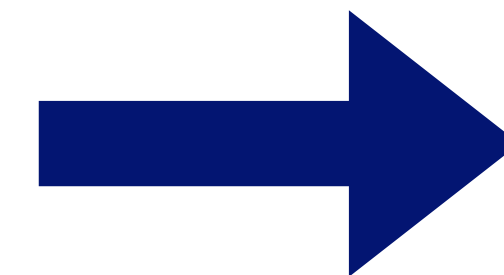
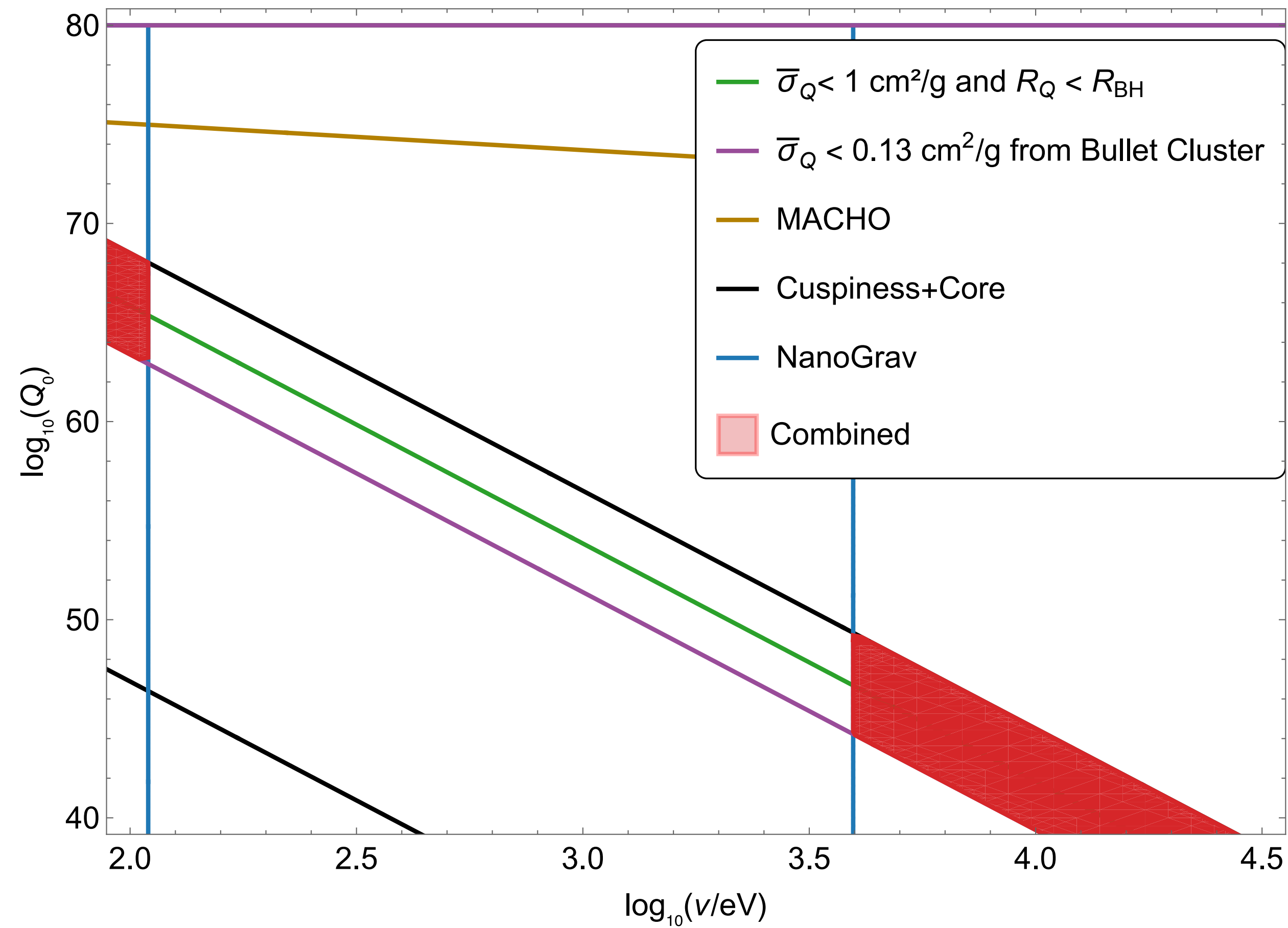
$$C = \frac{3}{32} \sqrt{\frac{\pi}{2}} \frac{t_0 r_N \rho_N^{3/2} \sqrt{G}}{v^3 Q_0^{1/4}}.$$



$$0.8 \text{ MeV} \leq v Q_0^{1/12} \leq 51 \text{ MeV}. \quad (37)$$

# Parameter space

It is convenient to select free parameters of the model not individually, but as a combination  $vQ_0^{1/12}$ .

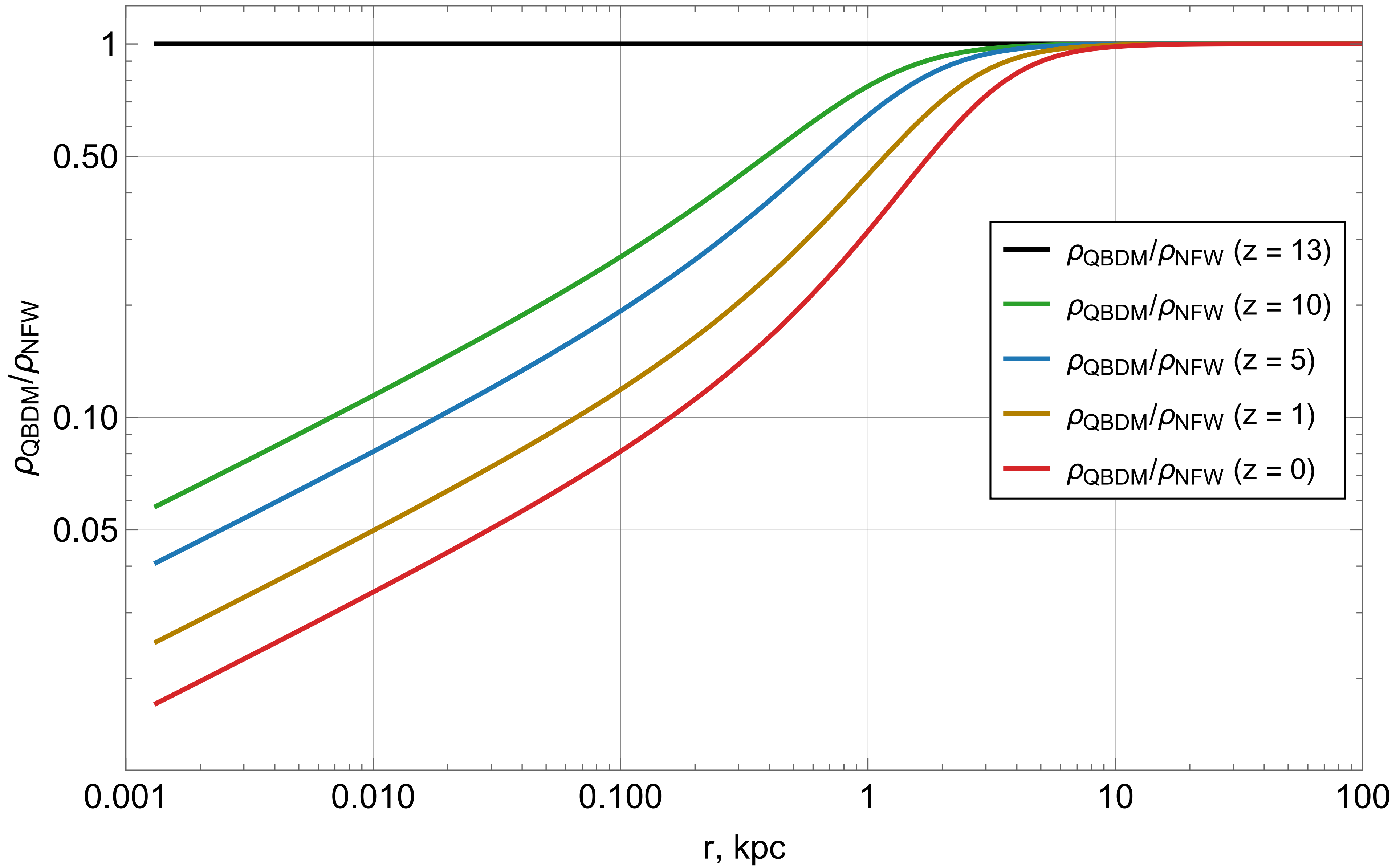


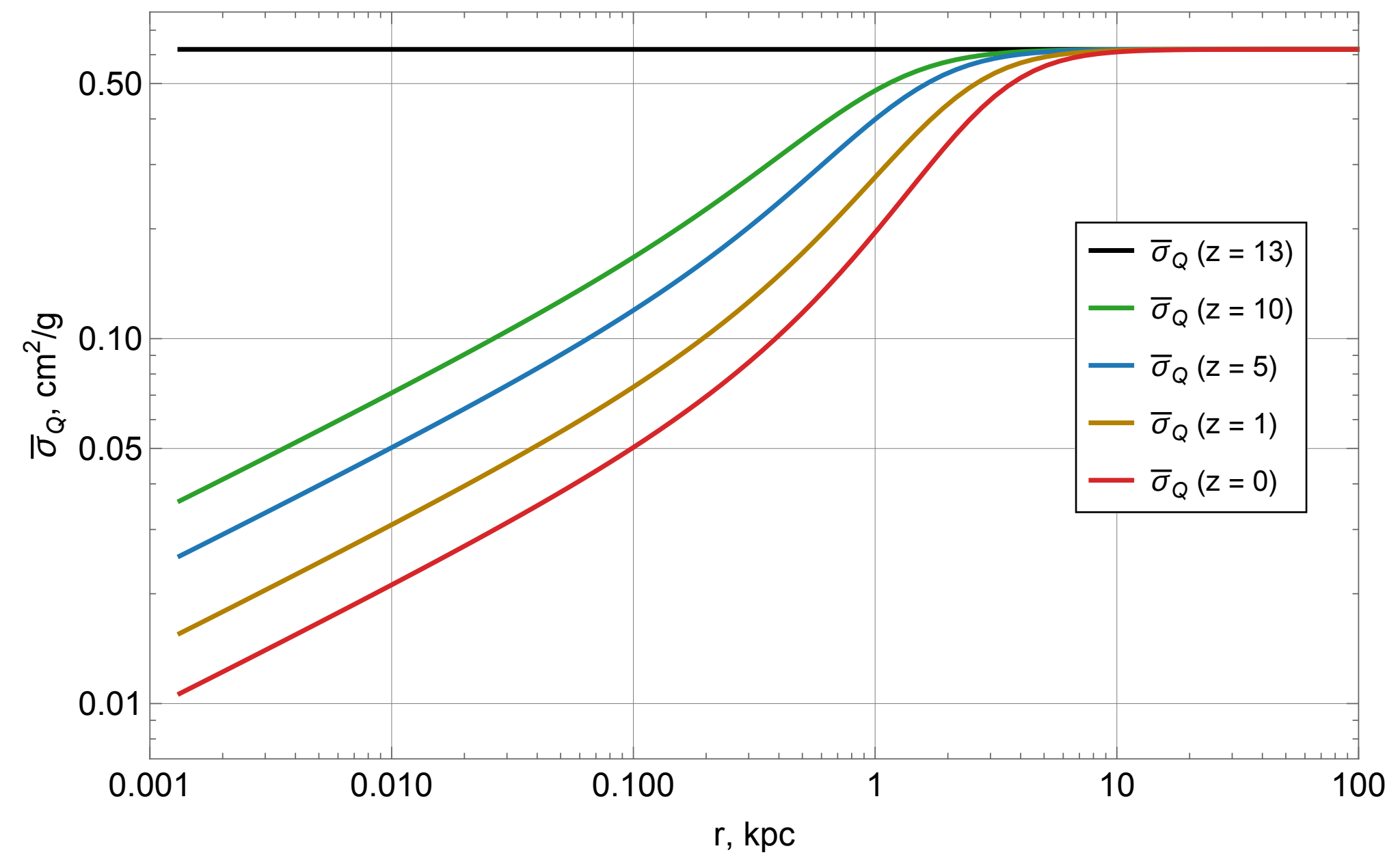
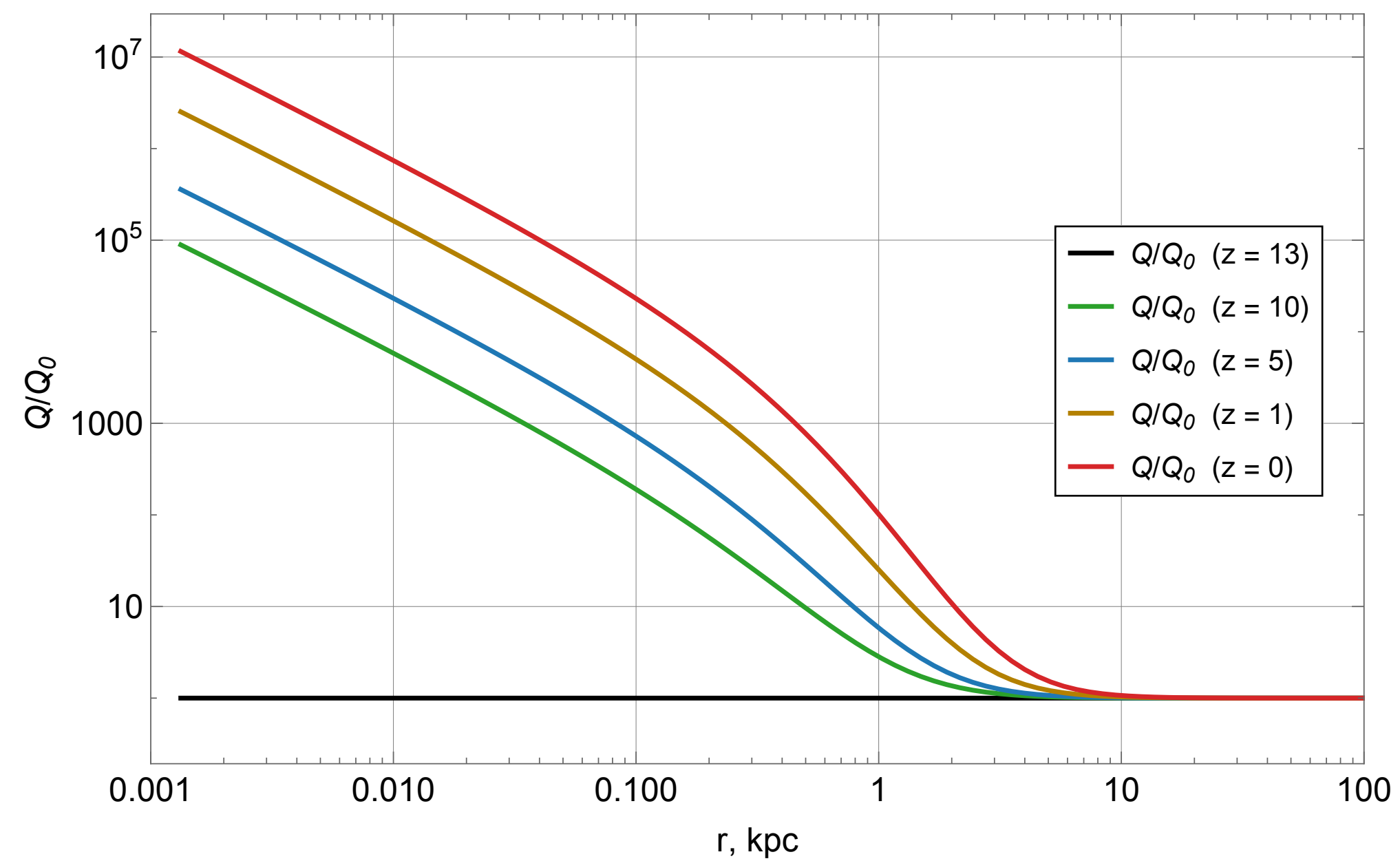
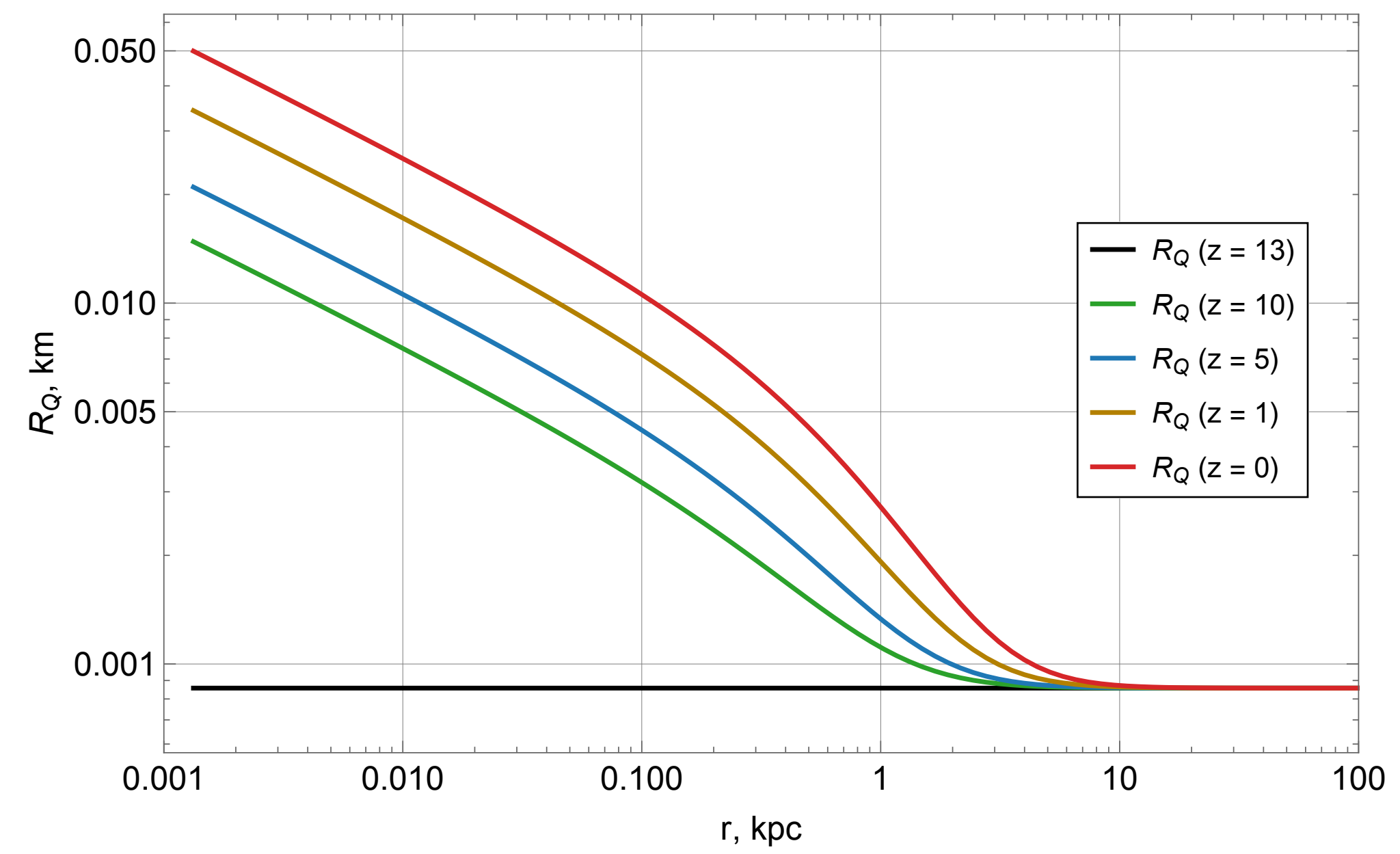
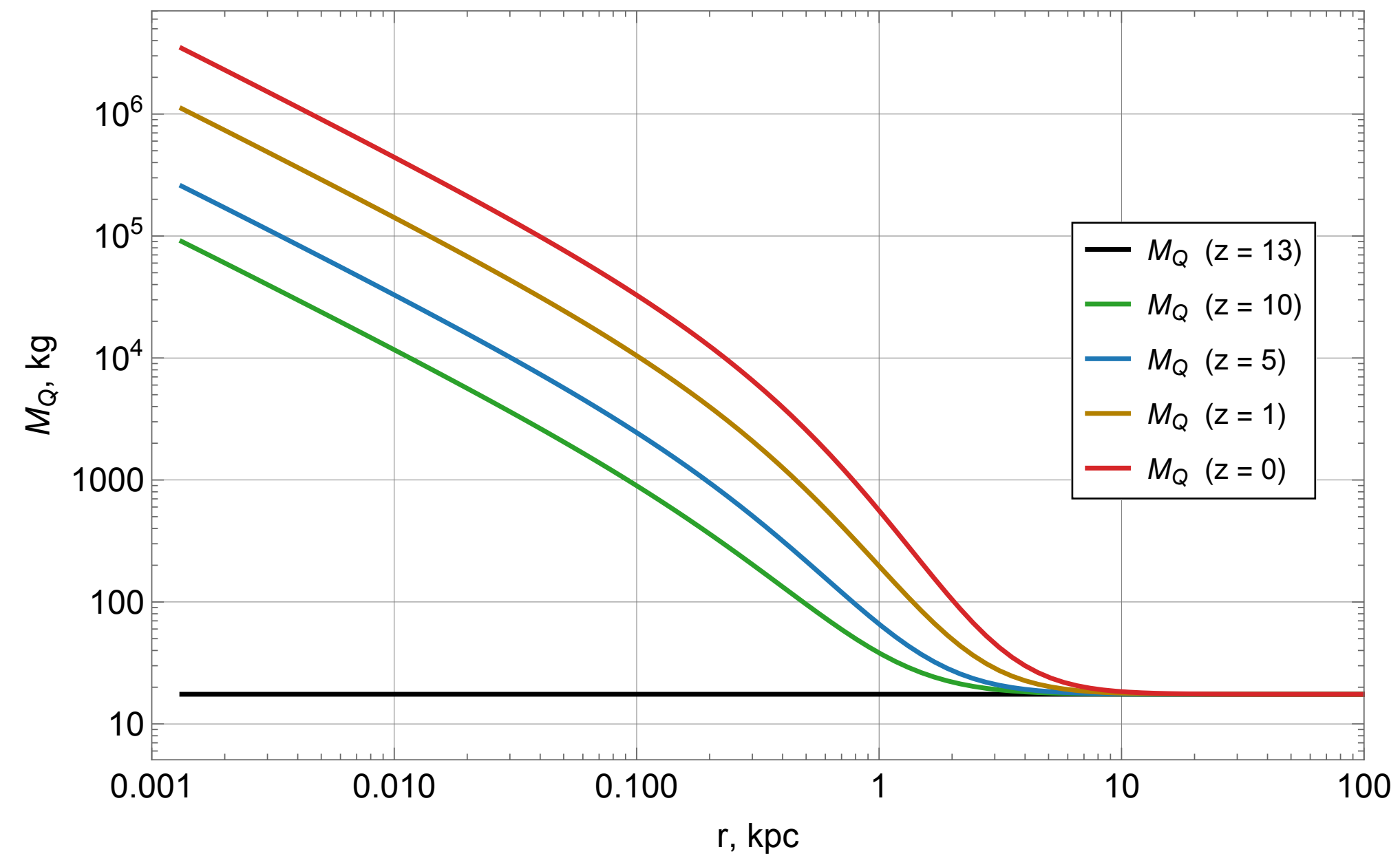
$$19 \text{ MeV} \lesssim vQ_0^{1/12} \lesssim 52 \text{ MeV}$$

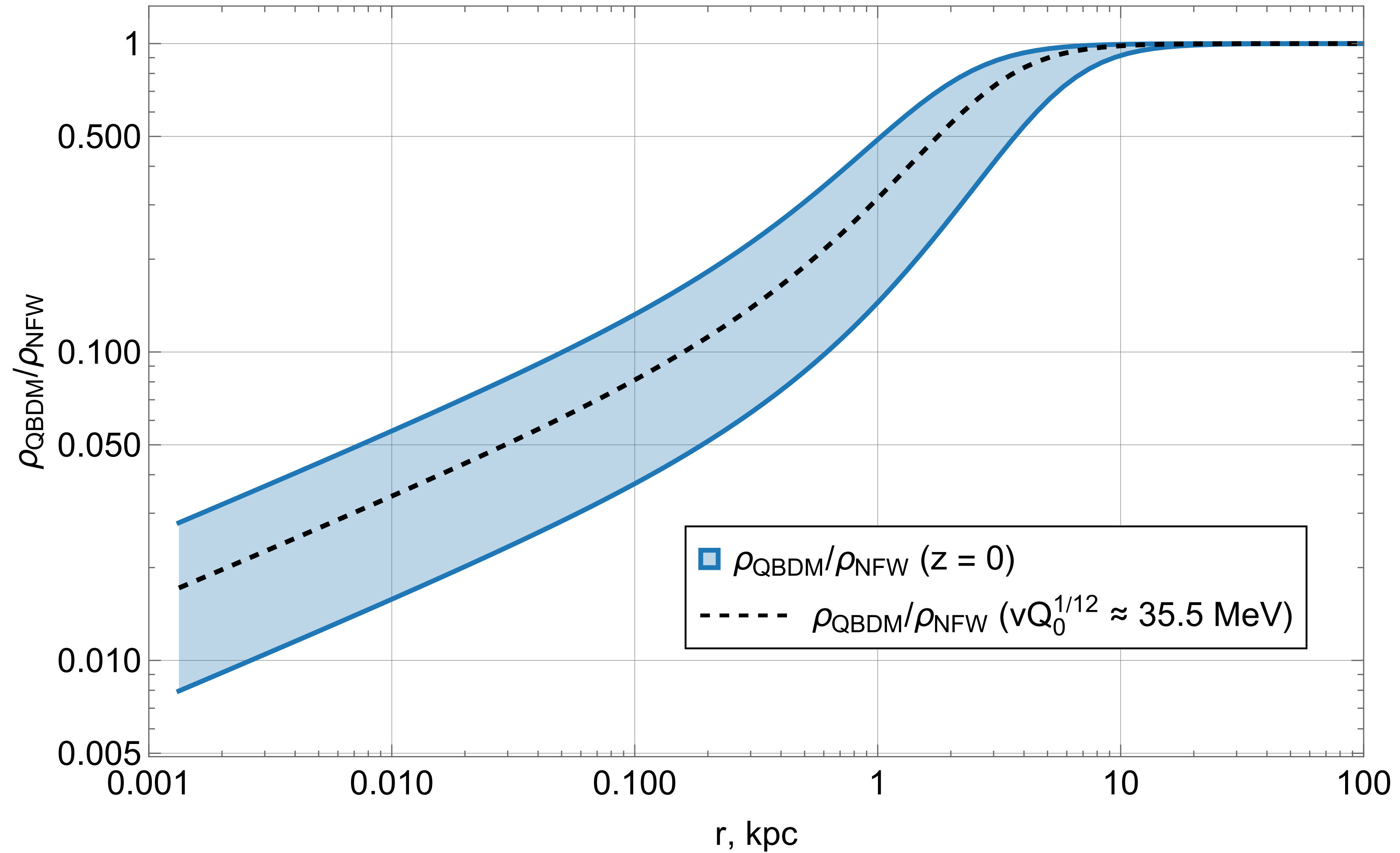
# Numerical solution

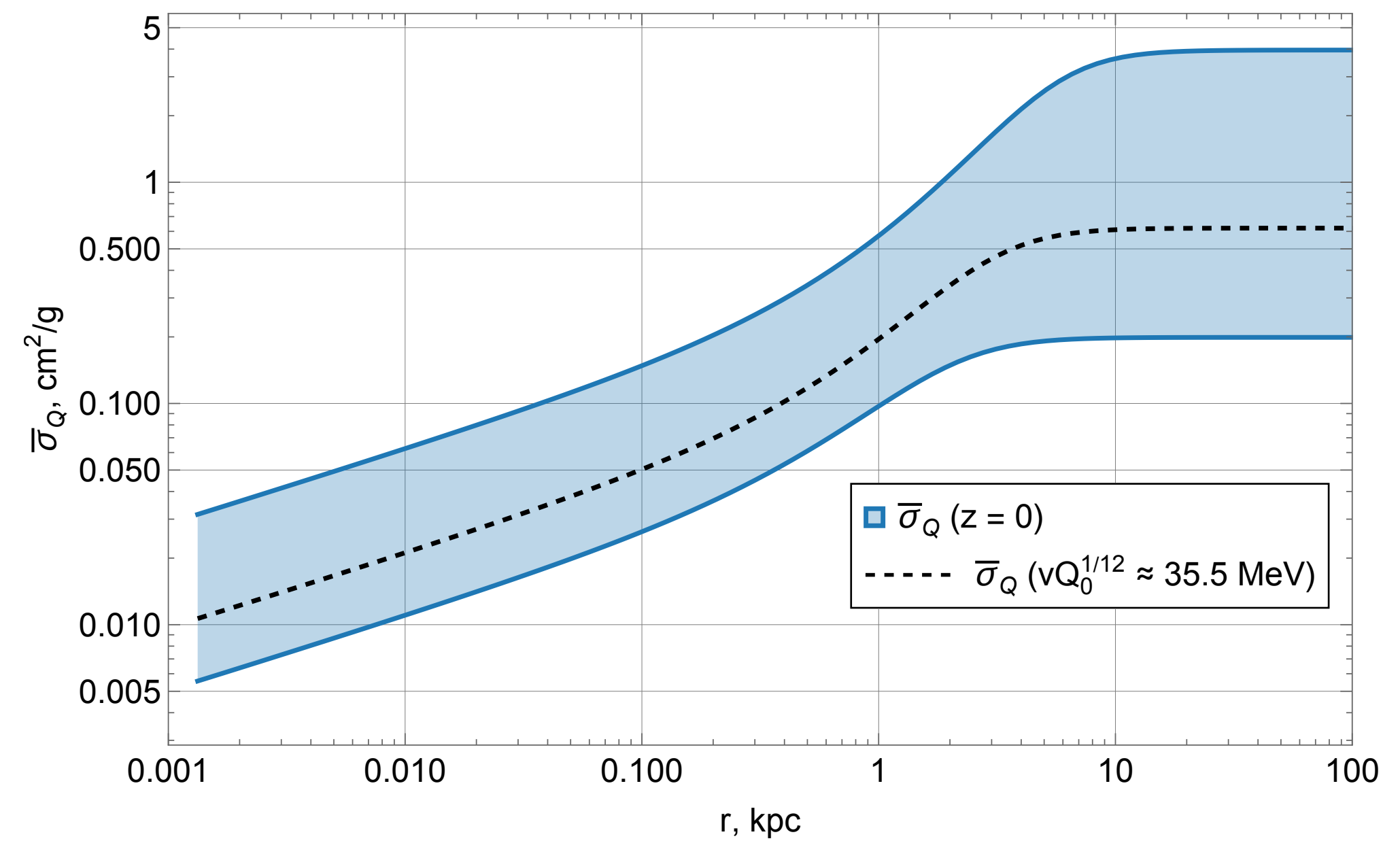
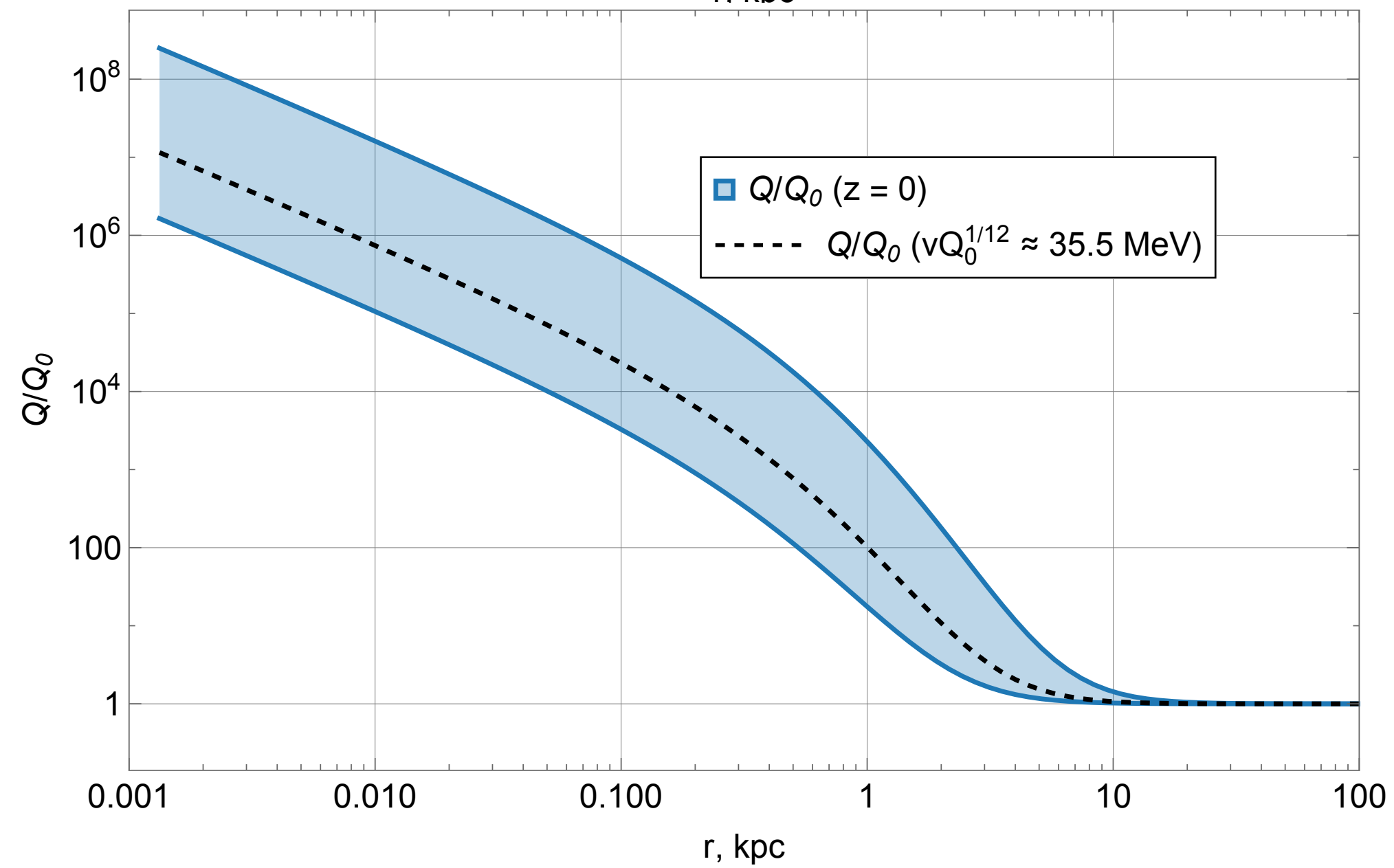
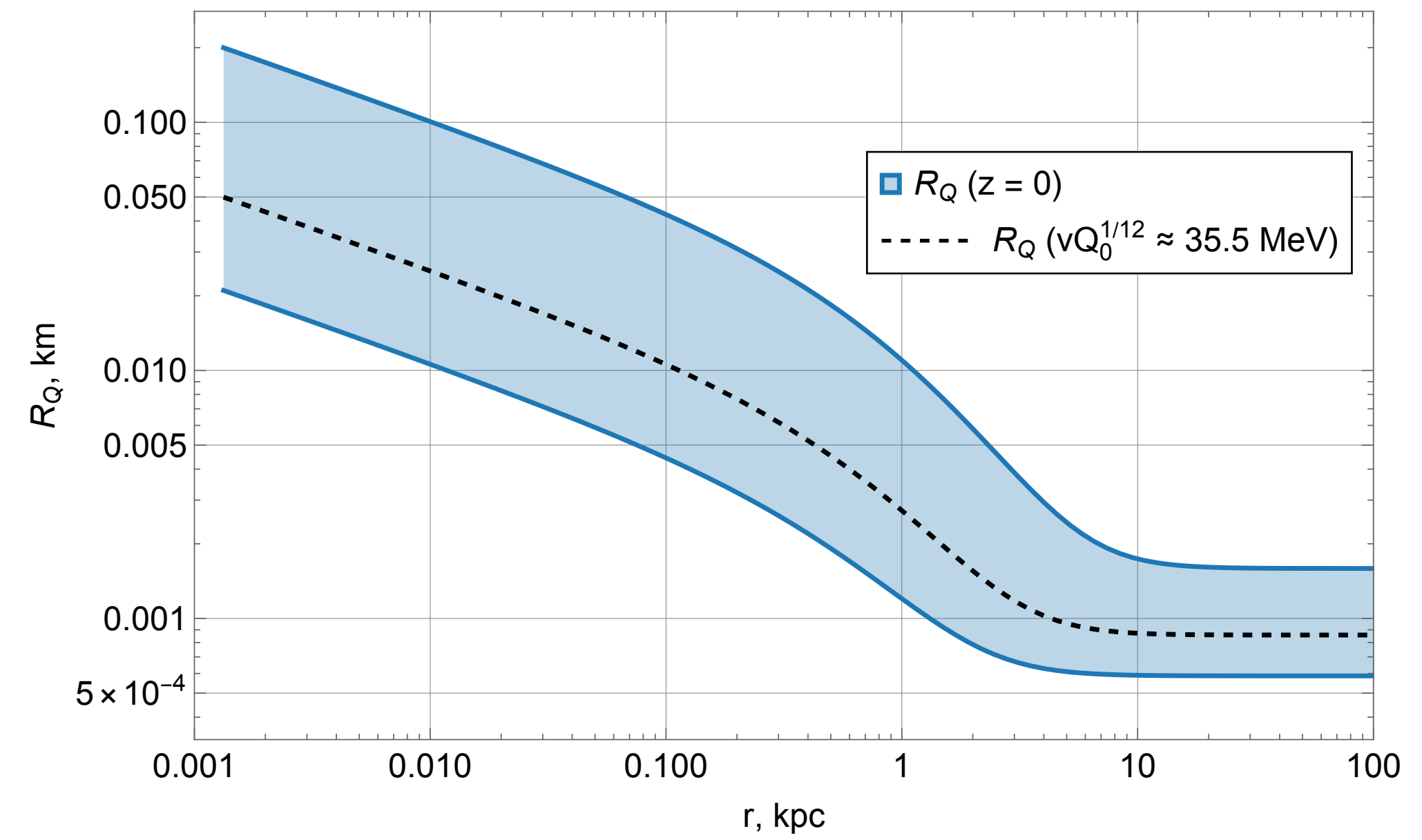
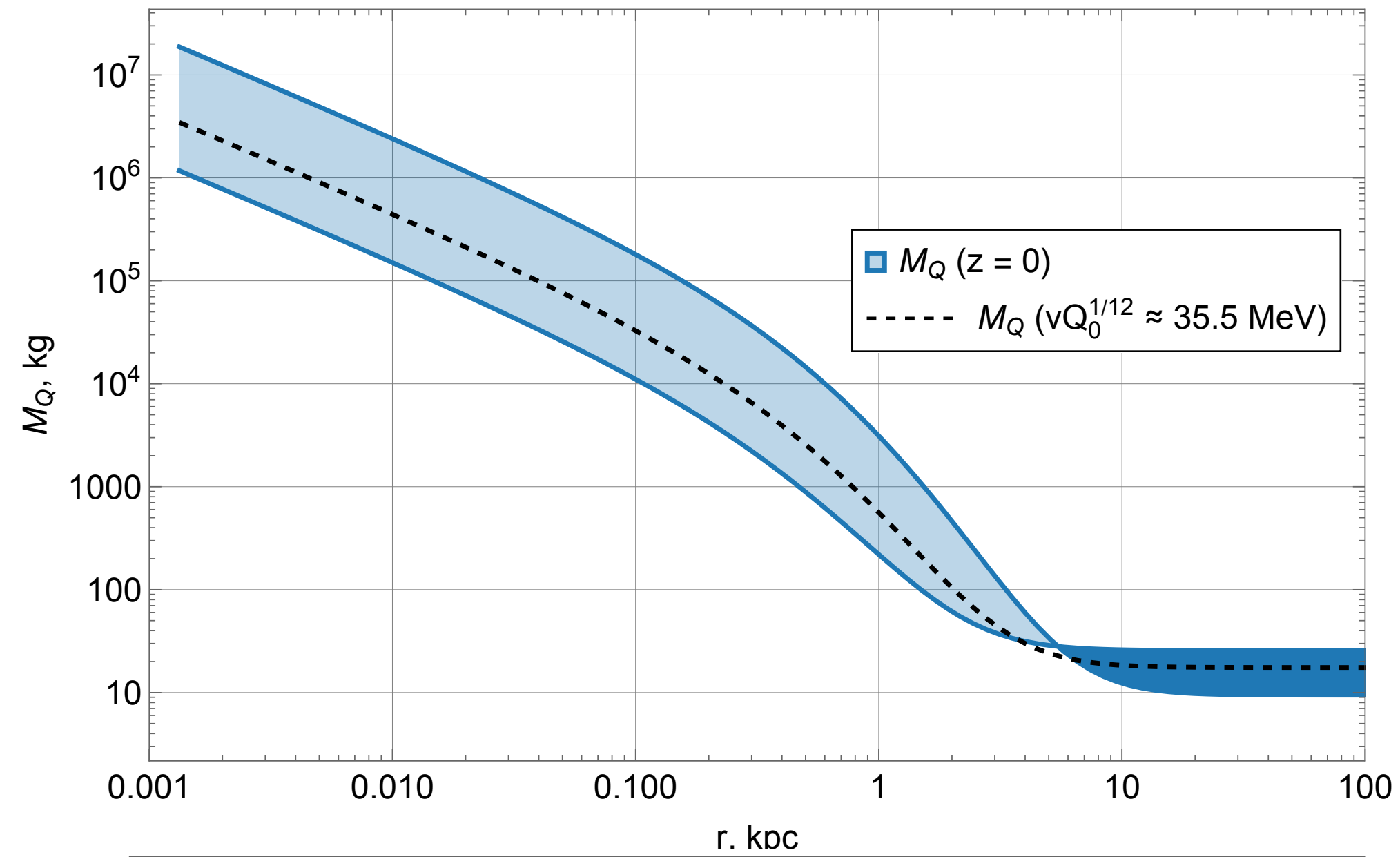
$$\left\{ \begin{array}{l} \frac{\partial \rho_{QBDM}(t, r)}{\partial t} = - \frac{3}{64\sqrt{2}} \frac{1}{v^3 Q_0^{1/4}} \frac{1}{\rho_{NFW}(r)} \sigma_r(t, r) \rho_{QBDM}^3(t, r), \quad t \in [0; t(z)] \text{ Gyr}, \\ \rho_{NFW}(r) = \rho_{QBDM}(t, r) + \rho_{GW}(t, r), \\ \rho(0, r) = \rho_{NFW}(r), \\ \rho_{NFW}(r) = \frac{\rho_0}{\frac{r}{r_s} (1 + \frac{r}{r_s})^2}, \end{array} \right. \iff \left\{ \begin{array}{l} \frac{\partial \xi}{\partial \theta} = - C \frac{\xi^{5/2}}{x^{1/2}(1+x)} \left( \int_x^\infty \frac{\xi \int_0^x x(x+1)^{-2} \xi dx}{x^3(1+x)^2} dx \right)^{1/2}, \quad \theta \in [0; \theta_{max}], \\ \xi(0, x) = 1, \\ C = \frac{3}{32} \sqrt{\frac{\pi}{2}} \frac{\tau_0 r_s \rho_0^{3/2} \sqrt{G}}{v^3 Q_0^{1/4}}. \end{array} \right.$$

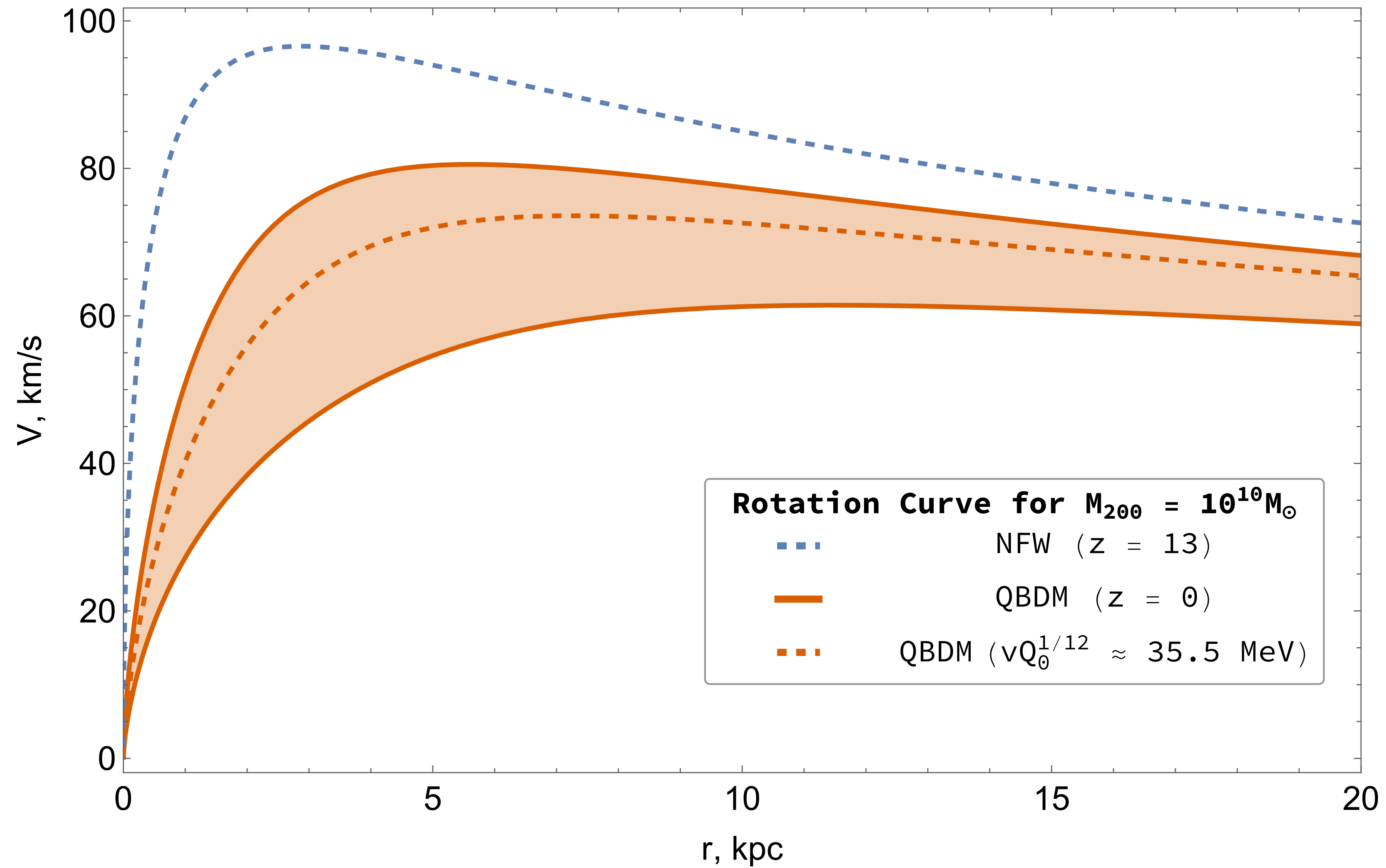
Next, the calculations will be based on the example of the mass of the:  $M_{200} = 10^{10} M_\odot$ . Free parameters of the model:  $v Q_0^{1/12} = 35.5 \text{ MeV}$ .

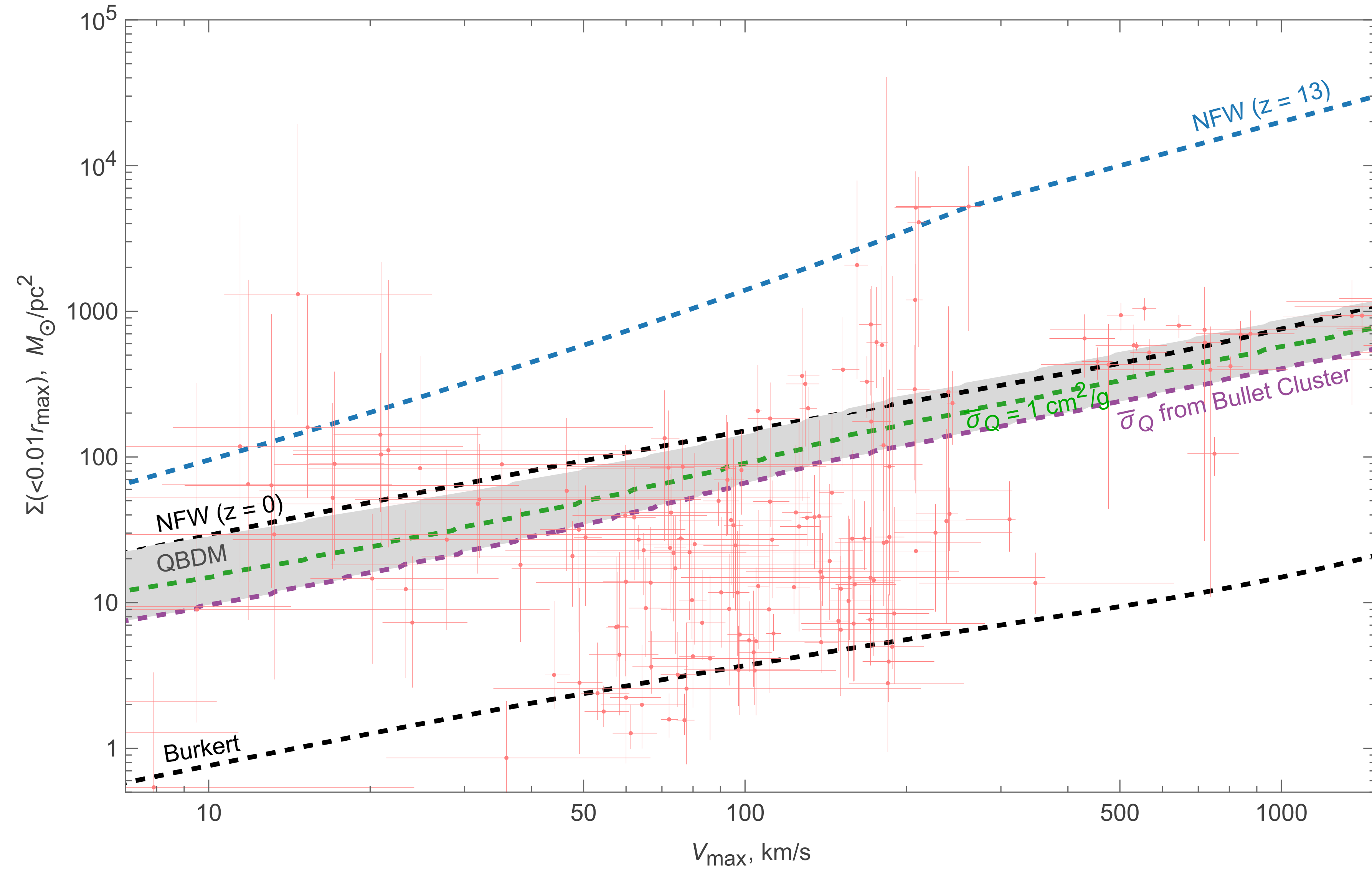












# Conclusion and discussion

- ✓ A simple model is constructed that allows for the flattening of the cusps, and an equation for the change in the density of the dark matter halo in a model with mutual mergers of Q balls is obtained.
- ✓ Parameter space for  $v$  and  $Q_0$  is obtained.
- ✓ A numerical solution has been obtained describing the evolution of the density profile of the dark matter halo and the evolution of the mass, radius, and cross-section of the Q-balls.
- ✓ The dependence of the surface density of dark matter on the maximum velocity in such a model is obtained for wide range of halo masses.
- ? Another energy emitting channel (dark radiation)?
- ? Halo formation mechanism?

# THANK YOU FOR YOUR ATTENTION!

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