Pion bremsstrahlung as a possible source for dark photons

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supported by RScF grant 25-12-00309

based on D. Gorbunov, **E. Kriukova** ***JHEP* 03 (2026) 192**

XXIII International Seminar "Quarks–2026",
Petrozavodsk, 19 May 2026

Dark photons

Portal formalism

$$\mathcal{L}_{\text{portal}} = \sum \mathcal{O}_{\text{SM}} \mathcal{O}_{\text{DS}}$$

The lowest dimension portals

- **Vector:** dark photon A'_μ, $-\frac{\epsilon}{2\cos\theta_W} \tilde{F}'_{\mu\nu} B^{\mu\nu}$
- **Scalar:** dark scalar S, $(\mu S + \lambda S^2) H^\dagger H$
- **Fermion:** heavy neutral lepton N, $Y_N L \tilde{H} N$
- **Pseudoscalar:** axion-like particle a, $\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$

Minimal dark photon model Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{1}{4} \tilde{F}'_{\mu\nu} \tilde{F}'^{\mu\nu} - \frac{\epsilon}{2\cos\theta_W} \tilde{F}'_{\mu\nu} B^{\mu\nu} + \frac{m_{\gamma'}^2}{2} \tilde{A}'_\mu \tilde{A}'^\mu,$$

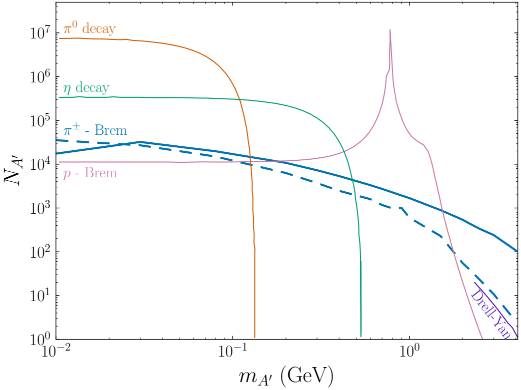
Simultaneous rotation $(W^3_\mu, B_\mu, \tilde{A}'_\mu) \rightarrow$

$\rightarrow$ interaction with charged SM particles $-\epsilon e J^\mu_{\text{em}} A'_\mu$

Dark photon production in fixed-target experiments

Production mechanisms

1. **meson decays** $m \to \gamma'\gamma$ (m: π^0, η)
2. **proton bremsstrahlung** $pp \to \gamma' X$
3. **Drell–Yan-like process in** pp**-collisions** $q\bar{q} \to \gamma'$
4. **NEW pion bremsstrahlung** $\pi p \to \gamma' X$

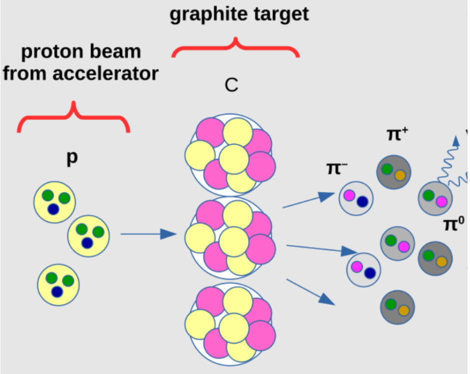


for $P = 120$ GeV, SpinQuest, Fermilab

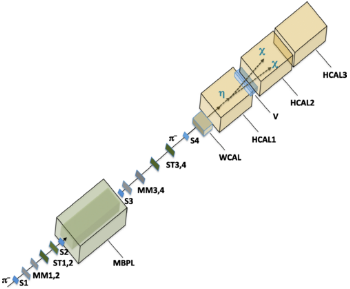
D. Curtin, Y. Kahn, R. Nguyen Phys.Rev.D 108 (2023) 9, 095039

Pion bremsstrahlung

Secondary pions in the fixed-target experiments with proton beam: T2K, DUNE, SHiP et al.



NA64h experiment: prepared π^--beam with $P = 50$ GeV from CERN SPS. Dark photon searches in meson decays are planned

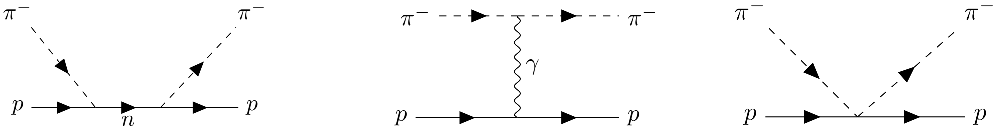


NA64 Collaboration Phys.Rev.Lett. 133 (2024) 12, 121803,
S. N. Gninenko, D. V. Kirpichnikov et al. Phys.Rev.D 109 (2024) 7, 075021

Elastic $\pi^- p$-scattering in LO ChPT

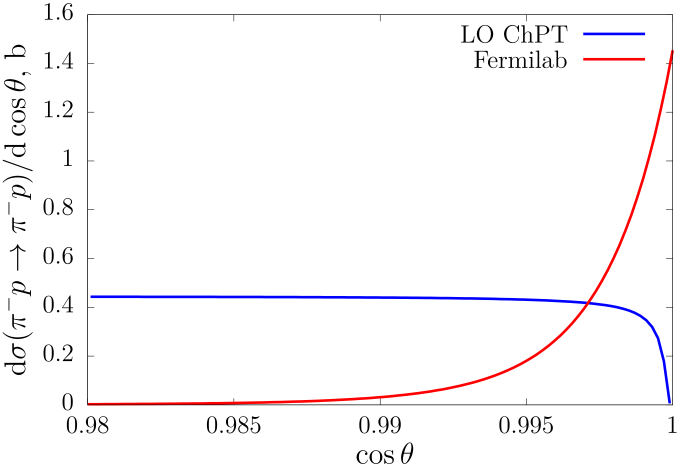
In the leading order in $Q/(4\pi F)$, where Q is a characteristic momentum scale, pion decay constant $F = 93\,\text{MeV}$, axial coupling $g_A = 1.27$,

$$\begin{aligned}\mathcal{L}_{\text{int}} = &- ieA^\mu\left(\pi^-\partial_\mu\pi^+ - \pi^+\partial_\mu\pi^-\right) - eA_\mu\bar{p}\gamma^\mu p\\ &- \frac{g_A}{F\sqrt{2}}\left(\bar{p}\gamma^\mu\gamma^5 n\partial_\mu\pi^+ + \bar{n}\gamma^\mu\gamma^5 p\partial_\mu\pi^-\right) + \frac{i}{4F^2}\bar{p}\gamma^\mu p\left(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+\right)\end{aligned}$$



(a) (b) (c)

Comparison with Fermilab data



Naively: one should limit energy transfer $E_4^{\mathrm{lab}} - M \leq 4\pi x F \Rightarrow$ constraint on **scattering angle** θ

Numerically in NA64h with $P = 50\,\mathrm{GeV}$:
$\cos\theta \geq 0.995$ for $x = 0.1$ and
$\cos\theta \geq 0.95$ for $x = 1$

Elastic πp-scattering can be described in **Regge regime**.
Elastic pion bremsstrahlung $\pi^- p \to \pi^- p \gamma'$ — by analogy?

D. Gorbunov, E. Kriukova JHEP 03 (2026) 192

Corrections to LO ChPT

For pions with $P = 50$ GeV (NA64h) c.o.m. energies $E_\pi^{cm} \simeq E_p^{cm} \simeq 5$ GeV

1. ChPT is the low-energy EFT $\Rightarrow$ for $Q \gtrsim 4\pi F$ NLO is important
2. For $Q \gtrsim 600\,$MeV corrections from **vector mesons** in t-channel

$$\mathcal{L}_{\rho\pi\pi} = \frac{i g_{\rho\pi\pi}\sqrt{2}}{F^2}\partial_{[\mu}\rho^0_{\nu]}\partial^{[\mu}\pi^-\partial^{\nu]}\pi^+, \quad g_{\rho\pi\pi} \simeq 0.05,$$
$$\mathcal{L}_{\rho pp} = g_{\rho NN}\bar{p}\gamma^\mu\rho^0_\mu p, \quad g_{\rho NN} \simeq 2.0$$

3. At Q from 1 GeV to 1.2 GeV the impact of **Δ-resonances** in s-channel

$$\mathcal{L}_{\Delta\pi p} = \frac{g_{\Delta\pi p}}{F}\left(\overline{\Delta^{++}_\mu}\Theta^{\mu\nu}\partial_\nu\pi^+ p - \frac{1}{\sqrt{3}}\overline{\Delta^0_\mu}\Theta^{\mu\nu}\partial_\nu\pi^- p\right) + h.c.,$$

where $g_{\Delta\pi p} \simeq 1.25$, $\Theta^{\mu\nu} \equiv g^{\mu\nu} + z_0\gamma^\mu\gamma^\nu$, and $z_0 \simeq -0.22$

It is impossible to include all resonances with masses up to 5 GeV in ChPT $\Rightarrow$ the cross section should be estimated in a different way

Half-off-shell pion electromagnetic form factor

When both pions are **on-shell**,

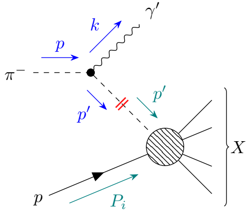
$$\left\langle \pi^-(p') \middle| J^\mu_{\mathrm{em}}(0) \middle| \pi^-(p) \right\rangle = \\ = -eF^\pi_{\mathrm{em}}((p'-p)^2)(p+p')^\mu \equiv -e\Gamma^\mu_0$$

Vertex function for **half-off-shell** case:
$p^2 = m_\pi^2,\ p'^2 \equiv t \neq m_\pi^2,\ Q^2 \equiv -(p'-p)^2$

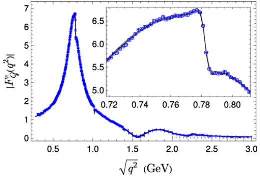
$$\Gamma^\mu_{1/2} = (p+p')^\mu F_1(Q^2,t) + \\ + (p-p')^\mu \frac{(t-m_\pi^2)}{Q^2}\left(F_1(0,t) - F_1\left(Q^2,t\right)\right)$$

Approximation using hadronic off-shell FF

$$F_1(Q^2,t) \simeq F^\pi_{\mathrm{em}}((p'-p)^2) F_{\mathrm{virt}}(z,k_\perp^2)$$



F^π_{em} from BaBar data on $e^+e^- \to \pi^+\pi^-$

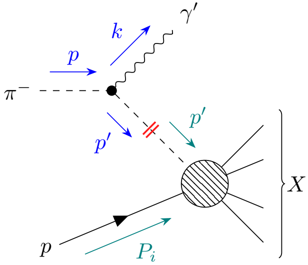


E. R. Arriola, P. Sanchez-Puertas
Phys.Rev.D 110 (2024) 5, 054003

Inelastic pion bremsstrahlung: factorization

Particles **4-momenta** in the lab frame

$$p = \{E_\pi, 0, 0, P\},$$
$$k = \{E_{\gamma'}, k_\perp \cos\varphi, k_\perp \sin\varphi, zP\},$$
$$P_i = \{M, 0, 0, 0\},$$
$$E_\pi \simeq P + \frac{m_\pi^2}{2P}, \quad E_{\gamma'} \simeq zP + \frac{m_{\gamma'}^2 + k_\perp^2}{2zP}$$



Idea of calculation:
factorize the input of subprocess $\pi^-(p) \to \gamma'(k)\pi^-(p')$
Amplitude of the inelastic pion bremsstrahlung

$$\mathcal{M}^{r\lambda}_{\pi p \to \gamma' X} = \mathcal{M}^{r}_{\pi p \to X} \frac{1}{p'^2 - m_\pi^2}(-\epsilon e) F^{\pi}_{\text{em}}(m^2_{\gamma'}) F_{\text{virt}}(z, k_\perp^2)(p + p')_\mu (\epsilon^\lambda)^{*\mu}$$

Quasireal approximation for $\pi^-(p) \to \gamma'(k)\pi^-(p')$

Standard calculation using Feynman technique

4-momentum is conserved in each vertex
$p' \equiv p - k$

Pion with momentum p' is virtual

Time-ordered perturbation theory

3-momentum is conserved in each vertex, but energy is not
$p' = p - k + \delta,$
$\delta \equiv \{\Delta E, \vec{0}\}, \quad \Delta E \equiv E'_\pi + E_{\gamma'} - E_\pi$
All particles are on-shell

Important: cross section should not depend on the calculation method

$$\mathcal{W} \equiv (2p - k + \delta)_\mu (2p - k + \delta)_\nu \sum_\lambda \left(\epsilon^\lambda\right)^{*\mu} \left(\epsilon^\lambda\right)^\nu$$

Example: massless vector boson, $m_{\gamma'}/(zP) \ll 1$
In gauge $(\epsilon^\pm)^\mu = \{0, \overrightarrow{\epsilon^\pm}\}$ non-conservation of energy ΔE cannot affect the final result, $\mathcal{W}^T = 4k_\perp^2/z^2$

Dawson correction for massive dark photon

Polarization sum $-g_{\mu\nu} + k_\mu k_\nu / m_{\gamma'}^2 \neq 0$, when μ or $\nu = 0$
Subtract **Goldstone mode** by hands

$$\epsilon_0^\mu \equiv \epsilon_L^\mu - \frac{k^\mu}{m_{\gamma'}} = \frac{m_{\gamma'}}{E_{\gamma'} + |\vec{k}|} \left\{ -1, \frac{\vec{k}}{|\vec{k}|} \right\}$$

Corrected **polarization sum**

$$\Sigma^{\mu\nu} \equiv \sum_{\lambda=\pm} (\epsilon_\lambda)^{*\mu} \epsilon_\lambda^\nu + (\epsilon_0)^{*\mu} \epsilon_0^\nu = -g^{\mu\nu} - \frac{k^\mu \epsilon_0^\nu + k^\nu \epsilon_0^\mu}{m_{\gamma'}}$$

Although $\Sigma^{0\mu} = \mathcal{O}\left(H_\pi / \left(z^2 P^2\right)\right) \neq 0$, where $H_\pi \equiv m_\pi^2 z^2 + m_{\gamma'}^2 (1-z) + k_\perp^2$,
in comparison with $\Sigma^{ij} = \mathcal{O}(1)$, $\Sigma^{0\mu} \ll \Sigma^{ij}$, so one can decompose in powers of δ

$$\mathcal{W} \simeq \mathcal{W}_0 \equiv (2p-k)_\mu (2p-k)_\nu \Sigma^{\mu\nu} = \frac{4H_\pi}{z^2} + m_{\gamma'}^2 - 4m_\pi^2$$

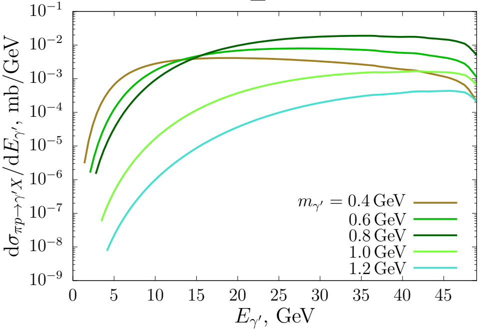
Inelastic pion bremsstrahlung: answer and spectra

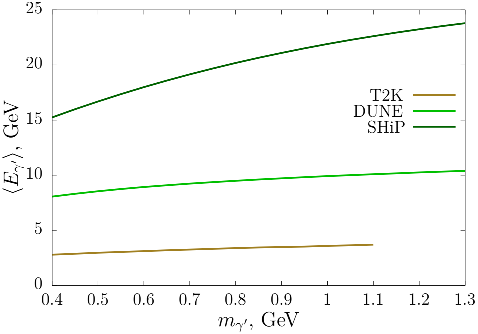
Pion splitting function

$$w_\pi(z, k_\perp^2) = \frac{\epsilon^2 \alpha_{\rm em}(1-z)}{4\pi H_\pi}\left(\frac{4}{z} + \frac{z}{H_\pi}\left(m_{\gamma'}^2 - 4m_\pi^2\right)\right)$$

allows to present differential cross section as

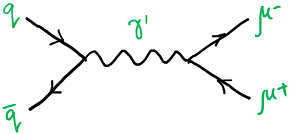
$$\frac{\mathrm{d}^2\sigma_{\pi p\to\gamma' X}}{\mathrm{d}z\mathrm{d}k_\perp^2} = w_\pi(z, k_\perp^2)|F_{\rm em}^\pi(m_{\gamma'}^2)|^2 F_{\rm virt}^2(z, k_\perp^2)\sigma_{\pi p\to X}(\bar{s})$$





Drell–Yan-like process at LO and NLO

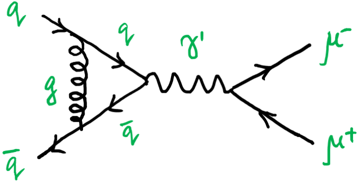
LO

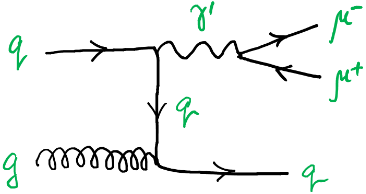


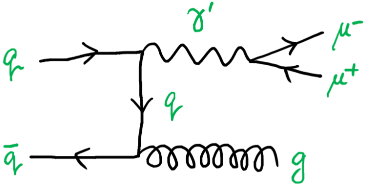
In **narrow width** approximation

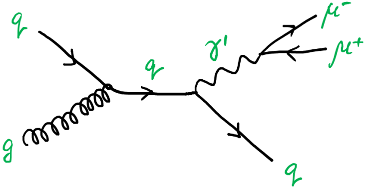
$$\sigma(q\bar{q} \to \gamma') = \sigma(q\bar{q} \to \mu^+\mu^-)\frac{\Gamma_{\text{tot}}}{\Gamma(\gamma' \to \mu^+\mu^-)}$$

NLO









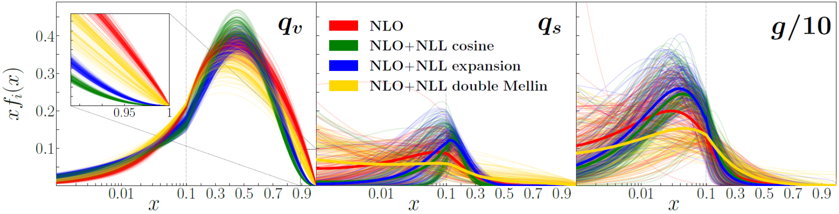
Drell–Yan cross section in parton model

$$\frac{d^2\sigma_{DY}}{dM^2 dx_F} = \frac{d^2\sigma_{LO}}{dM^2 dx_F} + \frac{d^2\sigma_{NLO}}{dM^2 dx_F},$$

$$\frac{d^2\sigma_{LO}}{dM^2 dx_F} = \frac{4\pi\alpha_{em}^2}{9}\frac{x_1 x_2}{x_1 + x_2}\frac{1}{(M^2 - m_{\gamma'}^2)^2 + m_{\gamma'}^2\Gamma_{tot}^2} \times$$

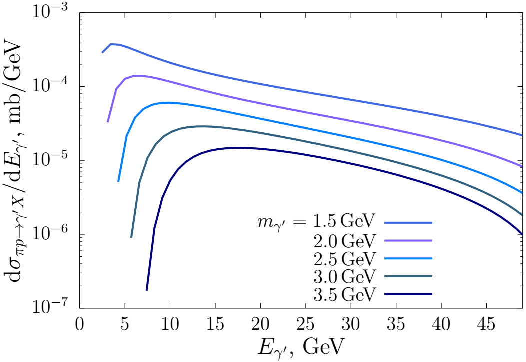
$$\times \sum_f e_f^2 \left(q_f(x_1, M^2)\bar{q}_f(x_2, M^2) + \bar{q}_f(x_1, M^2) q_f(x_2, M^2)\right),$$

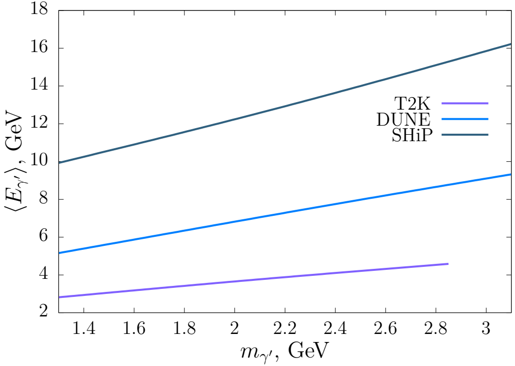
where M is the inv mass of $\mu^+\mu^-$, $x_F = x_1 - x_2$ is Feynman variable, $q_f(x, Q^2)$, $\bar{q}_f(x, Q^2)$ are parton distributions from LHAPDF library



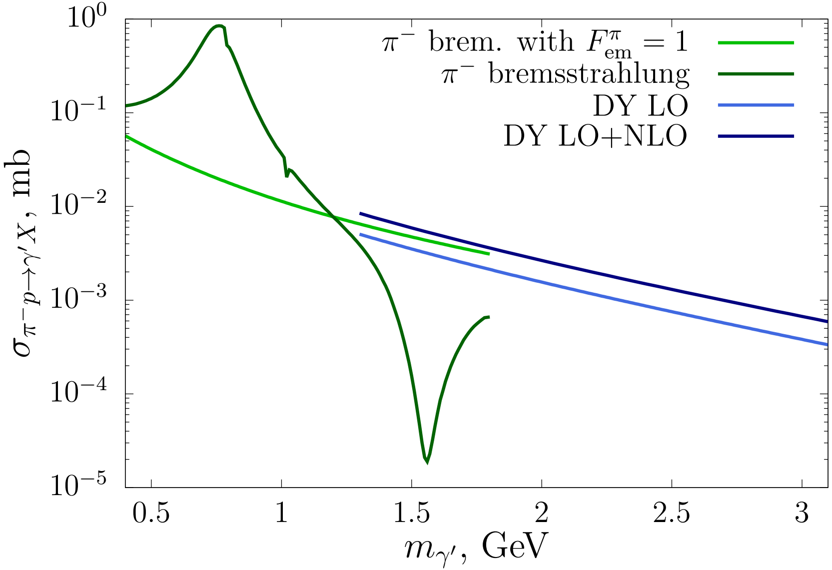
JAM Collaboration Phys.Rev.Lett. 127 (2021) 23, 232001

Spectra $\mathrm{d}\sigma/\mathrm{d}E_{\gamma'}$ and mean energy $\langle E_{\gamma'} \rangle$ of dark photons produced in Drell–Yan-like process (LO+NLO)





Full cross section of dark photon production



Results

- Using Dawson correction, in quasi-real approximation found **new splitting function** for dark photons production via inelastic pion bremsstrahlung
- Predicted **full cross section, spectra and mean energy** of dark photons with masses $0.4-3.5$ GeV, produced in NA64h, T2K, DUNE, SHiP experiments
- For $m_{\gamma'} = 0.4-1.2$ GeV pion bremsstrahlung is **enhanced** by electromagnetic form factor and gives the main contribution to full cross section