Recombination epoch, special relativity effects and Hubble tension

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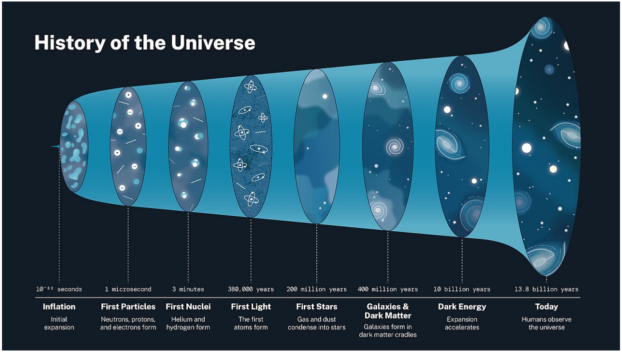
22.05.2026

Plan

Standard theory of the recombination

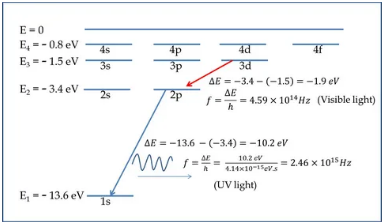


Recombination is affected by:

- primeval plasma composition: densities of $\gamma, e^-, p^+, He^+, \ldots$;
- Hubble parameter at the time of recombination $H(t)$;
- our understanding of the atomic energy levels, decay rates, etc.

Photoionization energy: 13.6 eV $= 1.6 \times 10^5$ K. L_α energy: 10.2 eV.
The temperature during the last photon scattering ~ 3000 K.
P.J.E. Peebles. Recombination of the Primeval Plasma. Published in: Astrophys.J. 153 (1968), 1

Standard theory of the recombination



"Effective three-level" model:

- $H_{n=2,l=2s} + \gamma \longleftrightarrow e^- + H^+$ – photoionization/recombination of the 2s level;
- $H_{n=1} + \gamma \longleftrightarrow H_{n=2,l=2p}$ – Lyman-α line;
- $H_{n=1} + 2\gamma \longleftrightarrow H_{n=2,l=2s}$ – two-quantum decay.

Sara Seager, Dimitar D. Sasselov, Douglas Scott. How exactly did the universe become neutral? Published in: Astrophys.J.Suppl. 128 (2000), 407-430

Standard theory of the recombination

$$\frac{d}{dt}\left(\frac{\nu n_\nu}{n}\right) = \frac{\nu J_\nu}{n} \tag{1}$$

$$n_a = \int_{\nu^-}^{\nu^+} n_\nu d\nu; \quad R = \int_{\nu^-}^{\nu^+} J_\nu d\nu \tag{2}$$

$$\frac{d}{dt}\left(\frac{n_a}{n}\right) = \frac{\nu_a}{n}\frac{1}{a}\frac{da}{dt}\left[n_{(\nu^+)} - n_{(\nu^-)}\right] + \frac{R}{n} \tag{3}$$

During recombination the number of resonance photons remains the same:

$$n_{(\nu^-)} = n_{(\nu^+)} + \frac{R}{\nu_a H(t)} \tag{4}$$

For convenience, define $\mathcal{N} = \frac{c^3 n_\nu}{8\pi\nu^2}$. Outside of the resonance, we have the thermal value of photons, so:

$$\mathcal{N}_\alpha = e^{-\frac{B_1 - B_2}{kT}} + \frac{R\lambda_a^3}{8\pi H} \tag{5}$$

Standard theory of the recombination

Net rate of recombination:

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \frac{\alpha_c n_e^2}{n} - \beta_c \frac{n_{2s}}{n} \qquad (6)$$

Three-level model:

$$\alpha_c n_e^2 - \beta_c n_{2s} = R + \Lambda_{2s.1s}\left[n_{2s} - n_{1s} e^{-(B_1-B_2)/kT}\right] \qquad (7)$$

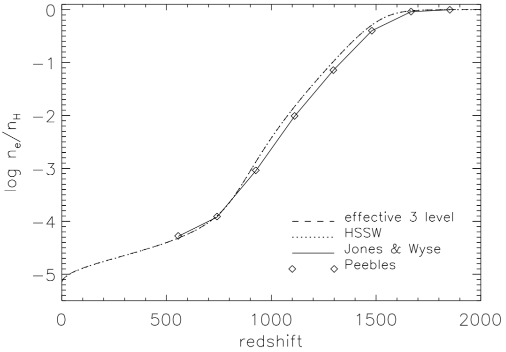
From that equation we obtain R and eliminate it from (5), obtaining for $\mathcal{N}_\alpha = \frac{n_{2s}}{n_{1s}}$:

$$\mathcal{N}_a = \frac{\alpha_c n_e^2 K + (1 + K\Lambda_{2s;1s} n_{1s}) e^{-\frac{B_1-B_2}{kT}}}{1 + K(\beta_c + \Lambda_{2s;1s}) n_{1s}} \qquad (8)$$

where $K = \frac{\lambda_\alpha^3}{8\pi H}$.
Substituting that expression in (6) we obtain the resulting electron density evolution:

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \left(\frac{\alpha_c n_e^2}{n} - \beta_c \frac{n_{1s}}{n} e^{-\frac{B_1-B_2}{kT}}\right) \frac{1 + K\Lambda_{2s;1s} n_{1s}}{1 + K(\beta_c + \Lambda_{2s;1s}) n_{1s}} \qquad (9)$$

Standard theory of the recombination

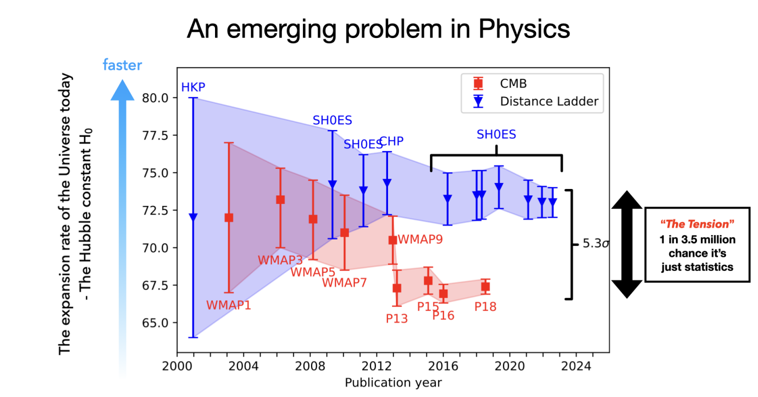


In terms of relative density $x_e = \frac{n_e}{n}$ and redshift z:

$$\frac{dx_e}{dz} = \frac{\left(\alpha_c x_e^2 n - \beta_c (1-x_e) e^{-\frac{B_1-B_2}{kT}}\right)(1 + K\Lambda_{2s;1s}(1-x_e)n)}{(1+z)H(z)(1+K(\beta_c + \Lambda_{2s;1s})(1-x_e)n)} \tag{10}$$

Hubble tension problem

Local Hubble parameter measurements give the value $H_0 = 73.04 \pm 1.04$ km/sec/MPc, and for parameter $S_8 = 0.790^{+0.018}_{-0.014}$.
Analysis of the cosmic microwave background data of *Planck* gives answer: $H_0 = 67.36 \pm 0.54$ km/sec/MPc, $S_8 = 0.832 \pm 0.013$



Hubble tension problem

The analysis of the *Planck* data heavily depends on our understanding of the recombination epoch. In particular, the acoustical horizon at the last scattering heavily affects the inferred value H_0:

$$r_* = \int_{z_*}^{\infty} \frac{c_s(z)}{H(z)} dz \tag{11}$$

Value z_* is defined as z for which visibility function $g(z) = \dot{\tau} e^{-\tau}$ takes the maximal value. Note, $\dot{\tau} = \sigma_T n_e(z) a(z)$.
Overall, if recombination ends earlier, that implies bigger values of z_* and, therefore, smaller horizon radius. This, in its place, means that during the fitting of the observable data one would need bigger value of H_0 and smaller value of S_8.
Other modifications of standard recombination epoch already addressed that approach for resolving Hubble tension: Seyed Hamidreza Mirpoorian (Simon Fraser U.), Karsten Jedamzik (U. Montpellier 2, LUPM), Levon Pogosian (Simon Fraser U.). Modified recombination and the Hubble tension. Published in: Phys.Rev.D 111 (2025) 8, 083519

Atomic recoil effect

Initially, we took interest in the topic because of the papers: A. V. Shepelev, Interaction of hydrogen atoms with resonant Ly-α radiation in the recombination epoch as a non-linear Brownian process
A.V. Shepelev. On the Refinement of the Theory of Primary Recombination and the Problem of the Hubble Tension. Published in: JETP Lett. 122 (2025) 9, 558-561
In these papers the author proclaims that Ly-α radiation exits resonance line due to the interaction with hydrogen atoms, rather than due to the redshift of the Universe, as is in the standard theory. In particular, they approximate the mean rate of momentum loss due to to the interactions with hydrogen atom as:

$$\frac{dP}{dt} = \frac{V}{c}\frac{h\nu}{c}B_{01}\rho = \left(\frac{g_1 cA\rho}{8\pi g_0 \nu^2}\right)V \tag{12}$$

And the time of leaving the resonance:

$$\Delta t = \frac{128\pi}{3} ln2 \frac{g_0}{g_1}\frac{\nu^3}{c^3}\frac{T}{\Delta T}\frac{1}{nA} \tag{13}$$

The author approximated the shift in z as: $\Delta z_L \sim 4 \times 10^{-4}$, while the standard theory gives $\Delta z_H \sim 2 \times 10^{-2}$.

Atomic recoil effect

The standard approach to calculate the contribution of atomic recoil is done in Fokker-Planck approximation, for example in these works:
J. Chluba and R.A. Sunyaev, Cosmological hydrogen recombination: influence of resonance and electron scattering
G. Rybicki, Improved Fokker-Planck Equation for Resonance Line Scattering
Their energy exchange formula has the following form:

$$\frac{\partial U}{\partial t} = -P \frac{h\nu_0}{m_H c^2} h\nu_0 \left(1 - \frac{T}{T_{R0}}\right) \tag{14}$$

We are in agreement that the leading special relativity correction to the energy exchange is suppressed by the factor $\frac{h\nu_0}{m_H c^2}$ and not by the factor V as is in the works of Shepelev.
These corrections of the equations dealing with resonance photon densities do not, to our knowledge, modify the equation pertaining to the photons leaving resonance. The corrections in one of the most frequently used codes for recombination computation, RECFAST, only affect the coefficient $C = \frac{1+K\Lambda_{2s;1s}n_{1s}}{1+K(\beta_c+\Lambda_{2s;1s})n_{1s}}$, not the coefficient K itself.

Our corrections

Let's estimate the time needed to cross the resonance line due to redshift:

$$\Delta t_0 = \frac{\Delta\nu_\alpha}{H\nu_\alpha} \tag{15}$$

In general, that time is:

$$\Delta t = \frac{\Delta\nu_\alpha}{v_\alpha}, \tag{16}$$

where

$$v_\alpha = p\overline{\Delta_\nu}\Gamma, \tag{17}$$

where p is the probability of relaxation before thermalization, $\overline{\Delta_\nu}$ is an average energy loss of the photon as a result of absorption and re-radiation, Γ is 1s atom radiation absorption rate.
For our initial estimate, let's take $p = 1$, $\overline{\Delta_\nu} = h\nu\frac{h\nu}{m_H} \sim 10^{-7}$eV
$\Gamma \sim A\frac{n_{1s}}{n} = \frac{n_{1s}}{n} \times 1sec^{-1}$. That nets $v_\alpha \sim \frac{n_{1s}}{n} \times 10^{-7}$eV/sec, while $H(z = 1000)\nu_\alpha \sim 10^{-13}$eV/sec. Even when the recombination epoch have only began and we have $\frac{n_{1s}}{n} \sim 10^{-3}$, the rates are incomparable!

Our corrections

We need to modify the equation (3):

$$\frac{d}{dt}\left(\frac{n_a}{n}\right) = \frac{\nu_a}{n}\frac{1}{a}\frac{da}{dt}\left[n_{(\nu^+)} - n_{(\nu^-)}\right] - \frac{v_\alpha}{n}n_{(\nu^-)} + \frac{R}{n} \tag{18}$$

That gives us an alternative equation to equation (5):

$$\mathcal{N}_a = e^{-\frac{B_1-B_2}{kT}}\frac{H\nu_\alpha}{H\nu_\alpha + v_\alpha} + \frac{Rc\lambda_a^2}{8\pi\left(H\nu_\alpha + v_\alpha\right)} \tag{19}$$

As in the standard theory, we can eliminate R by substituting it from the equation (7):

$$\mathcal{N}_a = \frac{\alpha_c n_e^2 K_{SR} + \left(\frac{H\nu_\alpha}{H\nu_\alpha + v_\alpha} + K_{SR}\Lambda_{2s;1s}n_{1s}\right)e^{-\frac{B_1-B_2}{kT}}}{1 + K_{SR}(\beta_c + \Lambda_{2s;1s})n_{1s}} \tag{20}$$

where $K_{SR} = \frac{c\lambda_\alpha^3}{8\pi H(1+v_{SR}/(H\nu_\alpha))}$.
Final expression:

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right) = \left(\frac{\alpha_c n_e^2}{n} - \beta_c\frac{n_{1s}}{n}e^{-\frac{B_1-B_2}{kT}}\frac{\frac{H\nu_\alpha}{H\nu_\alpha+v_\alpha} + K_{SR}\Lambda_{2s;1s}n_{1s}}{1 + K_{SR}\Lambda_{2s;1s}n_{1s}}\right)C_{SR} \tag{21}$$

Our corrections

Here $C_{SR} = \frac{1+K_{SR}\Lambda_{2s;1s}n_{1s}}{1+K_{SR}(\beta_c+\Lambda_{2s;1s})n_{1s}}$ is a modified Peebles coefficient that retains its form except our change of the K coefficient. In our estimate we have $\frac{K_{SR}}{K} = \frac{H\nu_\alpha}{v_{SR}} \ll 1$. In that case $C_{SR} \sim 1$. But $\beta_c \frac{n_{1s}}{n} e^{-\frac{B_1-B_2}{kT}}$ is suppressed compared to $\frac{\alpha_c n_e^2}{n}$ by a factor of $(\frac{H\nu_\alpha}{v_\alpha} + K_{SR}\Lambda_{2s;1s}n_{1s}) = \frac{K_{SR}}{K}(1 + K\Lambda_{2s;1s}n_{1s})$.

We modified RECFAST code and used CLASS to make our own estimations.

The result of code run for the standard theory:

-> recombination (maximum of visibility function) at z = 1088.792248 with comoving sound horizon = 144.508652 Mpc

-> sigma8=0.825008 for total matter (computed till k = 5.30504 h/Mpc)

The result of code run for the theory with our corrections:

-> recombination (maximum of visibility function) at z = 1534.938047 with comoving sound horizon = 114.347808 Mpc

-> sigma8=0.847122 for total matter (computed till k = 4.4399 h/Mpc)

Conclusions

- In standard recombination theory only redshift is responsible for photons exiting resonance line. That can be modified!
- Even without the introduction of new physics, special relativity corrections can significantly change the standard recombination history.
- All small and insignificant corrections should be reevaluated in this approach
- The modification we propose are easily implemented in existing recombination codes, such as RECFAST and can be further integrated in CMB analysis made with CAMB or CLASS code.

Thank you for your attention!

Standard theory of the recombination

All levels starting with $2p$ exist in thermal equilibrium:

$$n_{nl} = n_{2s}(2l+1)e^{-\frac{B_2 - B_n}{kT}} \tag{22}$$

From approximate thermodynamic equilibrium we have:

$$\frac{n_{2s}}{n_{1s}} = \frac{g_{2s}}{g_{1s}} \left(\frac{\mathcal{N}_\alpha}{1 + \mathcal{N}_\alpha} \right) \approx \mathcal{N}_\alpha \tag{23}$$

For nth line we have:

$$\mathcal{N}_n = \mathcal{N}_\alpha e^{-\frac{B_2 - B_n}{kT}} \tag{24}$$

Standard theory of the recombination

$$-\frac{d}{dt}\left(\frac{n_e}{n}\right)=\sum_{n>1}\left(\alpha_{nl}\frac{n_e^2}{n}-\beta_{nl}\frac{n_{nl}}{n}\right) \qquad (25)$$

$$\alpha_c=\sum_{n>1}\alpha_{nl}; \quad \beta_c=\sum_{n>1}(2l+1)\beta_{nl}e^{-\frac{B_2-B_n}{kT}} \qquad (26)$$

With Saha equation we have: β_c is related to α_c.

$$\beta_c=\alpha_c e^{-B_2/kT}(2\pi m_e kT)^{3/2}/h^3 \qquad (27)$$

Our corrections

Energy of the new photon in the laboratory system is related to the initial photon energy as:

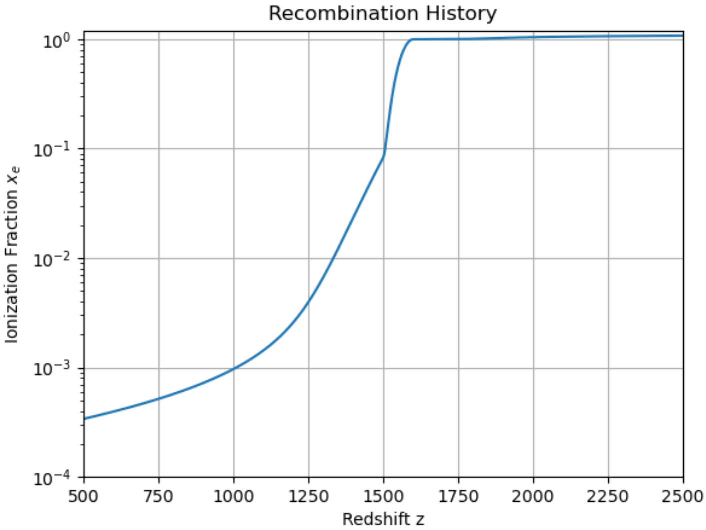
$$\frac{p'_\gamma}{p_\gamma} = \frac{1 + \frac{h\nu}{m_H} - V_H \cos\alpha - V_H^2 \frac{h\nu}{m_H}}{(1 + 2\frac{h\nu}{m_H})(1 - V_H^2)} \quad (28)$$

For the temperature 3000 K: $V_H = \sqrt{\frac{3kT}{m_H}} \sim 10^{-5}$, $\frac{h\nu}{m_H} \sim 10^{-8}$

Because the values $\frac{h\nu}{m_H}$ и V_H are approximately three orders of magnitude apart, we have

$$\frac{p'_\gamma}{p_\gamma}|_\alpha = 1 - V_H \cos\alpha - \frac{h\nu}{m_H} + o\left(\frac{h\nu}{m_H}\right) \quad (29)$$

Our corrections



Our corrections