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ON THE POSSIBILITY OF VORTEX LIGHT HEAVY QUARKONIUM SPECTROSCOPY

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Based on

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INTRODUCTION. QUARKONIUM OVERVIEW

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Term symbol $n^{2S+1}L_J$	$I^G(J^{PC})$	Particle	mass [MeV/c^2]
1^1S_0	$0^+(0^{-+})$	$\eta_c(1S)$	2983.4±0.5
1^3S_1	$0^-(1^{--})$	$J/\psi(1S)$	3096.900±0.006
1^1P_1	$0^-(1^{+-})$	$h_c(1P)$	3525.38±0.11
1^3P_0	$0^+(0^{++})$	$\chi_{c0}(1P)$	3414.75±0.31
1^3P_1	$0^+(1^{++})$	$\chi_{c1}(1P)$	3510.66±0.07
1^3P_2	$0^+(2^{++})$	$\chi_{c2}(1P)$	3556.20±0.09
2^1S_0	$0^+(0^{-+})$	$\eta_c(2S)$, or η'_c	3639.2±1.2
2^3S_1	$0^-(1^{--})$	$\psi(2S)$ or $\psi(3686)$	3686.097±0.025
1^1D_2	$0^+(2^{-+})$	$\eta_{c2}(1D)$	
1^3D_1	$0^-(1^{--})$	$\psi(3770)$	3773.13±0.35
1^3D_2	$0^-(2^{--})$	$\psi_2(1D)$	
1^3D_3	$0^-(3^{--})$	$\psi_3(1D)$[‡]	
2^1P_1	$0^-(1^{+-})$	$h_c(2P)$[‡]	
2^3P_0	$0^+(0^{++})$	$\chi_{c0}(2P)$[‡]	
2^3P_1	$0^+(1^{++})$	$\chi_{c1}(2P)$[‡]	
2^3P_2	$0^+(2^{++})$	$\chi_{c2}(2P)$[‡]	
$?^??_?$	$0^+(1^{++})$[*]	X(3872)	3871.69±0.17
$?^??_?$	$?^?(1^{--})$[†]	Y(4260)	4263±9

Notes:
[*] Needs confirmation.
[†] Interpretation as a 1^{--} charmonium state not favored.
[‡] Predicted, but not yet identified

Quarkonium: a bound state of a heavy quark and its antiquark — the simplest QCD system for testing quark interaction mechanisms.

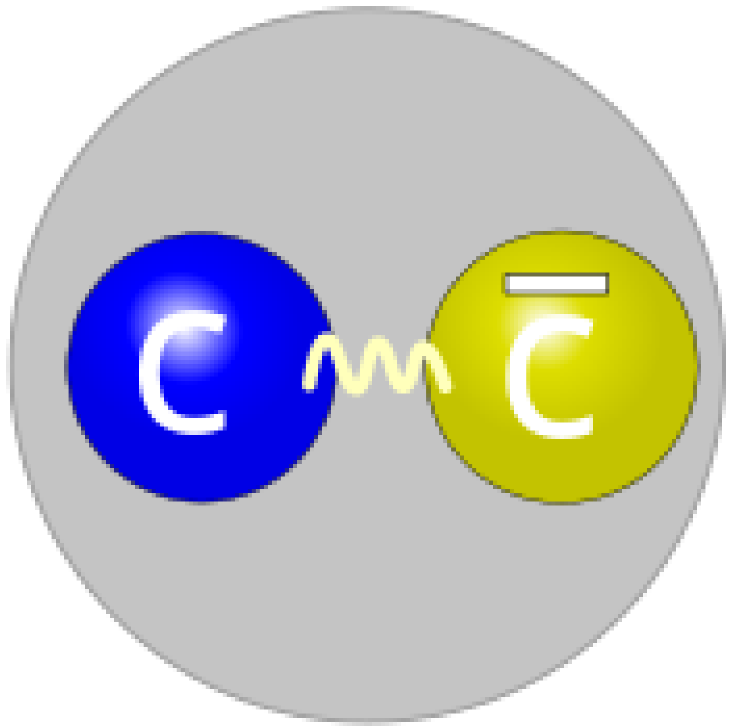

Figure 1

Key feature - none relativistic system:

- Charmonium $(c\bar{c})$: $\frac{v}{c} \approx 0.3$
- Bottomium $(b\bar{b})$: $\frac{v}{c} \approx 0.1$

Typical quark-antiquark separation:

- Charmonium $(c\bar{c})$: $r \approx 4$ fm
- Bottomium $(b\bar{b})$: $r \approx 0.25$ fm

Production:

- $e^+e^- \to \gamma^* \to Q\bar{Q}$ (vector states)
- $gg \to Q\bar{Q}$ (hadron colliders)
- $B \to J/\psi + X$ (feed down)
- $\gamma\gamma \to Q\bar{Q}$ (ultra-peripheral collisions)

Decay:

- $J/\psi, \Upsilon + l^+l^-$(clean, ~5%)
- $J/\psi, \Upsilon + (ggg)$ hadrons (~87%)
- $\eta_c \to (gg)$ hadrons (dominant)
- $\psi(2S) \to J/\psi + \pi^+\pi^-$ (main hadronic decay)
- $\chi_c \to J/\psi + \gamma$ (radiative transitions)

INTRODUCTION. QUARKONIUM SPECTROSCOPY

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None relativistic spinless quarkonium Hamiltonian

$$H = \frac{(\mathbf{p}_1 - q\mathbf{A}(\mathbf{r}_1))^2}{2m_1} + \frac{(\mathbf{p}_2 - \bar{q}\mathbf{A}(\mathbf{r}_2))^2}{2m_2} + V_0(|\mathbf{r}_1 - \mathbf{r}_2|) + H_{\mathrm{EM}},$$

$$H = H_0 + H^{(1)}_{\mathrm{int}} \qquad H_0 = H_{q\bar{q}} + H_{CM} + H_{\mathrm{EM}}, \qquad H_{q\bar{q}} = \frac{\mathbf{p}^2}{2\mu} + V_0(|\mathbf{r}|), \qquad \boxed{H_{CM} = \frac{\mathbf{P}^2}{2M_q},} \tag{1}$$

Center mass dynamics usually omitted

Quark-antiquark interaction term: Cornell potential

$$V_0(r) = -\frac{4\alpha_s}{3r} + \sigma r. \tag{2}$$

Here α_s is a running coupling constant and σ is string tension.

The Schrödinger equation gives the quarkonium spectrum. However, being spinless, it cannot distinguish between ortho and para states. Spin-dependent and relativistic corrections are included perturbatively via the Breit interaction

$$V^{\mathrm{Breit}}(\mathbf{r},\mathbf{p}) = \frac{\pi\alpha_s}{3\mu^2}\delta(\mathbf{r}) - \frac{\alpha_s}{6\mu^2}\left[\frac{\mathbf{p}^2}{r} + \frac{(\mathbf{r}\mathbf{p})^2}{r^3}\right] + \frac{\alpha_s}{3\mu}\left[\frac{8\pi}{3}\delta(\mathbf{r})(\mathbf{s}_1\mathbf{s}_2) + \frac{3(\mathbf{s}_1\mathbf{r})(\mathbf{s}_2\mathbf{r})}{r^5} - \frac{(\mathbf{s}_1\mathbf{s}_2)}{r^3}\right] + \frac{\alpha_s}{2\mu^2}\frac{(\mathbf{r}\times\mathbf{p})(\mathbf{s}_1+\mathbf{s}_2)}{r^3}, \tag{3}$$

Spectrum corrections

$$\delta E_{nlsj} = \langle nlm|V^{\mathrm{Breit}}(\mathbf{r},\mathbf{p})|nlm\rangle, \tag{4}$$

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Quarkonium excitation process

$$Q\bar{Q} + \gamma \to Q\bar{Q}'$$

Main issues:

- Short lifetime $< 10^{-20}$ s. makes it difficult to produce a beam of quarkonia.
- Where to obtain a sufficiently intense flux of photons at the required energy? Typical transition energies are hundreds of MeV.

$J/\psi \to \psi(2S)$: $\Delta E = 589$ MeV

$\eta_c \to \chi_{cJ}$: ΔE ~200-300 MeV

In the BaBar and Belle experiments, attempts were made to search for $J/\psi + \gamma \to \psi(2S)$ in ultra-peripheral collisions – with no success.

There are also many other technical problems. However, one of the main issues can be resolved (or circumvented). The large energy differences between states apply to dipole transitions. If one considers non-dipole transitions instead, the required photon energies can be significantly reduced.

***Proposal**: directly excite multipole transitions in quarkonia using **twisted photons**.*

INTRODUCTION. TWISTED LIGHT

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Twisted photons are the quantum states of light with definite energy k_0, longitudinal projection of momentum k_3, projection of the total angular momentum $m, m \in \mathbb{Z}$, and helicity $s, s = \pm 1$.

- The system of units is such that $\hbar = c = 1$.
- For narrow (paraxial) wave packets $m = l + s$, where l is the projection of the orbital angular momentum (OAM).
- The OAM l determines the topology of a phase front.
- The probability density for states with $l \neq 0$ possesses a typical annular form.

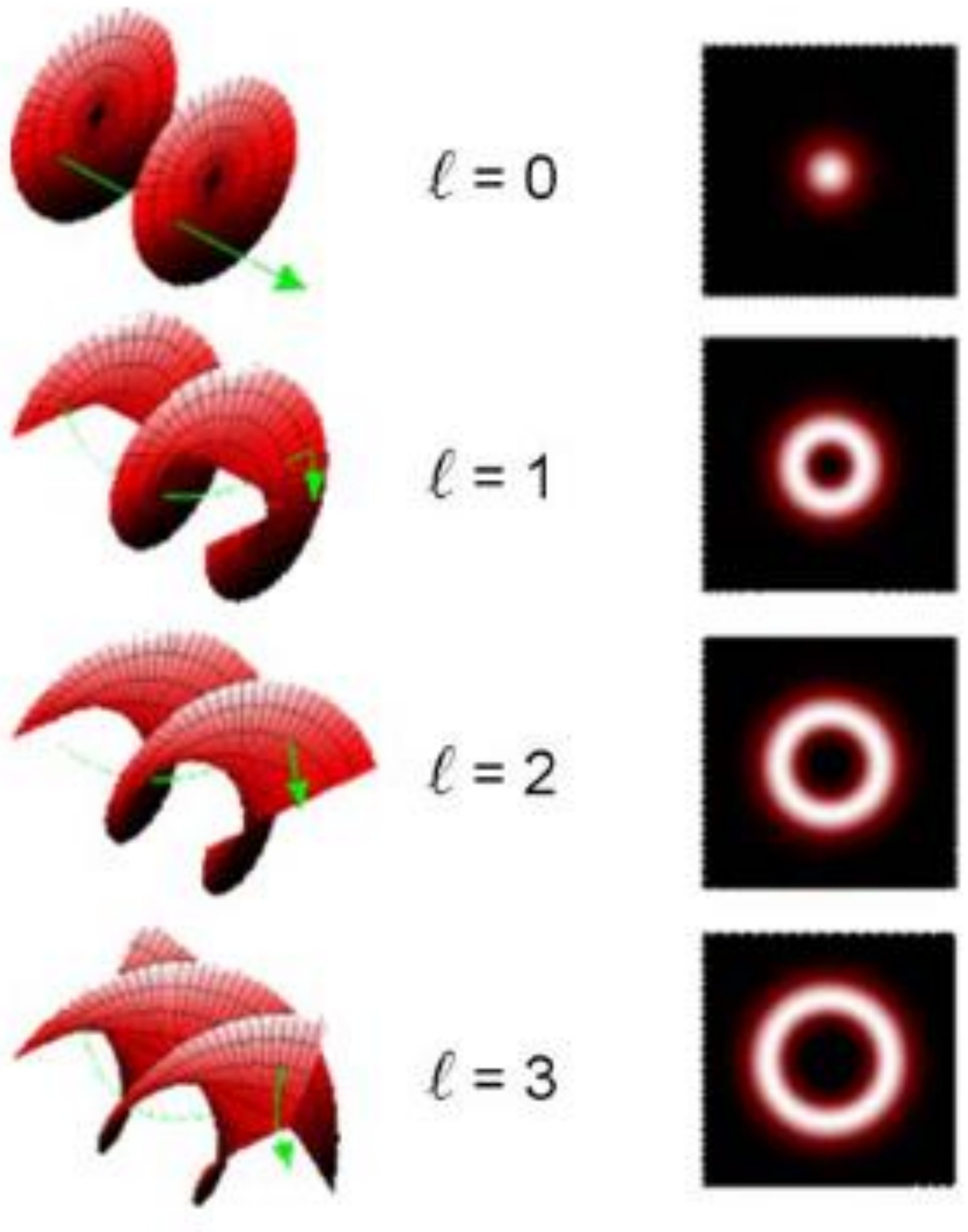

Figure 2

Some of applications:

- Parallel signal coding in (quantum) telecommunications;
- Quantum cryptography and quantum computing;
- Radiolocation;
- Particle traps and optical tweezers;
- High-contrast microscopy;
- Manipulating of rotational degrees of freedom of nano-particles, molecules, atoms, and nuclei.

Excitation of multipole transitions have been studied in

- Atoms: ***Matula*** *et.al. J. Phys. B (2013),* ***Afanasev****, et. al., Phys. Rev. A, (2013), J. Opt. (2016),* ***Kazinski****, et. al. Ann.Phys. (2026);*
- Nuclei: ***Lu*** *et. al. Phys. Rev. Lett. (2023),* ***Kazinski*** *&* ***Sokolov*** *Phys.At. Nucl. (2024);*
- Excitons: ***Kazinski*** *&* ***Ryakin*** *Ann. Phys. (2023).*

SETUP

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Quarkonium excitation process

$$Q\bar{Q} + \gamma \to Q\bar{Q}'$$

$$|\text{in}\rangle = \sum_{\gamma} \chi_{\gamma} e^{-ik_{0\gamma}t_1} \hat{c}_{\gamma}^{\dagger}|0\rangle \int \frac{V^{1/2}d\mathbf{P}}{(2\pi)^{3/2}} e^{-iE_{\text{in}}t_1} g(\mathbf{P})|\mathbf{P}\rangle|nlm\rangle,$$
$$|\text{out}\rangle = e^{-iE_{\text{out}}t_2}|0\rangle|\mathbf{P}'\rangle|n'l'm'\rangle, \quad (5)$$

Wave packet profile of the photon carrying **orbital angular momentum** $l_{\gamma} = m_{\gamma} - \lambda$

$$\chi_{\gamma} = C_{\gamma} k_{\perp}^{|l_{\gamma}|} e^{-\frac{(k_{\perp}^2 - (k_{\perp}^0)^2)^2}{4\sigma_{\perp}^4}} e^{-\frac{(k_3 - k_3^0)^2}{4\sigma_3^2}} \boxed{e^{il_{\gamma}\phi_k}} \delta_{\lambda\lambda_0}, \quad (6)$$

where m_{γ} is a projection of the total angular momentum and λ is a photon helicity.

Wave packet of the center mass of the quarkonium with impact parameter $\boldsymbol{b}_{\perp}$

$$g(\mathbf{P}) = C_{q\bar{q}} e^{-\frac{\mathbf{P}^2}{4\sigma_c^2}} e^{-i\mathbf{P}\mathbf{b}_{\perp}}, \quad |C_{q\bar{q}}|^2 = (\sqrt{2\pi}\sigma_c)^{-3} \quad (7)$$

PROBABILITY OF PHOTOABSORPTION

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The probability of photoabsorption of a photon by quarkonium in the non-relativistic approximation has the form

$$P_{n'l'm',nlm} = \int \frac{V d\mathbf{P}'}{(2\pi)^3} |\langle \text{out} | \hat{U}_{\infty,-\infty} | \text{in} \rangle|^2 = q^2 \int \frac{V d\mathbf{P}'}{(2\pi)^3} \Big| \sum_{\gamma} 2\pi\delta(E_{\text{out}} - E_{\text{in}} - k_{0\gamma})(K_\gamma^- + \kappa_\gamma^-) \Big|^2, \quad (8)$$

The key idea is to use the ***multipolar expansion*** *of a plane wave to factorize the angular variables when evaluating matrix elements with twisted photon wave packets*

$$\mathbf{f}_\lambda(\mathbf{k}) e^{-ib\mathbf{kr}} = 4\pi \sum_{JM\Lambda} (-i)^\Lambda i^J \left(\mathbf{f}_\lambda(\mathbf{k}) \mathbf{Y}_{JM}^{(\Lambda)*}(\theta_k, \phi_k) \right) \boldsymbol{\psi}_{JM}^\Lambda(k_0, -b\mathbf{r}), \quad (9)$$

here J is multipolarity and $J = 1, \dots, \infty$, $M = 0, \pm 1, \dots, \pm J$, $\Lambda = 1$ for electric (E) transitions, $\Lambda = 0$ for magnetic (M) transitions.

$$\kappa_\gamma^- = \frac{\chi_\gamma e^{i\lambda\phi_k}}{\sqrt{2k_{0\gamma}}} \int \frac{d\mathbf{P}}{(2\pi)^{3/2}} g(\mathbf{P}) \langle \mathbf{P}' | e^{i\mathbf{R}\mathbf{k}_\gamma} | \mathbf{P} \rangle 4\pi \sum_{J\Lambda M} i^{J+\Lambda} \left(\mathbf{f}_\lambda(\mathbf{k}_\gamma) \mathbf{Y}_{JM}^{(\Lambda)*}(\theta_k, \phi_k) \right) C_{lmJM}^{l'm'} B_J^{(\Lambda)},$$

$$K_\gamma^- = \frac{\chi_\gamma e^{i\lambda\phi_k}}{\sqrt{2k_{0\gamma}}} \int \frac{d\mathbf{P}}{(2\pi)^{3/2}} g(\mathbf{P}) \langle \mathbf{P}' | e^{i\mathbf{R}\mathbf{k}_\gamma} | \mathbf{P} \rangle 4\pi \sum_{J\Lambda M} i^{J+\Lambda} \left(\mathbf{f}_\lambda(\mathbf{k}_\gamma) \mathbf{Y}_{JM}^{(\Lambda)*}(\theta_k, \phi_k) \right) \left(\mathbf{W}_{JM}^{(\Lambda)} \mathbf{P} \right), \quad (10)$$

PROBABILITY OF PHOTOABSORPTION

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- Neglect quantum recoil and Doppler effects $\frac{\sigma_c}{M_q} \ll 1, \quad \frac{k_0}{M_q} \ll 1, \quad k_{0\gamma} \approx \varepsilon$ (11)

- Using narrow wave packets and additional assumptions

$$\frac{k^0_\perp \sigma_\perp}{(k^0_3)^2} \ll 1, \quad \frac{k^0_\perp \sigma_\perp}{\sigma_c^2} \ll 1, \quad \frac{k^0_\perp \sigma_\perp \sigma_3}{\sigma_c^2 k^0_3} \ll 1, \quad \sigma_\perp b_\perp \ll 1, \quad \frac{(|m_\gamma| + J + 1)\sigma_\perp}{k^0_\perp} \ll 1.$$

$$\frac{2(|m_\gamma| + 1)\sigma_\perp}{k^0_\perp} \ll 1 \qquad k^0_\perp \ll 2\sigma_c \qquad k^0_\perp b_\perp \ll 1 \tag{12}$$

- Sum over undetectable quantum number

$$P^{(Tw)}_{n'l',nl} := \frac{1}{2l+1} \sum_{m=-l}^{l} \sum_{m'=-l'}^{l'} P^{(Tw)}_{n'l'm',nlm}. \tag{13}$$

Probability of photoexcitation can be evaluated and simplified

$$P^{(Tw)}_{n'l',nl} = \frac{8\pi^2 \varepsilon q^2 \sigma_\perp^2}{\sigma_3 \tilde{k}_3^2 (k^0_\perp)} \frac{2l'+1}{2l+1} e^{-\frac{(k^0_\perp)^2}{4\sigma_c^2}} e^{-\frac{(\tilde{k}_3(k^0_\perp) - k^0_3)^2}{2\sigma_3^2}} \sum_{J=\max(|m_\gamma|,1)}^{\infty} \sum_{\Lambda=0,1} \frac{1}{2J+1} |\Theta^{(\Lambda)}_{Jm_\gamma;\lambda_0}(\theta^0_k)|^2 \left[|B^{(\Lambda)}_J|^2 + \sigma_c^2 |W^{(\Lambda)}_{Jm_\gamma}|^2\right]. \tag{14}$$

SELECTION RULES

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Probability of photoexcitation can be evaluated and simplified

$$P^{(Tw)}_{n'l',nl} = \frac{8\pi^2 \varepsilon q^2 \sigma_\perp^2}{\sigma_3 \tilde{k}_3^2(k_\perp^0)} \frac{2l'+1}{2l+1} e^{-\frac{(k_\perp^0)^2}{4\sigma_c^2}} e^{-\frac{(\tilde{k}_3(k_\perp^0)-k_3^0)^2}{2\sigma_3^2}} \sum_{J=\max(|m_\gamma|,1)}^{\infty} \sum_{\Lambda=0,1} \frac{1}{2J+1} |\Theta^{(\Lambda)}_{Jm_\gamma;\lambda_0}(\theta_k^0)|^2 \left[|B_J^{(\Lambda)}|^2 + \sigma_c^2 |W^{(\Lambda)}_{Jm_\gamma}|^2\right].$$

For $|m_\gamma| > 1$ only the multipole transitions with $J \geq |m_\gamma|$ are realized.

The dominance of the multipole $J = |m_\gamma|$ reflects angular momentum conservation in the complete system.

Projection of the total angular momentum onto the quantization axis are also conserved $m + m_\gamma = m'$. However we summed over these quantum numbers and obtained consequences of these rules.

For low-energy transitions, i.e., $\varepsilon r_B \ll 1$ $\qquad |l' - l| = |m_\gamma|$

The matrix element $W^{(\Lambda)}_{Jm_\gamma}$ comes from the center-of-mass dynamics and has different parity from $B_J^{(\Lambda)}$

$$J + \Lambda \quad \begin{cases} \text{odd for } W^{(\Lambda)}_{Jm_\gamma}, \\ \text{even for } B_J^{(\Lambda)}. \end{cases}$$

RESULTS AND EXAMPLES

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State	Experiment	Model
$\eta_c(1^1S_0)$	2984 ± 0.4	2980.47
$J/\psi(1^3S_1)$	3096.900 ± 0.006	3097.29
$\eta_c'(2^1S_0)$	3637.7 ± 0.9	3698.96
$\psi'(2^3S_1)$	3686.097 ± 0.011	3792.77
$\eta_c(3^1S_0)$		4260.57
$\psi(3^3S_1)$	4040 ± 4	4346.83
$\eta_c(4^1S_0)$		4749.33
$\psi(4^3S_1)$	4415 ± 5	4831.52
$h_c(1^1P_1)$	3525.37 ± 0.14	3526.25
$\chi_0(1^3P_0)$	3414.71 ± 0.3	3424.11
$\chi_1(1^3P_1)$	3510.67 ± 0.05	3500.72
$\chi_2(1^3P_2)$	3556.17 ± 0.07	3562.
$h_c(2^1P_1)$		4108.74
$\chi_0(2^3P_0)$	3922.1 ± 1.8	4010.92
$\chi_1(2^3P_1)$	3871.64 ± 0.06	4084.28
$\chi_2(2^3P_2)$	3922.5 ± 1.0	4142.98
$h_c(3^1P_1)$		4611.82
$\chi_0(3^3P_0)$		4516.23
$\chi_1(3^3P_1)$	4146.5 ± 3.0	4587.92
$\chi_2(3^3P_2)$		4645.28

State	Experiment	Model
$\eta_{c2}(1^1D_2)$		3908.19
$\psi(1^3D_1)$	3773.7 ± 0.7	3869.84
$\psi_2(1^3D_2)$	3823.51 ± 0.34	3900.52
$\psi_3(1^3D_3)$	3842.71 ± 0.20	3930.1
$\eta_{c2}(2^1D_2)$		4429.72
$\psi(2^3D_1)$	4191 ± 5	4392.03
$\psi_2(2^3D_2)$		4422.18
$\psi_3(2^3D_3)$		4451.27
$h_{c3}(1^1F_3)$		4235.94
$\chi_2(1^3F_2)$		4212.39
$\chi_3(1^3F_3)$		4232.26
$\chi_4(1^3F_4)$		4251.9
$h_{c3}(2^1F_3)$		4717.49
$\chi_2(2^3F_2)$		4694.14
$\chi_3(2^3F_3)$		4713.84
$\chi_4(2^3F_4)$		4733.3
$\eta_{c4}(1^1G_4)$		4532.46
$\psi_3(1^3G_3)$		4515.47
$\psi_4(1^3G_4)$		4530.29
$\psi_5(1^3G_5)$		4545.04

Octupole transitions

$$|l' - l| = 3$$

$$E_{1^3G_3 \to 3^3P_0} \approx 0.756 \text{ MeV}$$

$$E_{1^3F_3 \to 3^3S_1} \approx 114.56 \text{ MeV}$$

$(c\bar{c})$ masses [MeV]. $m_q = 1.285$ GeV, $\alpha_s = 0.29$ and $\sigma = 1.306$ GeV/fm

RESULTS AND EXAMPLES

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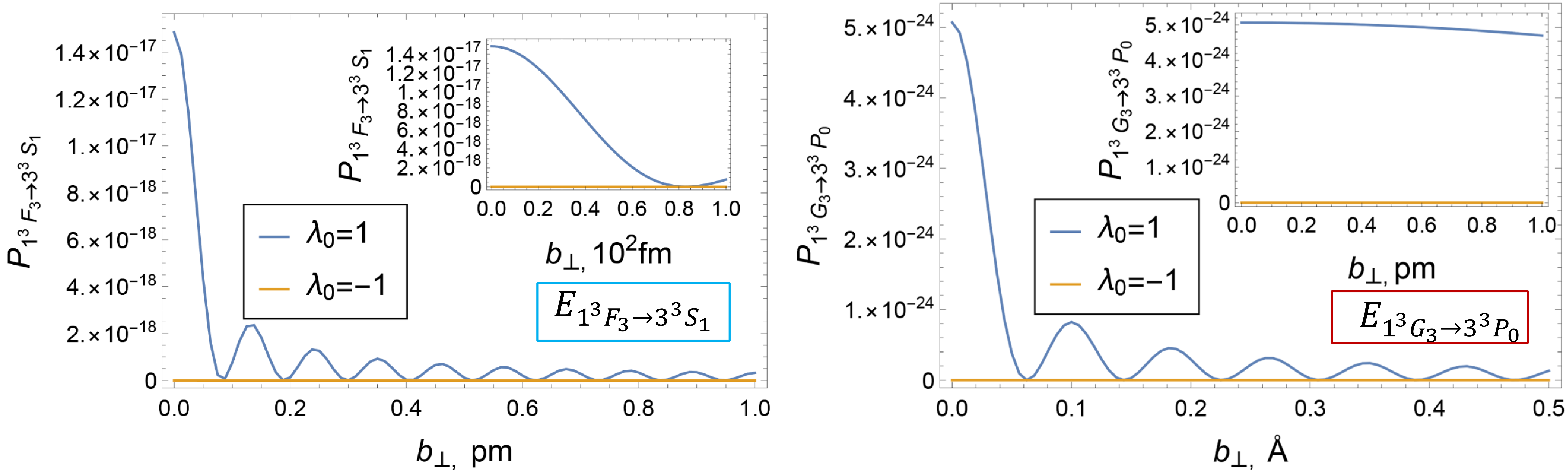

Figure 3 Photoexcitation probability for the charmonium transition $1^3F_3 \to 3^3S_1$ on the left panel and $1^3G_3 \to 3^3P_0$ on the right one by a twisted photon with $m_\gamma = 3$ for the different circular polarizations with respect to the impact parameter $b_\perp$. The parameters for the process on the left panel: $\varepsilon = 114.56$ MeV, $k_3^0 = 114.42$ MeV, $k_\perp^0 = 5.7$ MeV, $\sigma_3 = 1.14$ MeV, and $\sigma_\perp = 57$ keV. The parameters for the process on the right panel: $\varepsilon = 0.756$ MeV, $k_3^0 = 0.753$ MeV, $k_\perp^0 = 0.0756$ MeV, $\sigma_3 = 10$ keV, and $\sigma_\perp = 1$ keV. The common parameters: $m_q = 1.2185$ GeV, $\alpha_s = 0.29$, $\sigma = 1.306$ GeV/fm, $\sigma_c = 0.1$ GeV.

RESULTS AND EXAMPLES

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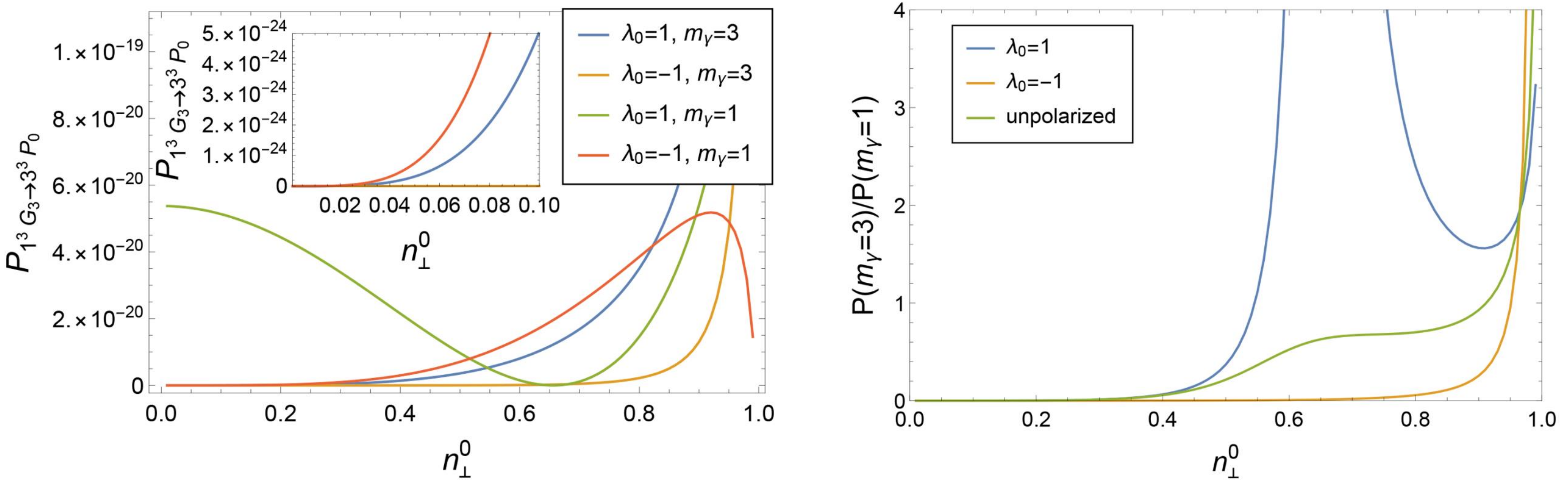

Figure 4 Comparison of photoexcitation probabilities for the charmonium transition $1^3G_3 \to 3^3P_0$ by photons of the same energy 0.756 MeV with $m_\gamma = 3$ and $m_\gamma = 1$ and different polarizations with respect to $n^0_\perp$. On the left panel: The absolute values of the probabilities. On the right panel: The ratio of the probability of photoexcitation by a twisted photon ($m_\gamma = 3$) to the probability of photoexcitation by a plane-wave photon ($m_\gamma = 1$). The parameters are: $m_q = 1.2185$ GeV, $\alpha_s = 0.29$, $\sigma = 1.306$ GeV/fm, $\sigma_c = 0.1$ GeV, $\sigma_3 = 10$ keV, $\sigma_\perp = 1$ keV, and $b_\perp = 100$ fm.

SUMMARY

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- In the framework of potential non-relativistic QCD, we have investigated the process of excitation of multipole transitions in the quarkonium system by a photon with a non zero projection of the orbital angular momentum described by the regularized Bessel state.
- We have derived the explicit expressions for the amplitude and probability of photoexcitation of quarkonium by a twisted photon taking into account the dynamics of the quarkonium center of mass.
- We have obtained the selection rules for transitions induced by twisted photons for the contributions to the probability amplitude coming from the relative motion of quarks and from the dynamics of the center of mass.
- The fulfillment of these selection rules imposes constraints on the parameters of the wave packet of the quarkonium center of mass, since, in general, the angular momentum can be transferred to the center of mass.
- The transition $J = |m_\gamma|$ can also be excited by a plane-wave photon, but in the case of the absorption of a twisted photon with $|m_\gamma| > 1$, the multipole transitions with $J = |m_\gamma|$ do not overlap with the transitions of lower multipolarity and can be studied separately.
- As illustrative applications of the formalism, we have considered two octupole transitions in the charmonium system, $1^3F_3 \to 3^3S_1$and $1^3G_3 \to 3^3P_0$.

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Thank you for your attention.

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