“Multiparticle production in electron-positron annihilation”

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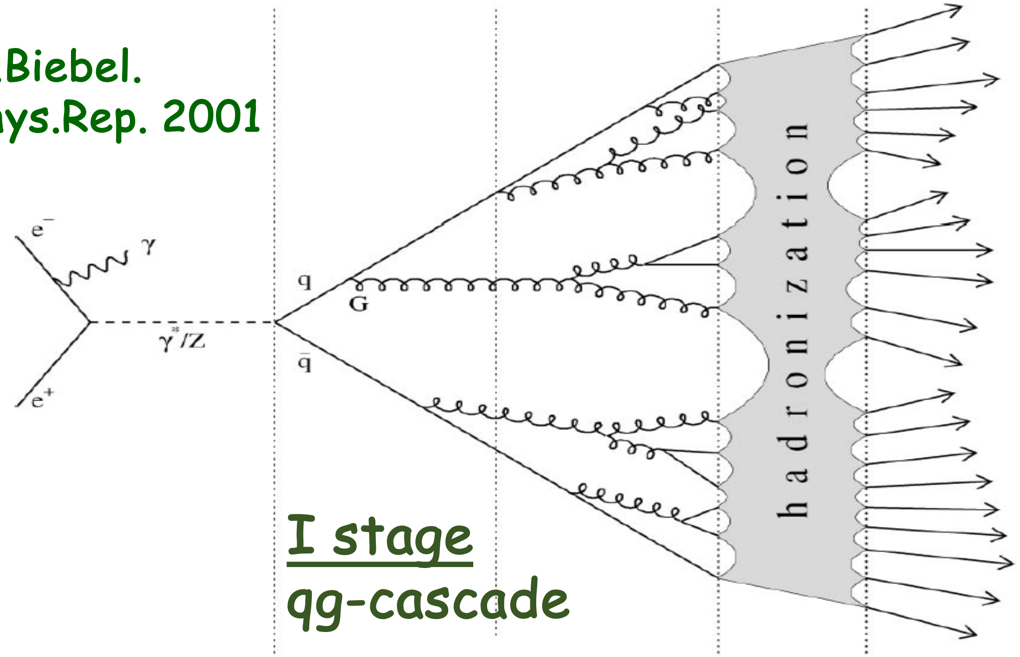
Multiparticle processes in HEP

Electron-positron annihilation (e^+e^-).
Proton-antiproton annihilation ($p\bar{p}$).
Hadron-hadron (pp) interactions.
Relativistic heavy ion collisions (AA)
Three-gluon decay of quarkonium (Υ).

e^+e^- - annihilation

$$e^+e^- \to \gamma(Z^0) \to q\bar{q} \to (q,g) \to ? \to hadrons$$

O.Biebel. Phys.Rep. 2001



I stage
qg-cascade

II stage
(hadronization)

Multiplicity Distribution (MD) $$P_n(s) = \frac{\sigma_n}{\sum_m \sigma_m}$$

Generation function (GF): $$Q(s,z) = \sum_n P_n(s) z^n$$

GF <-> MD $$P_n(s) = \frac{1}{n!}\frac{\partial^n}{\partial z^n} Q(s,z)\Big|_{z=0}$$

Correlative moments, F_k:

$$F_k(s) = \overline{n(n-1)\ldots(n-k+1)} = \frac{\partial^k}{\partial z^k} Q(s,z)\Big|_{z=1}$$

e^+e^- - annihilation

Konishi et al. & Giovannini [NP, 1979] described the qg-cascade in pQCD as Markovian branching processes of elementary events:

1) quark bremsstrahlung - $q \to q + g$, ($\tilde{A}$)
2) gluon fission - $g \to g + g$, (A)
3) quark-antiquark pair creation from gluon - (B) $g \to q + \bar{q}$.

$$\begin{cases} \frac{\partial G}{\partial Y} = -AG + AG^2, \\ \frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG. \end{cases}$$

System of diff. eq. describing branching processes, leads to Pólya (NBD) for q-jet and Yule-Furry MD for g-jet:

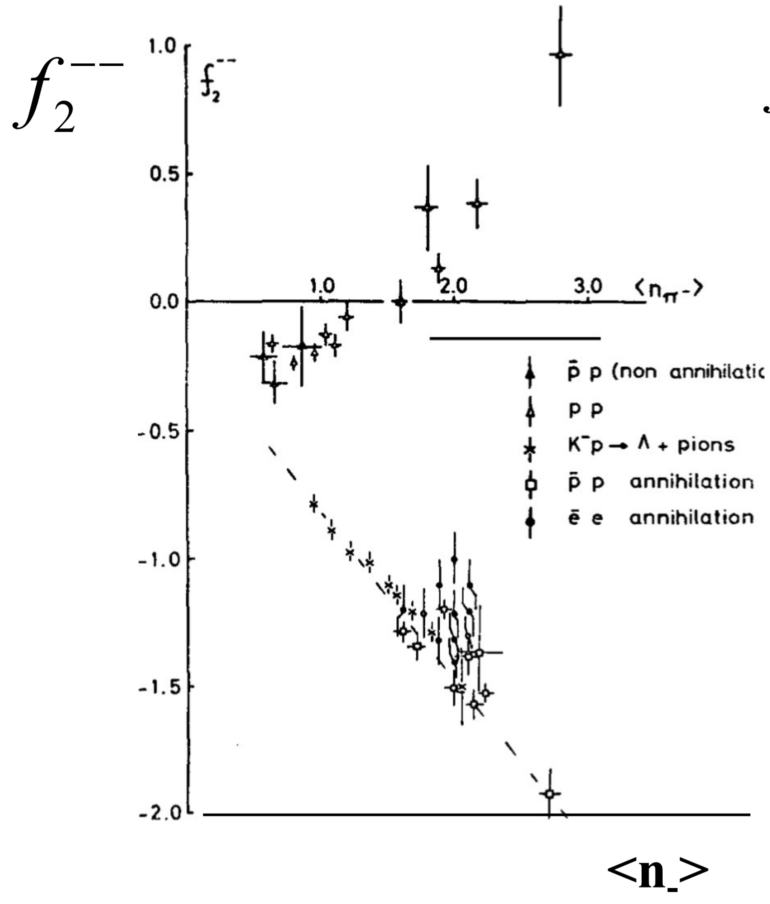
$$P_m^g = \frac{1}{\bar{m}}\left(1 - \frac{1}{\bar{m}}\right)^{m-1}, \quad P_m^q = \frac{k_p(k_p+1)\dots(k_p+m-1)}{m!}\left(\frac{\bar{m}}{\bar{m}+k_p}\right)^m \left(\frac{k_p}{\bar{m}+k_p}\right)^{k_p}.$$

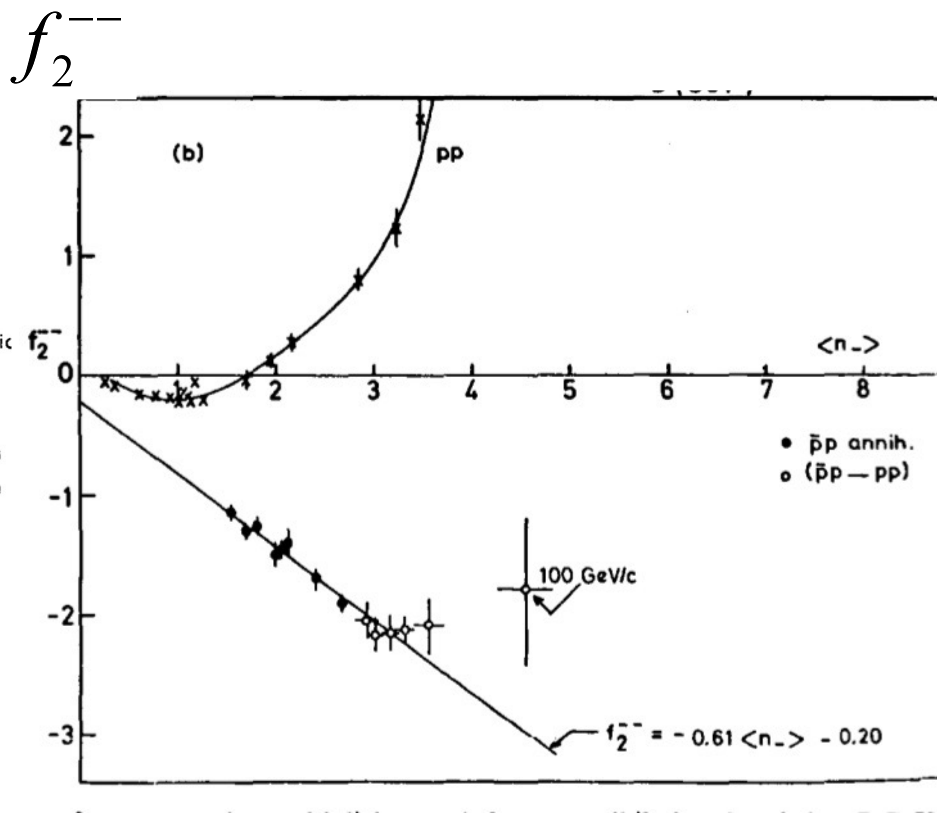
Evolutinary parameter:

$$Y = \frac{1}{2\pi b}\ln[1 + ab\ln(Q^2/\mu^2)],$$

$\tilde{A}$ и A – probabilities of 1) и 2) events, $k_p = \tilde{A}/A$.

Variation of f_2 with <n_> for annihilation & non-annihilation data





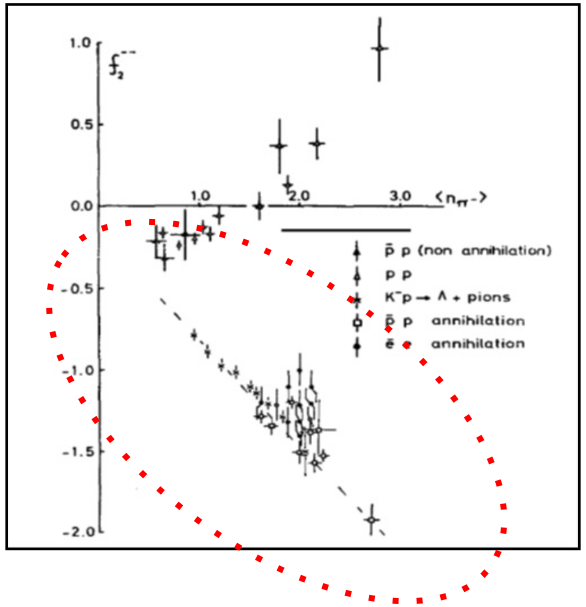
$f_2 = \langle n(n-1)\rangle - \langle n\rangle^2 =$
$= D_2 - \langle n\rangle < 0$ at low energy;

J.G. Rushbrooke, B.R. Webber. Phys. Rep. 44 (1978) 1

e^+e^- annihilation - II stage

pQCD is unable to describe hadronization. The choice of MD at this stage is based on experimental behavior of the second correlative moment f_2. It is always positive for Pólya (NBD) (and Furry also):

NBD → $$f_2 = \overline{n(n-1)} - \overline{n}^2 \rightarrow \frac{\overline{m}^2}{k_p} > 0$$



J.G. Rushbrooke, B.R. Webber. Phys. Rep. 44 (1978) 1

We chose binomial MD (Bernoulli) for II-stage:

$$P_P^H(n) = C_{N_p}^n \left(\frac{\overline{n}_p^h}{N_p}\right)^n \left(1 - \frac{\overline{n}_p^h}{N_p}\right)^{N_p - n}, P = q, g.$$

$$f_2 = -\frac{(\overline{n}_p^h)^2}{N_p} < 0.$$

Convolution of two stages. Two-stage model

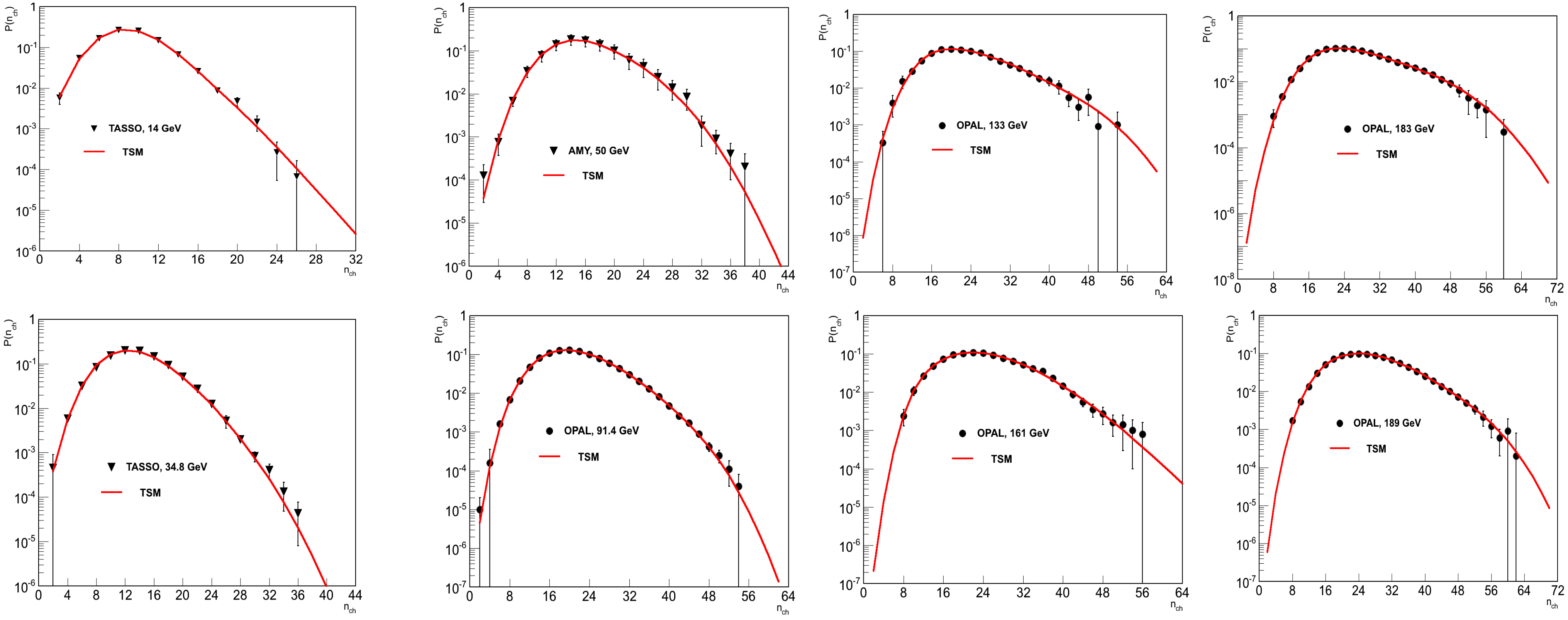
$$Q(s,z) = \sum_m P_m^P Q^H(m,s,z)$$ (soft discoloration).

$$P_n(s) = \Omega \sum_{m=0}^{M_g} P_m^P C_{(2+\alpha m)N}^n \left(\frac{\bar{n}^h}{N}\right)^n \left(1 - \frac{\bar{n}^h}{N}\right)^{(2+\alpha m)N-n}$$

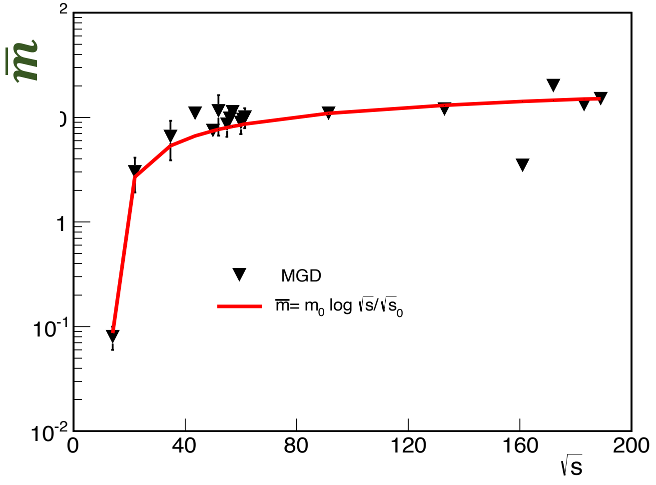
$$Q_p^H = \left[1 + \frac{\bar{n}_p^h}{N_p}(z-1)\right]^{N_p},$$ p = q, g,

Model parameters: k_p, $\bar{m}$, $N_q = N$, $\bar{n}_q^h$, $N_g = \alpha N$, $\bar{n}_g^h = \alpha \bar{n}_q^h$.

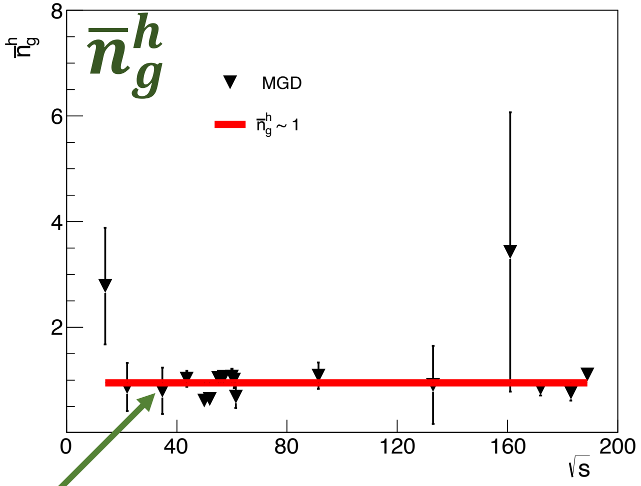
MD in e+e- annihilation (14 -189 GeV)



TSM parameters (2002)

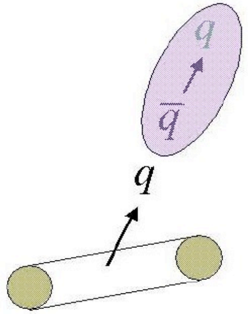


$\overline{m} \sim \log s$.



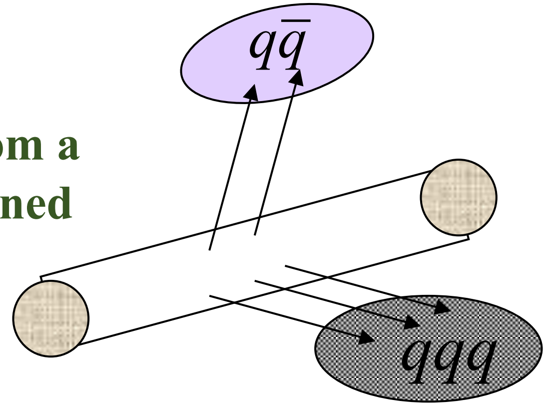
Average number of hadrons, $\overline{n}_g^h$, formed from gluon, is close to 1, which testifies the fragmentation mechanism of hadronization.

Hypothesis of Local Parton-Hadronic Duality (LoPHD)
$\langle m \rangle = \rho \langle n \rangle, \rho \sim 1.$



Fragmentation mechanism (in vacuum)

Recombination mechanism (from a thermal, deconfined medium)



[B. Muller. 2004]

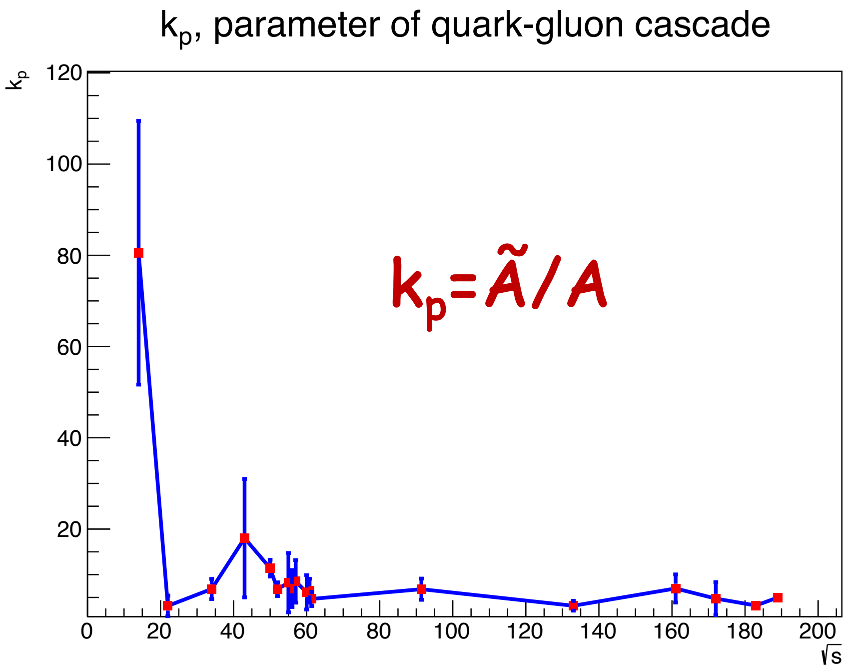
Second correlative moment, f_2

$$\overline{n}(s) = \frac{\partial Q(s,z)}{\partial z}\Big|_{z=1} = (2+\alpha\bar{m})\bar{n}^h$$

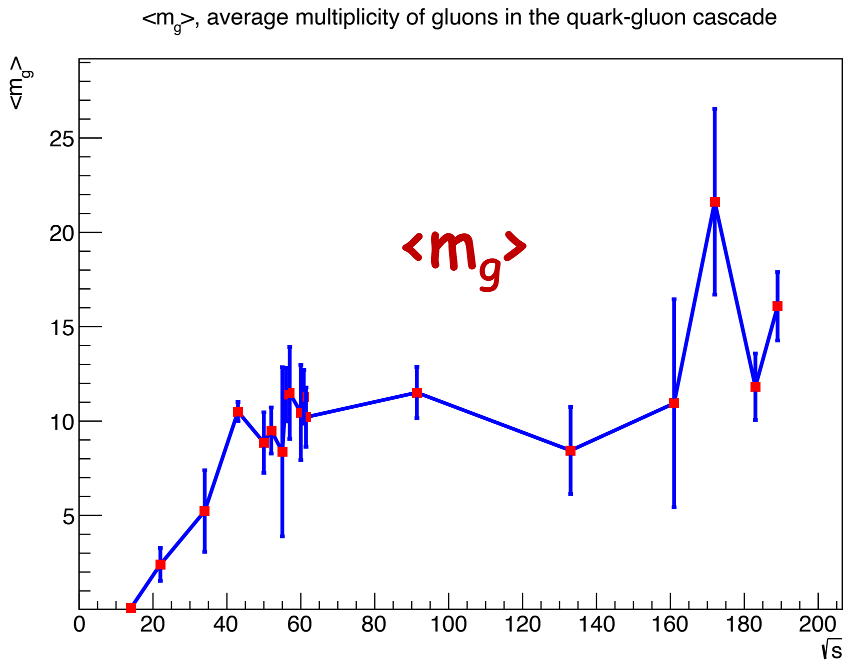
$$f_2 = F_2 - F_1^2 = \left[\alpha^2 \frac{\overline{m}^2}{k_p} + \alpha^2 \overline{m} - \frac{2+\alpha\overline{m}}{N}\right] (\overline{n}^h)^2$$

$$f_2 < 0 \quad \text{at} \quad \bar{m} << 1 \quad (\alpha < 1, k_p > 1)$$

TSM parameters: k_p, $\langle m_g \rangle$



$k_p = \tilde{A}/A$



$\langle m_g \rangle$

Low energies:

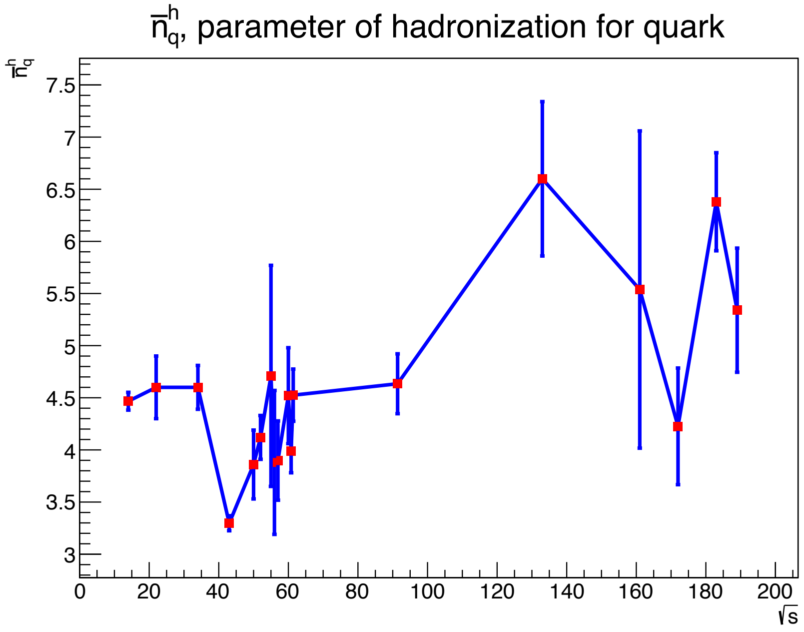
q-bremsstrahlung, g-fission is suppresed

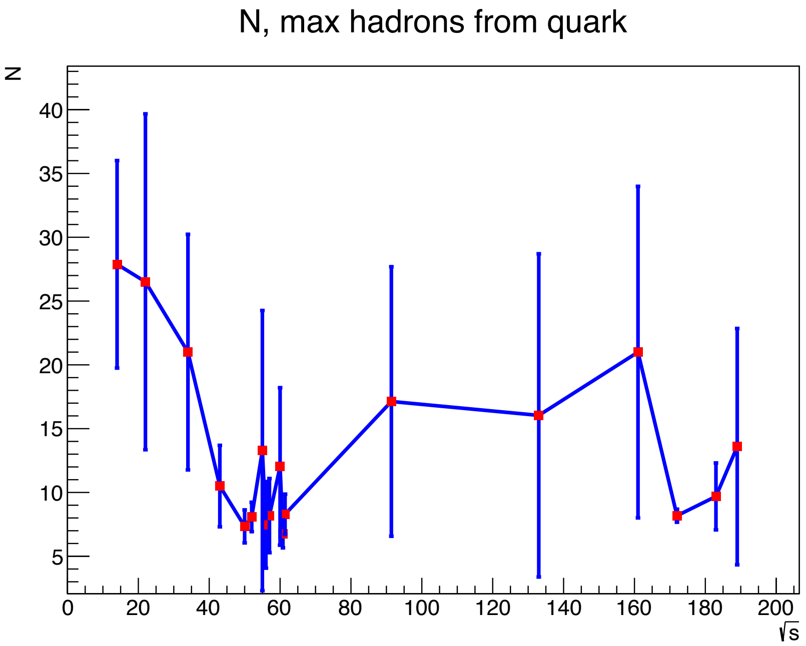
High energies:

q-bremsstrahlung/ g-fission ~ 6÷ 8

Steady growth trend

TSM parameters: $\bar{n}_q^h$, N_q

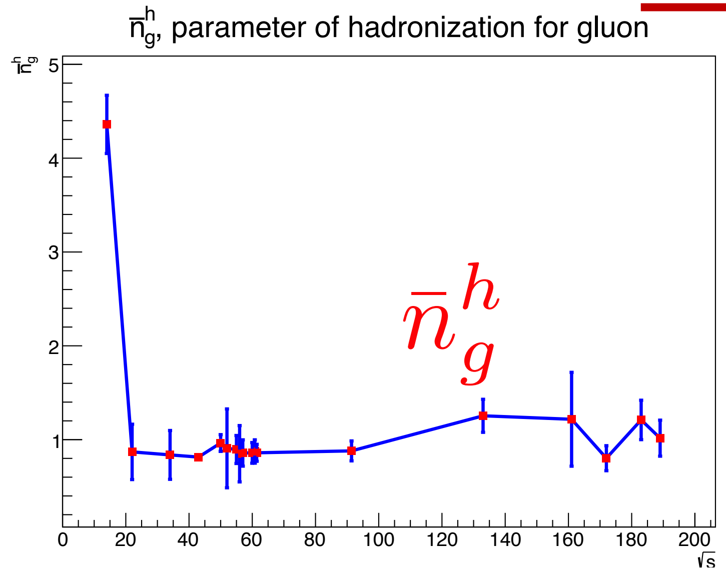


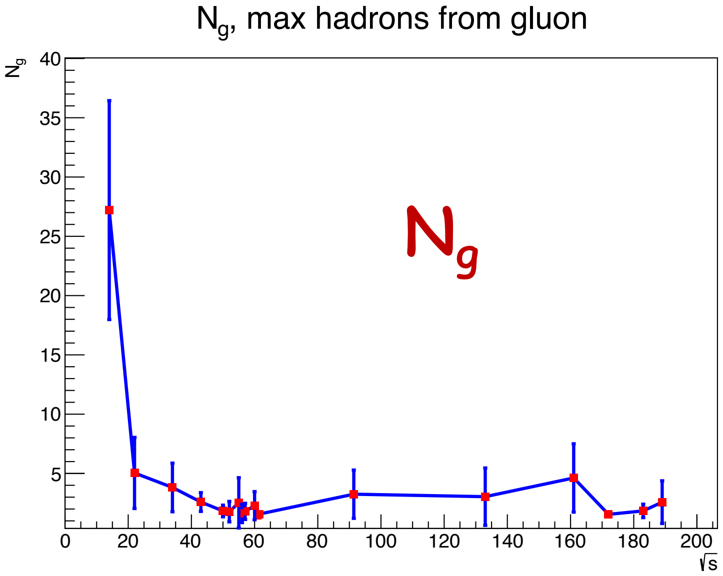


$\bar{n}_q^h$ Almost constant $\longrightarrow$ growth with oscillations

N_q steady decline $\longrightarrow$ almost constant

TSM parameters: $\bar{n}_g^h$, N_g

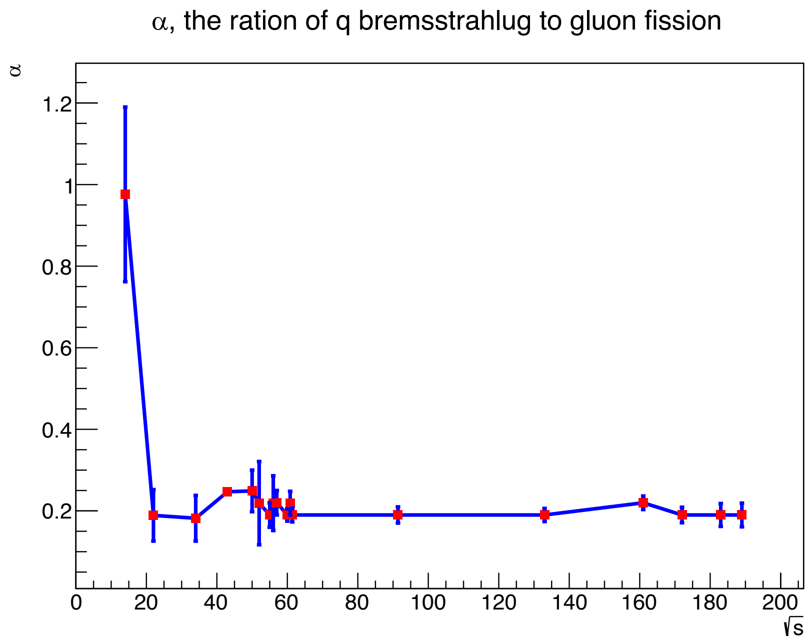


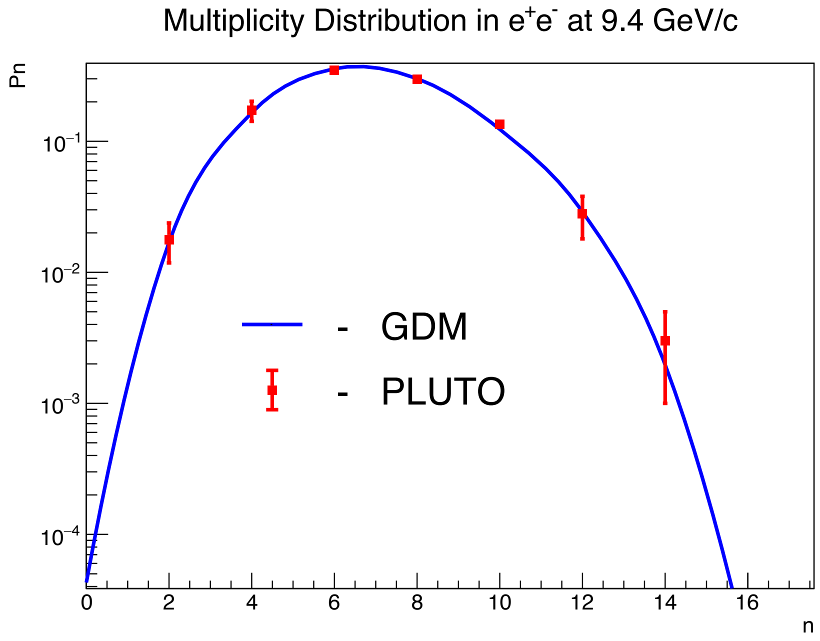


√s <10 GeV. $\bar{m} \ll 1$, g–fission are virtually absent .

√s > 10 GeV. $\bar{m} \geq 1$, multiplicity of g–fission increases, $\bar{n}_g^h$ remains constant and smaller than 1 up to ~100 GeV (fragmentation mechanism of hadronization), then it starts to grow and gently exceeds 1 (recombination mechanism of hadronization) at ≳130 GeV. Data from the 90s have weak indication of such behavior (HERA).

TSM parameter α & MD at 9.4 GeV

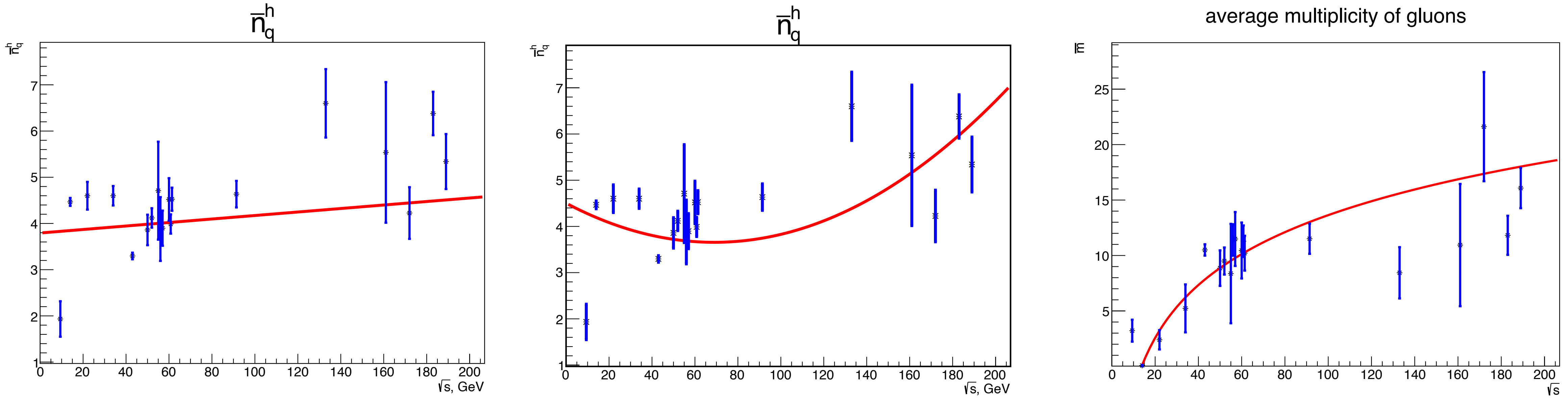




(On the left) Parameter **α** as the ration $\bar{n}_g^h / \bar{n}_q^h$. It almost stays constant and close to 0.2 (one fifth) as a steady process of qg-cascade up to ~ 200 GeV

(On the right) MD at low energy where f_2 is negative. Only q-bresstrahlung conrtibutes.

TSM parameter's fits for average multiplicity:



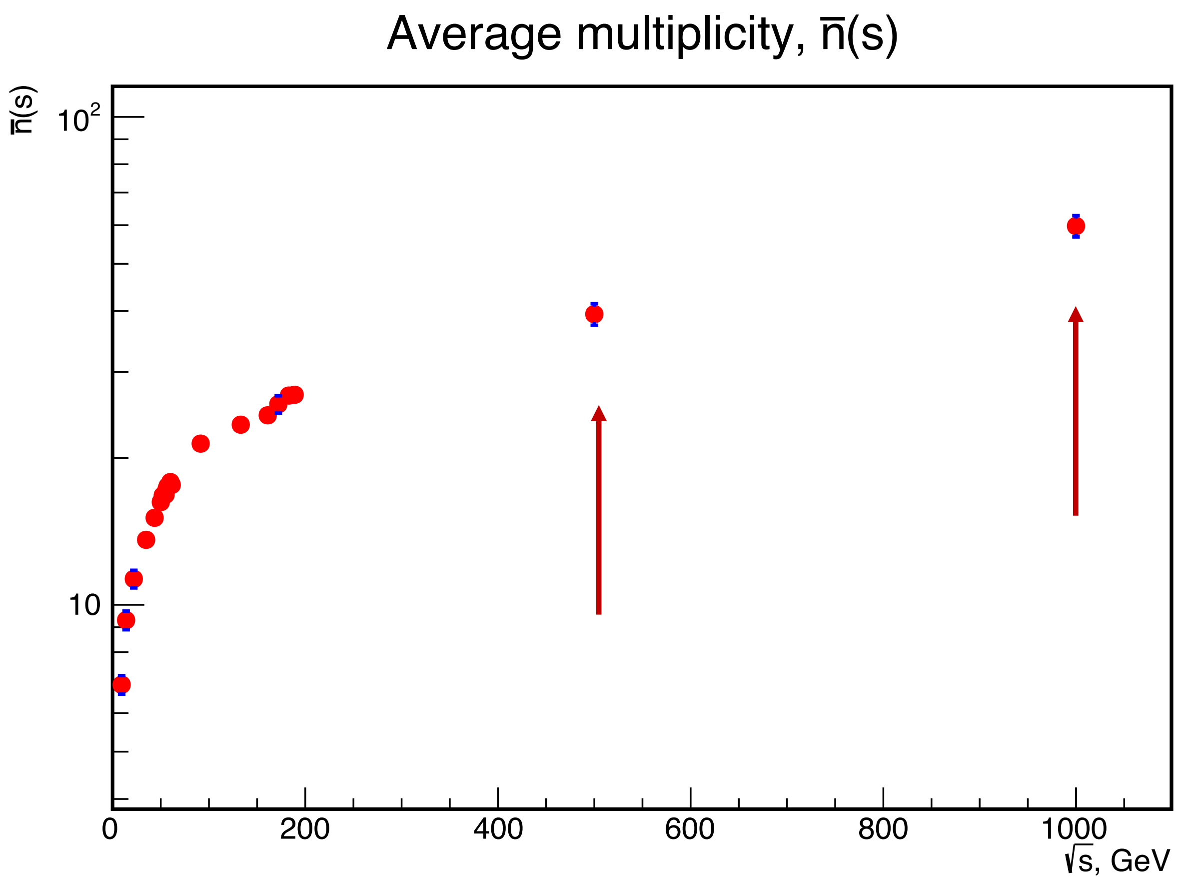
$$\bar{n}(s) = (2 + \alpha\bar{m})\bar{n}^h$$

(left) Linear growth for $\bar{n}_q^h$

(middle) quadratic growth for $\bar{n}_q^h$

(right) logarithmic growth for $\bar{m}$

Prediction for mean multiplicity at 500 GeV and 1 TeV:



Prediction: $\bar{n}(500\ \mathrm{GeV}) = 32 \div 39$ & $\bar{n}(1\ \mathrm{TeV}) = 37 \div 60$.

Summary from e^+e^--annihilation study:

1. The process of e^+e^- -annihilation is the simple one for the QCD description for the first (parton) stage because the creation of gluons and following its fission occurs in a vacuum at $\sqrt{s} < \sim 100$ GeV.
2. pQCD is not applied at the 2nd stage (hadronization). Our choice of MD at the hadronization is based on the data: f_2 for quark and gluon jets is always positive. Experimental f_2 is negative at low energies and becomes positive at higher emerges.
3. At low energy, hadronization dominates over the qg-cascade. We use Binomial distribution for hadronization. Convolution of those two stages describes well experimental MD up to 200 GeV including low energies.

Summary from e^+e^--annihilation study:

4. Average number of hadrons formed from single gluon ($\bar{n}_g^h$) at the second stage is close to 1 at the wide energy region (~ 10-100 GeV): 1 gluon -> 1 hadron (confirmation of the fragmentation mechanism of hadronization in vacuum).
5. Average gluon multiplicity ($\overline{m}$) rises logarithmically.
6. Gluon fission (g -> g + g) commences prevailing over bremsstrahlung (q -> q + g) with increasing of energy and f_2 changes sign.
7. Hadronization of gluons is softer than quark one at the hadronization.

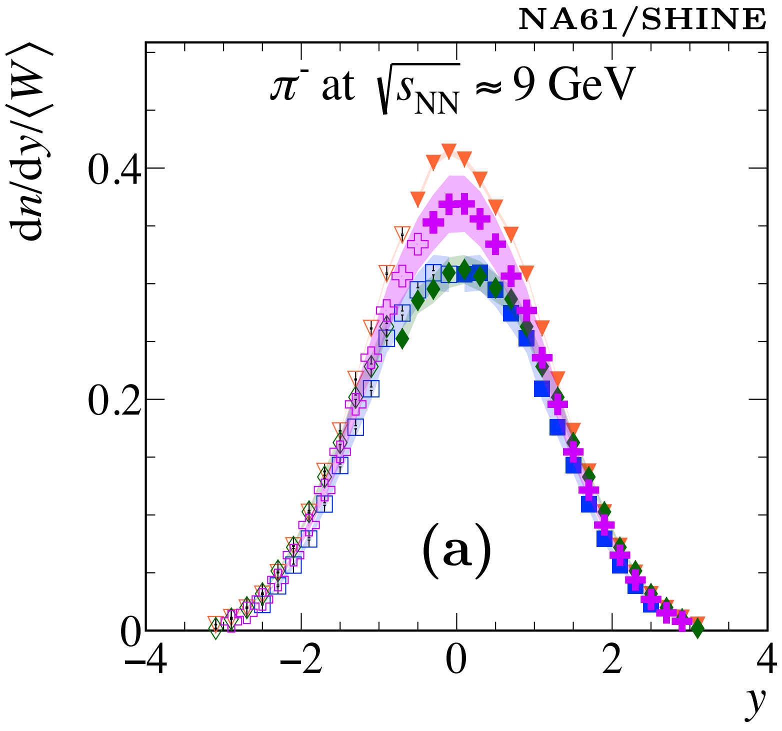
Future e+e- accelerators
(CEPC and others)
will answer questions about
hadronization stage,
its mechanism and
gluon fission

Thank you for attention

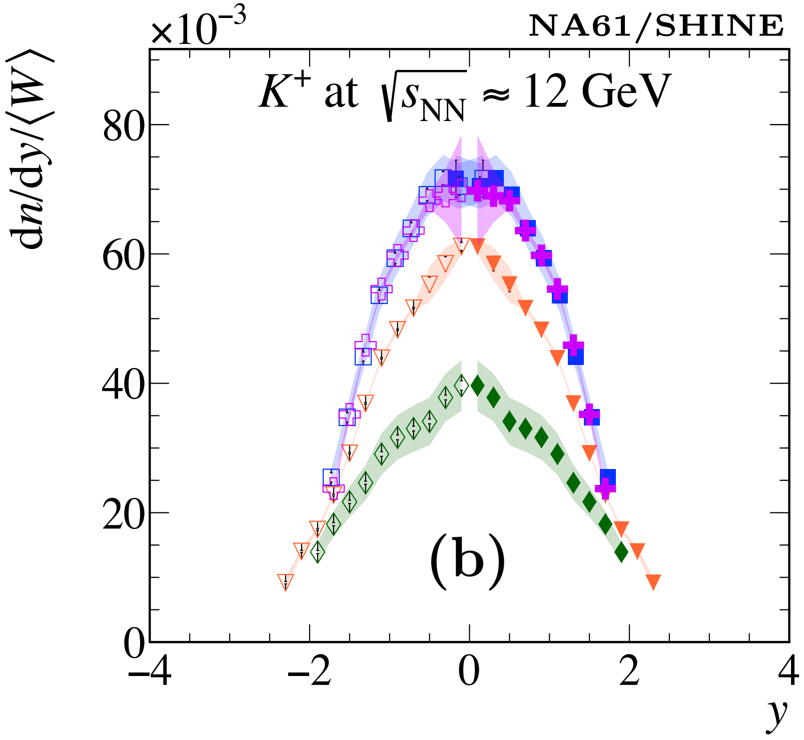
Spare slides

Overview of results from NA61/SHINE.

A. Rybicki for the NA61/SHINE Coll. arXiv:2604.20616 [nucl-ex]



Be + Be (4+4)
Ar + Sc (18+21)
Xe +La (54 +57)
Pb +Pb (82 +82)



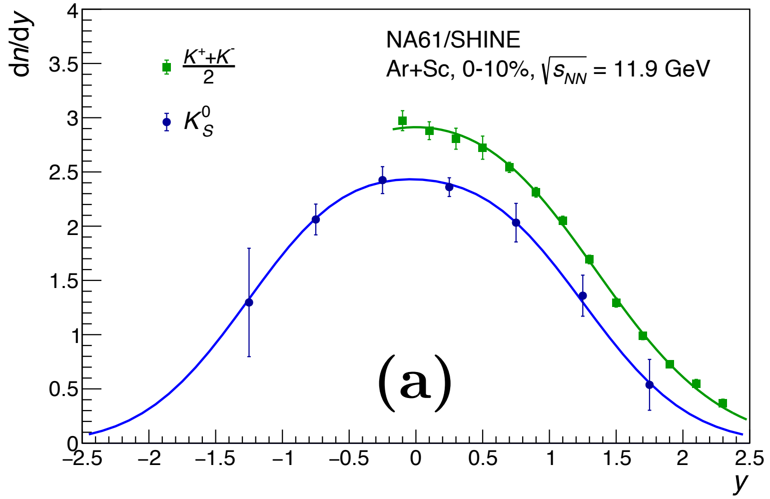
(a) Be+Be -> Pb+Pb -> Xe+La -> Ar+Sc

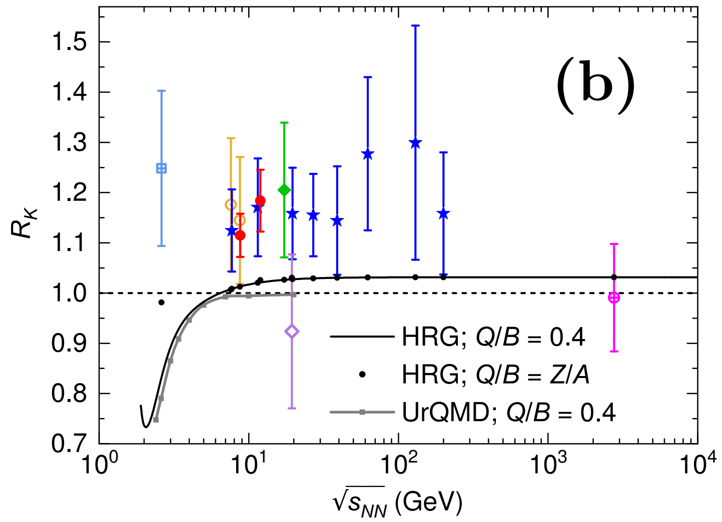
(b) Be+Be -> Ar+Sc -> Xe+La -> Pb+Pb

K+/π+ ratio -> (b)

Kaon sector of MP: unexpectedly strong violation of isospin (flavor) symmetry [arXiv:2604.20616]:

$$\frac{K^+ + K^-}{2} \approx \frac{K^0 + \bar{K}^0}{2} = K_S, \qquad R_K = \frac{K^+ + K^-}{2K_S}.$$





Flavor symmetry (QCD) in the limit of equal quark masses, the remnant approximate light u/d quark isospin symmetry implies equal production of charged and respective neutral kaons in case of colliding charge-symmetric (Z = N ≡ A – Z) nuclei .

Data from Ar+Sc nuclei collisions at √sNN=11.9 GeV do not follow this prediction, and exhibit a charged-over-neutral kaon excess reaching 18.4±6.1% at mid-rapidity.

Possible explanation of these findings

Gluons are active sources of secondaries, they are formed ar high energy collisions (q-breamsstrahlung and g-fission). At the hadronization stage they decay into quark pairs: **u**, $\bar{\mathbf{u}}$, **d**, $\bar{\mathbf{d}}$, **s**, $\bar{\mathbf{s}}$. Probabilities of their creation depend of the mass value. It is known, quark masses differ from each other. The d-quark has slightly greater mass than the u-quark (by a couple of MeV), but is already noticeably less than the s-quark. So, at hadronization, u, $\bar{u}$ quarks will be predominant, then d, $\bar{d}$ (a little less) and $s, \bar{s}$ quarks will be even fewer.

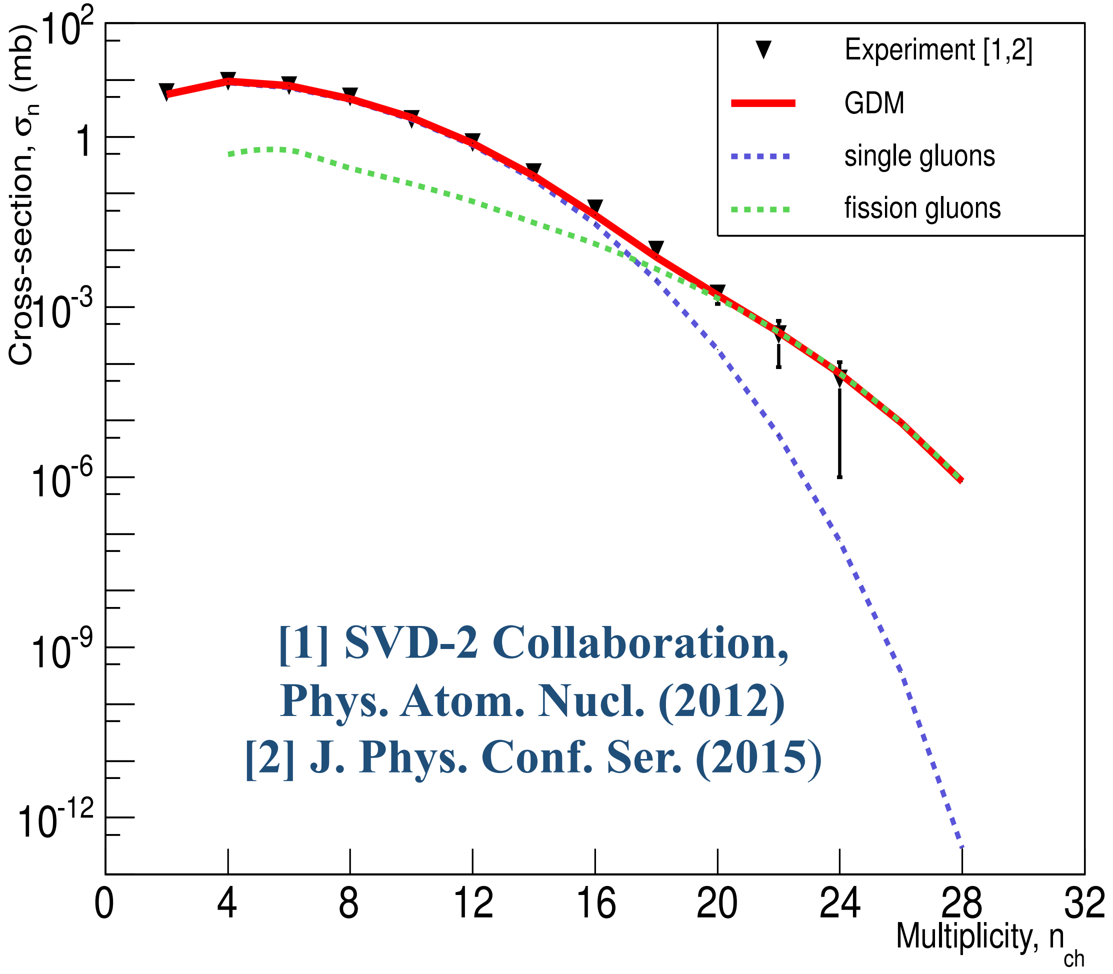
Possible explanation of $K^{\pm}$ excess

Simple combinatorics shows: the number of ways in which quarks can form $K^{\pm}$ must be greater than formation of K_S. $K^{+}\{u\bar{s}\}$, $K^{-}\{s\bar{u}\}$, $K^{0}\{d\ \bar{s}\}$ and $\bar{K}^{0}\{s\ \bar{d}\}$. Let us define N_u, N_d and N_s multiplicities of corresponding quarks and antiquarks.
Experimental excess of $K^{\pm}$ over K_S

$$R_K \approx 18.4 \pm 6.1\ \%,$$

As $R_K = (Nu - Nd)/(Nu + Nd)$ we get the ratio between u and d quarks, $Nd \sim 0.69\ Nu$. The fraction of d-quarks in u-quarks is close to 2/3 Nu.

GDM with gluon fission



SVD-2 and Mirabelle (50 GeV) have been stitched along σ_n. GDM in HM region takes into account 2 types of contributions: without g-fission (blue line) and with (green line). Their superposition is shown by red line. HM stipulates namely by gluon fission. Ratio of bremsstrahlung to gluon fission is equal to ~ 1/9. More than 64% of E is converted into a mass of newborn pions.

$p\bar{p}$ annihilation

GDM offers description of $p\bar{p}$ anihilation by the formation of 3 and more intermediate charged quark topologies with corresponding contributions c_0, c_2 and c_4, which are stipulated by all kinds of permutations of valence quarks with antiquarks and the formation of three leading pions.

Variants of q-topologies

2 variants of "0"- topology (3 π^0):

$u_1\bar{u}_1 + u_2\bar{u}_2 + d\bar{d}$ and $u_1\bar{u}_2 + u_2\bar{u}_1 + d\bar{d}$;

4 variants of "2"- topology (π^0, π^+, π^-):

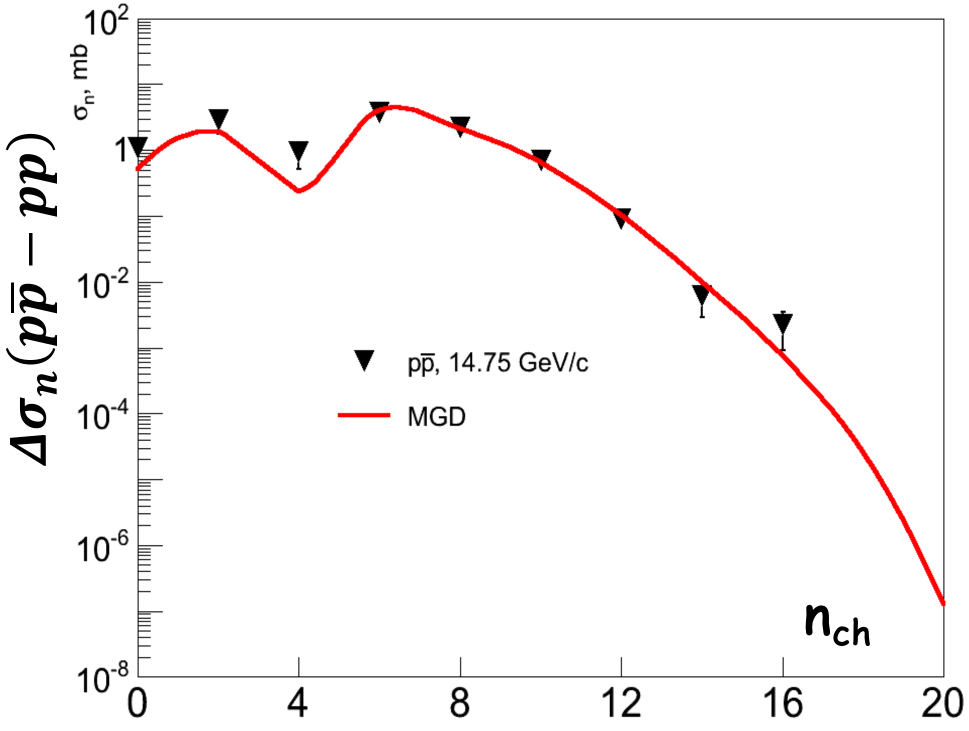
$u_1\bar{d} + u_2\bar{u}_1 + d\bar{u}_2$, $u_1\bar{u}_2 + u_2\bar{d} + d\bar{u}_1$

$u_1\bar{u}_1 + u_2\bar{d} + d\bar{u}_2$, and $u_1\bar{d} + u_2\bar{u}_2 + d\bar{u}_1$;

Topology "4" (and higher) is formed by adding to a valence quark (an antiquark) the corresponding antiquark (quark), which are born from active gluons (g->q+$\bar{q}$):

$u_1\bar{d} + \bar{u}_1 d + u_2\bar{d} + d\bar{u}_2 + \ldots$ ($\pi^+, \pi^-, \pi^-, \pi^+$):

$p\bar{p}$ annihilation



Generation function in GDM:

$$Q(z) =$$

$$= \; c_0 \; \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N}(z-1)\right]^{mN} +$$

$$+ \; c_2 z^2 \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N}(z-1)\right]^{mN} +$$

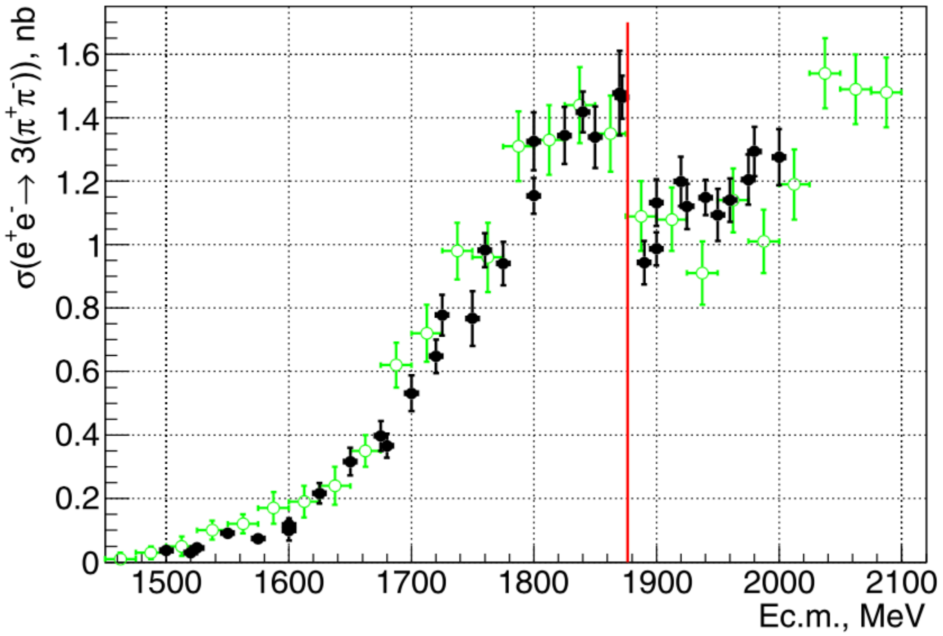
$$+ \; c_4 z^4 \sum_m P_m^G \left[1 + \frac{\bar{n}^h}{N}(z-1)\right]^{mN} .$$

The “4” topology is responsible for the tail of HM.

Study of the process $e^+e^- \to 3(\pi^+\pi^-)$ in the c.m. energy range 1.5–2.0 GeV with the CMD-3 detector

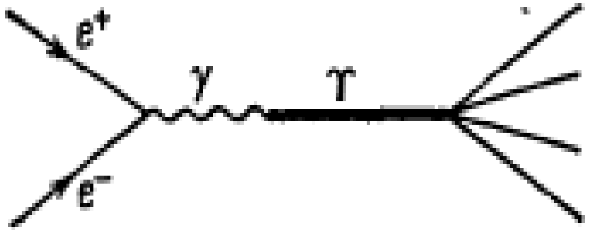
"σ(e^+e^- -> $3(\pi^+\pi^-)$) is measured at the CMD-3 (VEPP-2000) in the c.m. energy range 1.5–2.0 GeV. The measured σ exhibits a sharp drop near the $p\bar{p}$ threshold (~1.88 GeV). A first study of dynamics of 6-pion production has been performed".

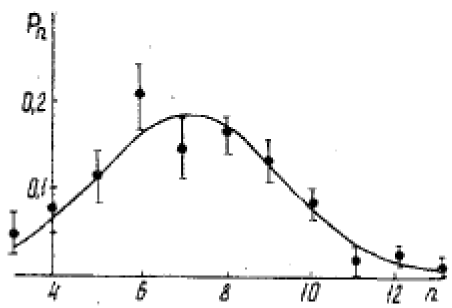
R.R. Akhmetshin et al. Phys. Lett. B 723 (2013) 82–89.

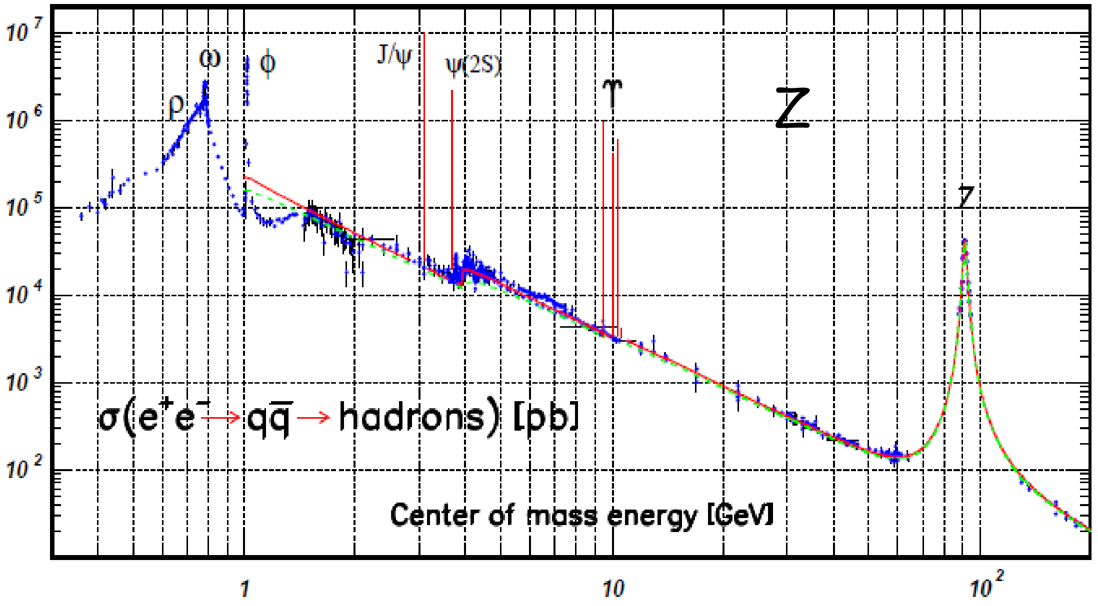


σ($e^+e^- \to 3(\pi^+\pi^-)$) measured with the CMD-3 at VEPP-2000 (black). BaBar data are shown by (green). The red line shows the $p\bar{p}$ threshold.

Three-gluon decay of quarkoniums ϒ(9.46), ϒ (10.02)







$$\Delta\bar{n}_{TSM}(s) = \left[\alpha(\overline{m}' - \overline{m}_{(q)}) - 3(\alpha - 2/3)\right]\bar{n}_q^h \qquad \Delta\bar{n}_{\exp}(s) \approx \Delta\bar{n}_{TSM}(s) \approx 0.8 \qquad \overline{m} = \overline{m'} + 3.$$

And as afterword, I would like to compare the formation of secondary hadrons from gluons with DNA replication. Nature unexpectedly repeats itself ~ gluons -> hadrons