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The work was supported by state contract Project No. FWNF-2026-0034.

Generalized Vector Dominance Model describing the data up to 2 GeV

arXiv:2109.02994

Simultaneous analysis of the reactions $e^+e^- \to \pi^+\pi^-, \omega\pi^0, \eta\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0$

Introduction

- **Generalized Vector Dominance Model (GVDM) is commonly used to describe experimental data involving vector mesons in the low energy region.**

- **In the energies below 2 GeV we take into account** $\rho(770)$, $\omega(782), \phi(1020), \rho_1'(1450), \rho_2'(1700), \omega_1'(1420), \omega_2'(1650), \phi_1'(1680)$, **and** $\phi_2'(2170)$.

- **Different modifications of GVDM are used to fit each data set on each process separately, resulted parameters sometimes even contradict each other.**

Questions:

- **Is it possible to describe the data simultaneously in the frame of GVDM?**

- **What is the GVDM accuracy?**

In N.N. Achasov and A.A. Kozhevnikov, Phys. Rev. D55, 2663 (1997); Phys. Rev. D57, 4334 (1998) a description of $e^+e^- \to \pi^+\pi^-, \omega\pi^0, \eta\pi^+\pi^-, \pi^+\pi^-\pi^+\pi^-, K^+K^-, \pi^+\pi^-\pi^0, \omega\pi^+\pi^-, K^0_S K^\pm\pi^\mp$, $K^{*0}K^-\pi^+ +$ **c.c and some other processes was presented. Results of our work are based mainly on the formulae of that works with some improvements.**

For the first step we take into consideration $e^+e^- \to \pi^+\pi^-, \omega\pi^0$, $\eta\pi^+\pi^-$, **going through isovectors** $\rho(770), \rho'_1(1450)$, $\rho'_2(1700)$, **and** $e^+e^- \to K^+K^-, \pi^+\pi^-\pi^0$, **going through isoscalars** $\omega(782), \omega'_1(1420)$, $\omega'_2(1650)$, $\phi(1020), \phi'_1(1680), \phi'_2(2170)$ **(**$e^+e^- \to K^+K^-$ **has isovector contribution also).**

Reactions $e^+e^- \to \pi^+\pi^-, \omega\pi^0$, $\eta\pi^+\pi^-$

$$\sigma(e^+e^- \to \rho+\rho'_1+\rho'_2 \to f) = \frac{4\pi\alpha^2}{s^{3/2}}|(\frac{m^2_\rho}{f_\rho}, \frac{m^2_{\rho'_1}}{f_{\rho'_1}}, \frac{m^2_{\rho'_2}}{f_{\rho'_2}})G^{-1}_\rho(s)\begin{pmatrix} g_{\rho f} \\ g_{\rho'_1 f} \\ g_{\rho'_2 f} \end{pmatrix}|^2 P_f, \qquad (1)$$

where $f = \pi^+\pi^-$, $\omega\pi^0$ **and** $\eta\pi^+\pi^-$, P_f – **phase volume:**

$$P_f \equiv P_f(s) = \frac{2}{3s}q^3_{\pi\pi},\ \frac{1}{3}q^3_{\omega\pi},\ \frac{1}{3}\langle q^3_{\rho\eta}\rangle \cdot \frac{2}{3}. \qquad (2)$$

Matrix of inverse propagators:

$$G_\rho(s) = \begin{pmatrix} D_\rho & -\Pi_{\rho\rho'_1} & -\Pi_{\rho\rho'_2} \\ -\Pi_{\rho\rho'_1} & D_{\rho'_1} & -\Pi_{\rho'_1\rho'_2} \\ -\Pi_{\rho\rho'_2} & -\Pi_{\rho'_1\rho'_2} & D_{\rho'_2} \end{pmatrix}. \quad (3)$$

$$D_{\rho_i}(s) = m^2_{\rho_i} - s - i\sqrt{s}\Gamma_{\rho_i}(s),$$

$$\mathbf{Im}\Pi_{\rho_i\rho_j} = \sqrt{s}[\frac{g_{\rho_i\pi\pi}g_{\rho_j\pi\pi}}{6\pi s}q^3_{\pi\pi} + \frac{g_{\rho_i\omega\pi}g_{\rho_j\omega\pi}}{12\pi}(q^3_{\omega\pi} + q^3_{K^*K} + \frac{2}{3}\langle q^3_{\rho\eta}\rangle) +$$
$$\frac{3}{2}g_{\rho_i\rho^0\pi^+\pi^-}g_{\rho_j\rho^0\pi^+\pi^-}W_{\pi^+\pi^-\pi^+\pi^-}(s) + g_{\rho_i\rho^+\rho^-}g_{\rho_j\rho^+\rho^-}W_{\pi^+\pi^-\pi^0\pi^0}(s)].$$

$$\sqrt{s}\Gamma_{\rho_i}(s) = \mathbf{Im}\Pi_{\rho_i\rho_i}$$

Reactions $e^+e^- \to K^+K^-, \pi^+\pi^-\pi^0$

$$\sigma_f = \frac{(4\pi\alpha)^2}{s^{3/2}} \Bigg| \left(\frac{m_\rho^2}{f_\rho}, \frac{m_{\rho_1'}^2}{f_{\rho_1'}}, \frac{m_{\rho_2'}^2}{f_{\rho_2'}}\right) G_\rho^{-1}(s) \begin{pmatrix} g_{\rho f} \\ g_{\rho_1' f} \\ g_{\rho_2' f} \end{pmatrix}$$

$$+ \left(\frac{m_\omega^2}{f_\omega}, \frac{m_{\omega_1'}^2}{f_{\omega_1'}}, \frac{m_{\omega_2'}^2}{f_{\omega_2'}}, \frac{m_\varphi^2}{f_\varphi}, \frac{m_{\varphi_1'}^2}{f_{\varphi_1'}}, \frac{m_{\varphi_2'}^2}{f_{\varphi_2'}}\right) G_{\omega\varphi}^{-1}(s) \begin{pmatrix} g_{\omega f} \\ g_{\omega_1' f} \\ g_{\omega_2' f} \\ g_{\varphi f} \\ g_{\varphi_1' f} \\ g_{\varphi_2' f} \end{pmatrix} \Bigg|^2 P_f.$$

$$P_f \equiv P_f(s) = W_{3\pi}(\sqrt{s}),\ \frac{2q_{KK}^3}{3s}.$$

$$G_{\omega\varphi}(s) = \begin{pmatrix} D_{\omega} & -\Pi_{\omega\omega_1'} & -\Pi_{\omega\omega_2'} & -\Pi_{\omega\varphi} & -\Pi_{\omega\varphi_1'} & -\Pi_{\omega\varphi_2'} \\ -\Pi_{\omega\omega_1'} & D_{\omega_1'} & -\Pi_{\omega_1'\omega_2'} & -\Pi_{\omega_1'\varphi} & -\Pi_{\omega_1'\varphi_1'} & -\Pi_{\omega_1'\varphi_2'} \\ -\Pi_{\omega\omega_2'} & -\Pi_{\omega_1'\omega_2'} & D_{\omega_2'} & -\Pi_{\omega_2'\varphi} & -\Pi_{\omega_2'\varphi_1'} & -\Pi_{\omega_2'\varphi_2'} \\ -\Pi_{\omega\varphi} & -\Pi_{\omega_1'\varphi} & -\Pi_{\omega_2'\varphi} & D_{\varphi} & -\Pi_{\varphi\varphi_1'} & -\Pi_{\varphi\varphi_2'} \\ -\Pi_{\omega\varphi_1'} & -\Pi_{\omega_1'\varphi_1'} & -\Pi_{\omega_2'\varphi_1'} & -\Pi_{\varphi\varphi_1'} & D_{\varphi_1'} & -\Pi_{\varphi_1'\varphi_2'} \\ -\Pi_{\omega\varphi_2'} & -\Pi_{\omega_1'\varphi_2'} & -\Pi_{\omega_2'\varphi_2'} & -\Pi_{\varphi\varphi_2'} & -\Pi_{\varphi_1'\varphi_2'} & D_{\varphi_2'} \end{pmatrix} \tag{4}$$

$$D_{V_i}(s) = m_{V_i}^2 - s - i\sqrt{s}\Gamma_{V_i}(s)$$

$$\begin{aligned} \mathbf{Im}\Pi_{V_iV_j}(s) \ = \ & \sqrt{s}\Big(g_{V_i\rho\pi}g_{V_j\rho\pi}P_{\pi^+\pi^-\pi^0} \\ & +2g_{V_iK^+K^-}g_{V_jK^+K^-}P_{K^+K^-} \\ & +4g_{V_iK^{*+}K}g_{V_jK^{*+}K^-}P_{K_S^0K^+\pi^-} \\ & +g_{V_iV_1\pi^+\pi^-}g_{V_jV_1\pi^+\pi^-}W_{VP\pi}\Big), \end{aligned} \quad (5)$$

$$W_{VP\pi} \equiv W_{VP\pi}(\sqrt{s}, m_{V_1}, m_\pi).$$

$$\sqrt{s}\Gamma_{V_i}(s) = \mathbf{Im}\Pi_{V_iV_i}(s). \quad (6)$$

Suppression of the width growth

$$C_{VP}(s) = \frac{1 + (R_{VP}m_0)^2}{1 + (R_{VP}\sqrt{s})^2}$$

Relations between constants

$$\begin{aligned}
g_{\rho_{1,2,3}K^{*+}K^{-}} &= \frac{1}{2} g_{\omega_{1,2,3}\rho\pi},\\
g_{\omega_{1,2,3}K^{*+}K^{-}} &= \frac{1}{2} g_{\omega_{1,2,3}\rho\pi},\\
g_{\varphi_{1,2,3}K^{*+}K^{-}} &= \frac{1}{\sqrt{2}} g_{\omega_{1,2,3}\rho\pi},
\end{aligned} \tag{7}$$

$$g_{\rho^0_{1,2,3}K^+K^-} = -\frac{1}{\sqrt{2}} g_{\varphi_{1,2,3}K^+K^-} = -g_{\rho^0_{1,2,3}K^0\bar{K}^0},$$

$$g_{\omega_{1,2,3}K^+K^-} = -\frac{1}{\sqrt{2}} g_{\varphi_{1,2,3}K^+K^-} = g_{\omega_{1,2,3}K^0\bar{K}^0}, \tag{8}$$

The data

$e^+e^- \to \pi^+\pi^-$: **BABAR-2012;**
$e^+e^- \to \omega\pi^0$: **SND-2016** $(e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma)$;
$e^+e^- \to \eta\pi^+\pi^-$: **SND-2017;**
$e^+e^- \to K^+K^-$: **SND-2016;**
$e^+e^- \to \pi^+\pi^-\pi^0$: **SND-2015.**

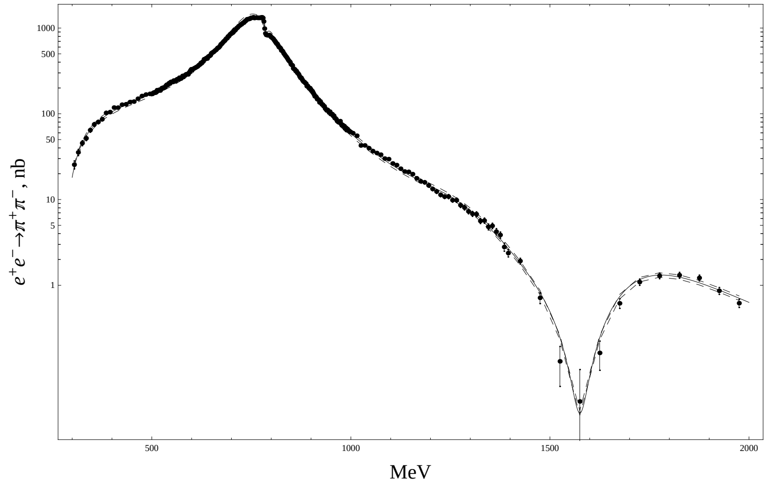


Рис. 1: $\sigma(e^+e^- \to \pi^+\pi^-)$, nb.

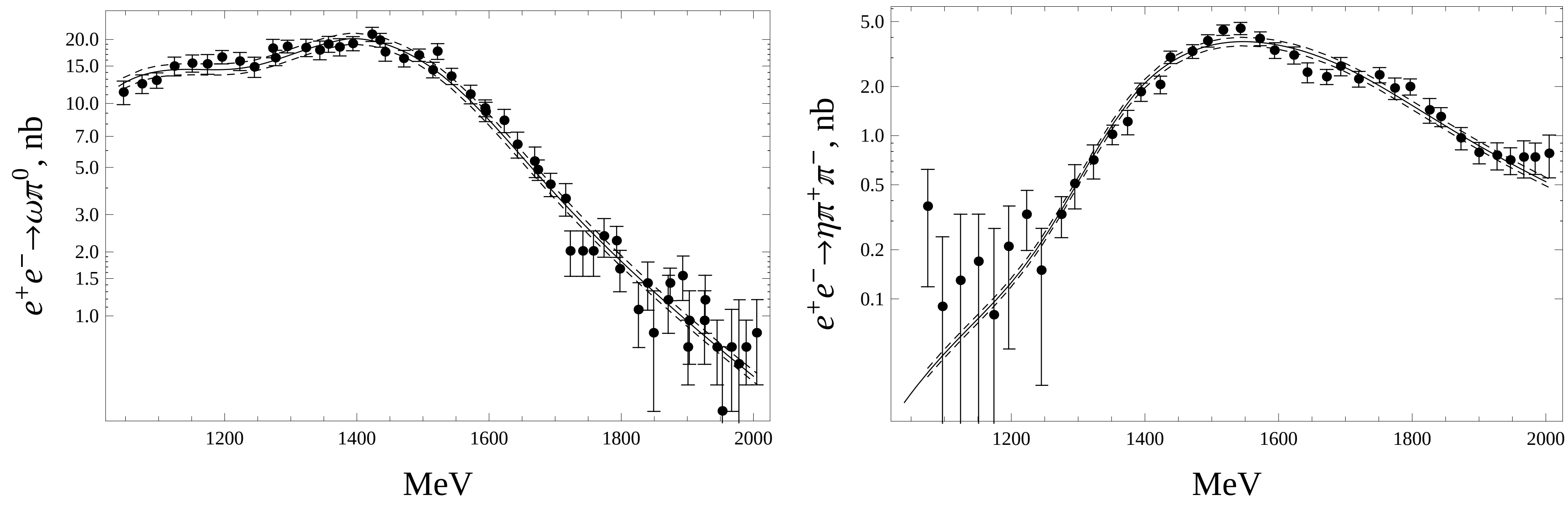


Рис. 2: $\sigma(e^+e^- \to \omega\pi^0)$ and $\sigma(e^+e^- \to \eta\pi^+\pi^-)$, nb.

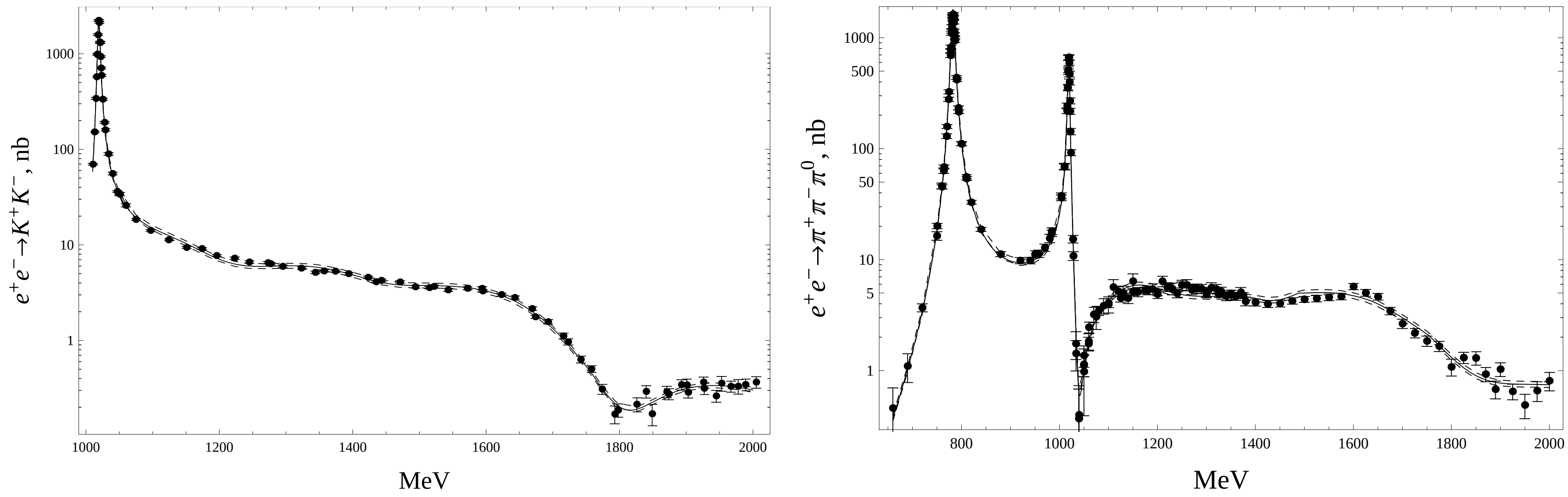


Рис. 3: $\sigma(e^+e^- \to K^+K^-)$ and $\sigma(e^+e^- \to \pi^+\pi^-\pi^0)$, nb.

Residual χ^2

Usual χ^2:

$$\sum_i (x_i^{exp} - x_i^{th})^2/\sigma_i^2 .$$

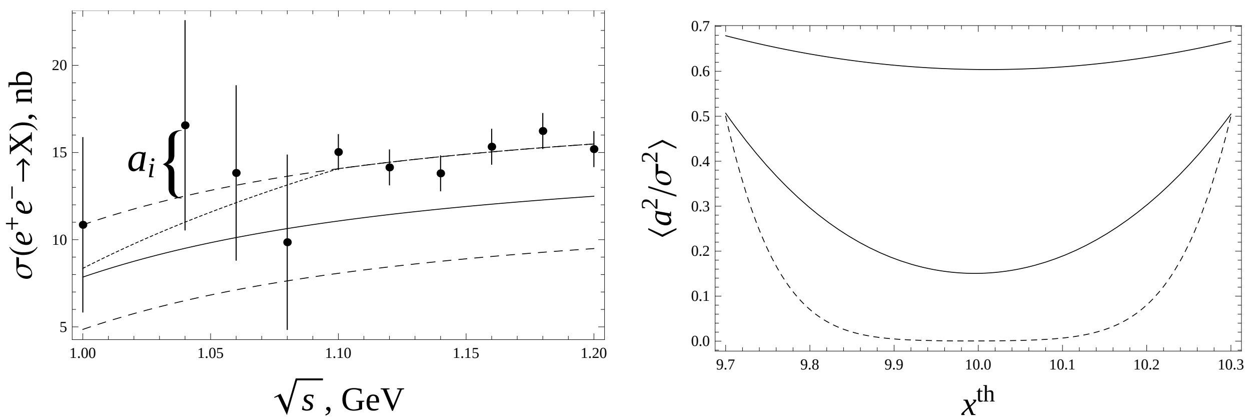
If we introduce residual χ^2 as

$$\bar{\chi}^2 = \sum_i \frac{a_i^2}{\sigma_i^2} \,,$$

$$a_i = \theta(x_i^{exp} - \frac{x_i^{th}}{1-\beta})\,(x_i^{exp} - \frac{x_i^{th}}{1-\beta}) + \theta(\frac{x_i^{th}}{1+\beta} - x_i^{exp})\,(x_i^{exp} - \frac{x_i^{th}}{1+\beta}) \,,$$

we will underestimate model error. So let's choose

$$\bar{\chi}^2 = \sum_i \frac{1}{\langle a_i^2/\sigma_i^2 \rangle} \frac{a_i^2}{\sigma_i^2} = \sum_i \frac{a_i^2}{\langle a_i^2 \rangle} \approx N_{df} \,;\;\; x_i^0 = \frac{x_i^{th}}{1-\beta} \,.$$



а) б)

Рис. 4: a) A fictional situation for an illustration: cross section of some process $e^+e^- \to X$ in the energy region (1 GeV, 1.2 GeV). Solid line is a theoretical curve, dashed lines are borders, dotted line is a true cross section, a_i is the distance to the corridor, points are experimental data. b) Average value $\langle a^2/\sigma^2 \rangle$ as a function of x^{th} with $\beta = 0.03$, $x^0 = 10$. The upper solid line is for $\sigma = 0.1$, the bottom solid line is for $\sigma = 0.3$, and the dashed line is for $\sigma = 1$. The shown x^{th} region is from $(1-\beta)x^0 = 9.7$ up to $(1+\beta)x^0 = 10.3$.

Of course, our approach can suffer "masking effect" – if in some point theoretical prediction is closer to true value than $x_i^{th}/(1-\beta)$, the contribution to $\bar{\chi}^2$ is less than estimation, it can compensate possible going beyond the corridor in some other point.

Fit	**1**	**2**
m_ρ, **MeV**	776.43	769.52
$g_{\rho\pi\pi}$	6.145	5.948
f_ρ	5.170	5.567
$m_{\rho'_1}$, **MeV**	1454.65	1450.59
$g_{\rho'_1\pi^+\pi^-}$	-3.568	-4.057
$f_{\rho'_1}$	2.163	2.363
$m_{\rho'_2}$, **MeV**	1760.15	1760.09
$g_{\rho'_2\pi^+\pi^-}$	1.558	1.805
$f_{\rho'_2}$	1.464	1.588
m_φ, **MeV**	1019.461	1028.283
$g_{\varphi K^+K^-}$	4.010	3.090
f_φ	-12.336	-13.229
$m_{\varphi'_1}$, **MeV**	1638.00	1635.37
$g_{\varphi'_1 K^+K^-}$	-7.957	-8.031
$f_{\varphi'_1}$	-4.092	-4.640

$m_{\varphi_2'}$, **MeV**	2000	2000
$g_{\varphi_2' K^+ K^-}$	3.379	3.219
$f_{\varphi_2'}$	-3.554	-3.929
$\mathbf{Re}\Pi_{\varphi\varphi_1'}$, $\mathbf{GeV}^2$	-0.009294	-0.112439
$\mathbf{Re}\Pi_{\varphi\varphi_2'}$, $\mathbf{GeV}^2$	0.010112	0.098679
$\mathbf{Re}\Pi_{\varphi_1'\varphi_2'}$, $\mathbf{GeV}^2$	-0.9	-0.9
$3f_\rho/f_\omega$	1.07	1.06
$3f_{\rho_1'}/f_{\omega_1'}$	0.90	0.87
$3f_{\rho_2'}/f_{\omega_2'}$	1.17	1.17
$-3f_\rho/(\sqrt{2}f_\varphi)$	0.89	0.89
$-3f_{\rho_1'}/(\sqrt{2}f_{\varphi_1'})$	1.12	1.08

$-3f_{\rho'_2}/(\sqrt{2}f_{\varphi'_2})$	0.87	0.86
$g_{\rho K^+K^-}/g_{\omega K^+K^-}$	1	1
$g_{\rho'_1 K^+K^-}/g_{\omega'_1 K^+K^-}$	0.98	0.98
$g_{\rho'_2 K^+K^-}/g_{\omega'_2 K^+K^-}$	1.52	1.63
$-\sqrt{2}g_{\rho K^+K^-}/g_{\varphi K^+K^-}$	0.83	0.94
$-\sqrt{2}g_{\rho'_1 K^+K^-}/g_{\varphi'_1 K^+K^-}$	0.98	0.94
$-\sqrt{2}g_{\rho'_2 K^+K^-}/g_{\varphi'_2 K^+K^-}$	1.36	1.39
$g_{\varphi\rho\pi}/g_{\omega\rho\pi}$	0.047	0.053
$g_{\varphi'_1\rho\pi}/g_{\omega'_1\rho\pi}$	-0.1	-0.1
$g_{\varphi'_2\rho\pi}/g_{\omega'_2\rho\pi}$	0.106	0.102
$\chi^2_{\pi^+\pi^-}$ (307 **n.d.f.**)	49.9	24.5
$\chi^2_{\omega\pi^0}$ (53 **n.d.f.**)	29.8	28.0
$\chi^2_{\eta\pi^+\pi^-}$ (39 **n.d.f.**)	39.6	39.8
$\chi^2_{K^+K^-}$ (78 **n.d.f.**)	62.9	52.5
$\chi^2_{\pi^+\pi^-\pi^0}$ (142 **n.d.f.**)	121.4	133.3
$\chi^2_{tot}/$**n.d.f.**	303.7/619	278.1/618

Radiative ϕ decays

Shift of the bare mass m_ϕ from the familiar value ≈ 1019.5 MeV has a low influence on the description of $\phi \to \eta\pi^0\gamma$ and $\phi \to \pi^0\pi^0\gamma$ decays. At first, visible cross section of the main K^+K^--loop mechanism is proportional to $e^+e^- \to K^+K^-$ cross section at $\sqrt{s}$, equal to generally accepted m_ϕ value:

$$\frac{d\sigma(e^+e^- \to K^+K^- \to (\sigma + f_0)\gamma \to \pi^0\pi^0\gamma, s, m)}{dm} =$$

$$\frac{6\pi s}{q^3_{K^+K^-}}\,\sigma(e^+e^- \to K^+K^-, s)\,\frac{1}{g^2_{\phi K^+K^-}}\frac{d\Gamma(\phi \to K^+K^- \to \pi^0\pi^0\gamma, m)}{dm}$$

Secondly, mass shift is mainly caused by ϕ mixing with φ_1' and φ_2', most probably they are also $s\bar{s}$ states. Thirdly, for Fit 2 real part of the $G_{\omega\varphi}(s)$ denominator vanishes at 1019.75 MeV, this position may be treated as an estimation of the physical ϕ mass.

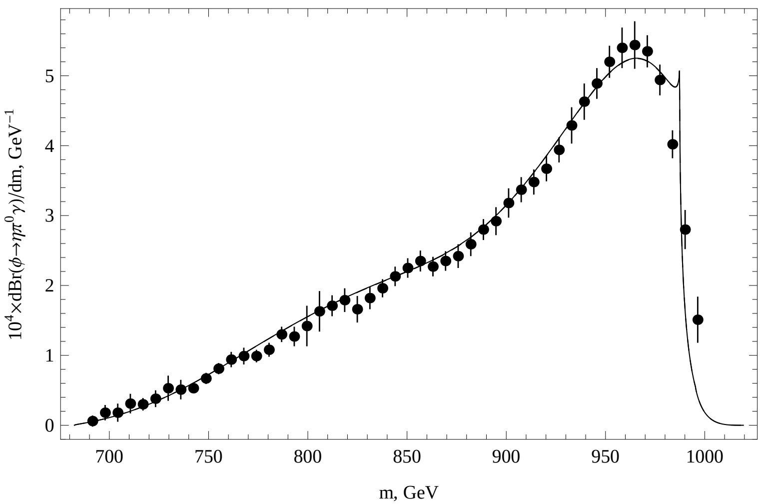


Рис. 5: The plot of mass spectra $\frac{1}{\sigma_\phi}\frac{d\sigma(e^+e^-\to\eta\pi^0\gamma)}{dm}$. Solid line is for Fit 1, dashed line is for Fit 2 (indistinguishable), the data on differential branching $dBr(\phi\to\eta\pi^0\gamma)/dm$ are from KLOE.

Conclusion

1. The procedure of model error estimation is suggested.
2. Cross sections under consideration could be described with not worse than 6% model accuracy, with no model error for $\pi^+\pi^-$ and $\omega\pi$ ones at the same time.
3. There is a possibility of bare ϕ mass shift from the familiar value, while physical ϕ mass keeps its familiar position.
4. There are ways to improve the model, including taking into account energy dependence of polarization operators real parts.