Spinning Q-balls in the Maxwell-Chern-Simons gauge model

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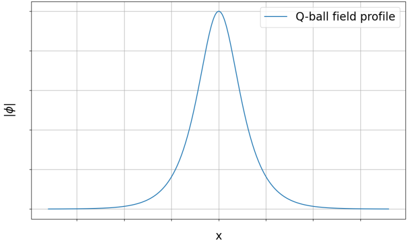
Non-topological Solitons

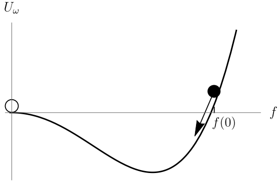
Stationary solution $\phi = e^{-i\omega t} f(x)$.

- Non-topological soliton (Q-ball): $f(\pm\infty) = 0$- trivial vacuum boundary condition.



$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(|\phi|^2)$, where $V\left(|\phi|^2\right)$ describes attractive self-interaction.

Finite energy functional E and global U(1) charge Q. Equation of motion allows for mechanical interpretation. The corresponding mechanical potential is $U_\omega = \omega^2 |\phi|^2 - V(|\phi|^2)$.



Rosen(1968), Coleman's conditions (1985). Overshoot and undershoot.

Gauged Q-balls

An additional gauge field A^μ can be introduced in a theory of scalar Q-balls (Lee et al., PRD 1989)

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \mathcal{D}_\mu\phi^*\mathcal{D}^\mu\phi - V(|\phi|^2). \tag{1}$$

This generalization remains correct in $(3+1)$ dimensions, since the field A^μ behaves at large r as $\sim \alpha/r$. Thus, both E and Q are finite. In lesser space-time dimensions, gauged Q-balls possess a divergent energy functional integral.

In $(2+1)$: $A^\mu \sim \ln(\beta r)$ at large r.

In $(1+1)$: $A^\mu \sim \gamma x$ at large x.

This issue can be solved for a planar theory by including the topological Chern-Simons term.

Maxwell-Chern-Simons Generalization

The most general $(2+1)$-dimensional Lorentz-invariant theory of the massive complex scalar field ϕ and the Abelian gauge field A^μ

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g}{2}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho + D_\mu\phi D^\mu\phi^* - m^2\phi\phi^* + \frac{\lambda}{2}(\phi\phi^*)^2 - \frac{\sigma}{3}(\phi\phi^*)^3,$$

To obtain spinning non-topological solitons[1], we introduce the following ansatz

$$\phi(t,\vec{x}) = e^{-i\omega t + in\theta}f(r), \qquad A^{(0)}(t,\vec{x}) = G(r), \qquad A^i(t,\vec{x}) = -\epsilon^{ij}\frac{a(r)x^j}{r^2}.$$

The electric and magnetic components of the field tensor $F_{\mu\nu}$ are expressed by the following formulas:

$$\mathcal{E}^i(r) = -G'(r)\frac{x^i}{r}, \qquad \mathcal{H}(r) = \epsilon_{ij}\partial_i A_j = \frac{a'(r)}{r}.$$

[1] Unwinded solitons were considered in M. Deshaies-Jacques, R. MacKenzie, PRD 2006

The energy of the field configuration

$$E = 2\pi \int_0^\infty \left[\frac{\vec{\mathcal{E}}^2 + \mathcal{H}^2}{2} + (f')^2 + \left(\omega^2 + m^2 + \frac{n^2}{r^2} \right) f^2 - \frac{\lambda}{2} f^4 + \frac{\sigma}{3} f^6 + \right.$$

$$\left. + eGf^2(eG + 2\omega) + e\frac{af^2}{r^2}(ea - 2n) \right] r\,dr$$

is finite only for $g \neq 0$. U(1) charge:

$$Q = 2\pi \int_0^\infty \left[2f^2(\omega + eG) \right] r\,dr\,.$$

In addition to the U(1) charge, the solution under consideration carries the magnetic flux

$$\Phi = \int_{\mathbb{R}^2} \mathcal{H}\,d^2x = 2\pi \int_0^\infty a'\,dr = 2\pi \cdot a\big|_{r=\infty},$$

that is also conserved.

Presence of the gauge field, as well as angular dependence of the ansatz for the scalar field, provides intrinsic angular momentum J:

$$J = \int_{\mathbb{R}^2} \epsilon^{ij} x^i T^{0j}\, d^2x = -2\pi \int_0^\infty \left[2f^2(\omega + eG)(n - ea) - G'a'\right] r\, dr\,.$$

After integrating by parts:

$$J = -nQ + \frac{e^2}{4\pi g} Q^2.$$

$$J \to -J \Longleftrightarrow \begin{cases} g \to -g, \\ n \to -n, \\ a \to -a. \end{cases}$$

The function $J = J(Q)$ has a global extremum at $Q_T = 2\pi g n / e^2$, which is a minimum in the current notation.

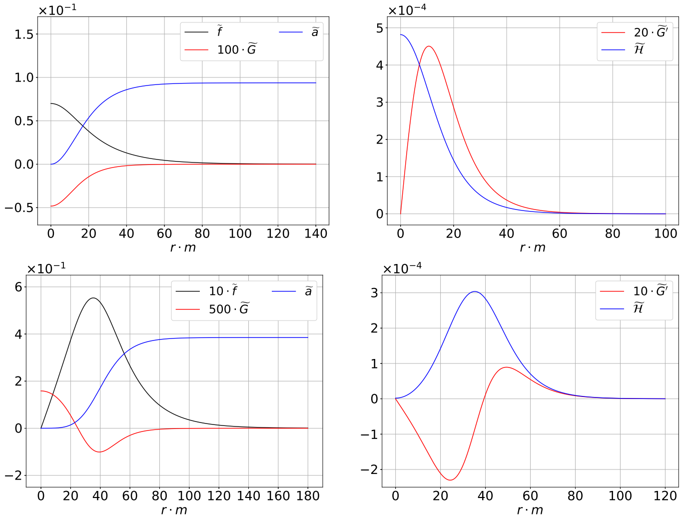


Figure 1: Profiles of the soliton with n = 0, 1. Left panels: Amplitudes of the fields φ, A^{μ}. Right panels: Amplitudes of the fields $\vec{\mathcal{E}}$, $\mathcal{H}$. Here g/m = 1, λ/m = 2, $\sigma = 3/2$, e/$\sqrt{\mathrm{m}}$ = 0.05, and ω/m = 0.999.

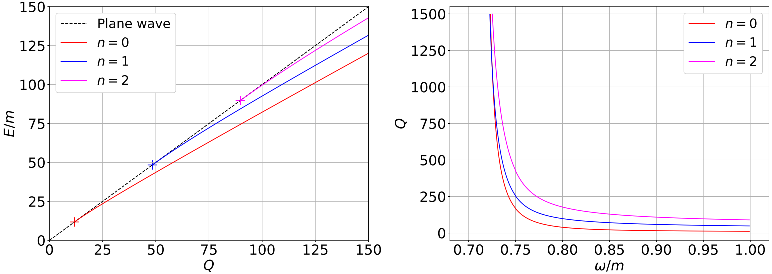


Figure 2: Left panel: Soliton energy E/m as a function of its U(1) charge Q for different winding numbers n. Right panel: Noether charge Q as a function of frequency ω/m for the solitonic solutions with different winding numbers n. Here g/m = 1, λ/m = 2, $\sigma = 3/2$, and $e/\sqrt{m} = 0.05$.

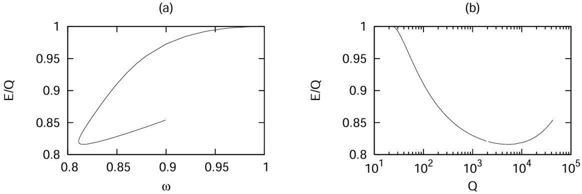
From the left panel of Fig. 2 it can also be seen that $\partial^2 E/\partial Q^2 < 0$, which is equivalent to the relation

$$\frac{\partial Q}{\partial \omega} = \frac{1}{\omega}\frac{\partial E}{\partial \omega} < 0. \tag{2}$$

- A model for kinematically stable gauged Q-balls in $(2+1)$ dimensions was discussed;
- Gauged model with additional Chern-Simons term preserves scale invariance of the non-relativistic theory.

To do:

A: Special point $dQ/d\omega = \infty$.
B: Maximal size of large thin-wall Q-balls.



(Deshaies-Jacques & MacKenzie, PRD 2006)

THANK YOU FOR ATTENTION!