# High-energy motivation and low-energy signatures of the Inverse Seesaw model

# (QUARKS-2026)

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# Outline

1. Main features of models with Inverse seesaw mechanism

2. Left-Right symmetric model with 2 Higgs doublets [(D)LRSM] as a source of Inverse seesaw

3. Possible warm Dark Matter candidate with ISS-scale mass

4. Conclusion and discussion

# Set of new fermionic fields $\mathrm{ISS}\,(\mathbf{p},\mathbf{q})$

**Field content of Inverse Seesaw (ISS):**

- $\nu_{L,\alpha}, \quad \alpha = e, \mu, \tau$ – ordinary left-handed flavor neutrino
- $N_{R,a} \quad a = \overline{1, \mathbf{p}}$ – right-handed neutrino with only Dirac mass term $\sim M_R$
- $S_{R,b} \quad b = \overline{1, \mathbf{q}}$ – chiral singlets with small Majorana mass term $\sim \mu$

$$-\mathcal{L}^{\mathrm{ISS}}_{\mathrm{mass}} = Y(\bar{l}_L \tilde{\Phi})\nu_R + M_R \overline{\nu^c_R} S_R + \frac{\mu}{2} \bar{S}^c_R S_R + \text{H.c.},$$

where $\quad m_D \sim f_\nu v_{\mathrm{EW}}, \qquad M_R = f_R \cdot \Lambda(M_R), \qquad \mu = f_S \cdot \Lambda(\mu)$

**Naturalness condition**

$$||M_R|| \gg ||m_D|| \gg ||\mu|| \qquad \longleftrightarrow \qquad \Lambda(M_R) \gg v_{\mathrm{EW}} \gg \Lambda(\mu)$$

where $\Lambda(M_R)$ - New Physics scale, $\Lambda(\mu)$ - small scale of lepton number soft-breaking

# Effective neutrino mass operator $m_\nu$

$$\mathcal{L}_{ISS} = \frac{1}{2}(\overline{\nu}_L, \ \overline{\nu_R^c}, \ \overline{S_R^c}) \begin{pmatrix} \mathbb{O}_{3\times 3} & m_{D\ 3\times\mathsf{p}} & \mathbb{O}_{3\times\mathsf{q}} \\ m^T_{D\ \mathsf{p}\times 3} & \mathbb{O}_{\mathsf{p}\times\mathsf{p}} & M_{R\ \mathsf{p}\times\mathsf{q}} \\ \mathbb{O}_{\mathsf{q}\times 3} & M^T_{R\ \mathsf{p}\times\mathsf{q}} & \mu_{\ \mathsf{q}\times\mathsf{q}} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \\ S_R \end{pmatrix} + \text{H.c.}$$

Write mass matrix in **seesaw I**-like form with notation: $\tilde{m}_D \equiv (m_D, \mathbb{O})$

$$\mathbf{M} = \begin{pmatrix} \mathbb{O}_{\ 3\times 3} & \widetilde{m}_{D\ 3\times(p+q)} \\ \widetilde{m}^T_{D\ (\mathsf{p}+\mathsf{q})\times 3} & \mathcal{X}_{(\mathsf{p}+\mathsf{q})\times(\mathsf{p}+\mathsf{q})} \end{pmatrix} \quad \text{where} \quad \mathcal{X} = \begin{pmatrix} \mathbb{O}_{\ p\times p} & M_{R\ p\times q} \\ M^T_{R\ q\times p} & \mu_{\ q\times q} \end{pmatrix}$$

$$m_\nu \simeq U_{\rm PMNS} \begin{pmatrix} m_{\nu_1} & 0 & 0 \\ 0 & m_{\nu_2} & 0 \\ 0 & 0 & m_{\nu_3} \end{pmatrix} U^\dagger_{\rm PMNS} = m_D \left(M_R \mu^{-1} M_R^T\right)^{-1} m_D^T \propto \Lambda(\mu)\left(\frac{v_{\rm EW}}{\Lambda(M_R)}\right)^2,$$

## Constraints on p and q $\rightarrow$ $2 \leq \mathsf{p} \leq \mathsf{q}$

- Model $\mathrm{ISS}\,(\mathsf{p},\mathsf{q})$ generates $\min\,(\mathsf{p},\mathsf{q},3)$ non-zero active neutrino masses on **tree-level**.
- If $\mathsf{p} > \mathsf{q}$ then $\det \mathcal{X} = 0$ and $\mathcal{X}$ is **non-invertible**. Numerical results shows that for $\mathsf{p} > \mathsf{q}$ there is heavy active neutrino with mass $\sim v_{\rm EW}$ and large mixing.

# Pseudo-Dirac heavy neutrino

$$\Psi \equiv \begin{pmatrix} \nu_L^c \\ \nu_R \\ \vdots \end{pmatrix}$$

$\Psi' = \mathbf{U}\Psi, \quad (\mathbf{U}^\dagger = \mathbf{U}^{-1})$ ← to phys. states

$\mathbf{U^T M U} = \mathrm{diag}\,(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_1, \dots)$ ← Takagi decomp.

$\boxed{\nu_i = \nu_{i\,L} + (\nu_{i\,L})^c}$ $\boxed{N_I = N_{I\,R} + (N_{I\,R})^c}$ ← Majorana

"Almost Dirac" structure of ISS mass matrix $\mathbf{M}$ leads to uniform mixing in pairs of Majorana states $\Psi_{1,2}^{\mathrm{Maj}}$ $((\Psi_{1,2}^{\mathrm{Maj}})^c = \Psi_{1,2}^{\mathrm{Maj}})$:

$$\Psi_\pm^{\mathrm{PD}} \simeq \frac{1}{\sqrt{2}}(\Psi_1^{\mathrm{Maj}} \pm i\Psi_2^{\mathrm{Maj}}), \qquad (\Psi_\pm^{\mathrm{PD}})^c = \Psi_\mp^{\mathrm{PD}} \qquad m(\Psi_1^{\mathrm{Maj}}) \approx m(\Psi_2^{\mathrm{Maj}})$$

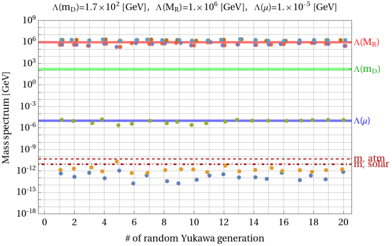
ISS $(\mathsf{p}, \mathsf{q})$ with $\mathsf{p} \leq \mathsf{q}$:

| | | |
|---|---|---|
| $3 \times \nu^{\mathrm{Maj}}$ | $\mathsf{p} \times N^{\mathrm{PD}}$ | $(\mathsf{p} - \mathsf{q}) \times N_{\mathrm{light}}^{\mathrm{Maj}}$ |
| $m(\nu^{\mathrm{Maj}}) \simeq 0.01$ eV | $m(N^{\mathrm{PD}}) \simeq \Lambda(M_R)$ | $m(N_{\mathrm{light}}^{\mathrm{Maj}}) \simeq \Lambda(\mu)$ |

# Neutrino spectrum for various $\mathrm{ISS}(\mathbf{p}, \mathbf{q})$

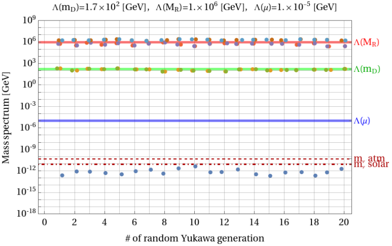
$p < q$

## ISS(2,3)

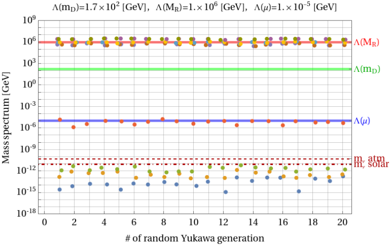



$p > q$

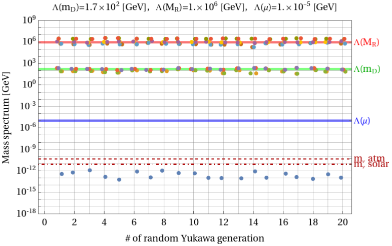
## ISS(3,2)




## ISS(3,4)




## ISS(5,3)



# Active-sterile mixing $\mathrm{ISS}\,(\mathbf{p},\mathbf{q})$, $\mathbf{p} \leq \mathbf{q}$

The mixing between active and sterile states can be encoded into $\tilde{\theta} \equiv (\theta_1, \theta_2) = \tilde{m}_D \mathcal{X}^{-1}$.

$$\mathcal{L}^{CC} = \frac{g}{\sqrt{2}} \bar{l}_\alpha \gamma_\mu P_L \left[ \sum_{i=1}^{3} U_{\alpha i} \nu_i + \sum_{J=1}^{\mathsf{p+q}} (\tilde{\theta}\mathcal{U})^*{}_{kJ} N_J \right] W^\mu + \text{H.c.}$$

$$\mathcal{L}^{NC} = \frac{g}{2c_w} \left[ \cdots + \sum_{J=1}^{\mathsf{p+q}} (\tilde{\theta}\mathcal{U})^*{}_{kJ} N_J \right] \gamma_\mu P_L \left[ \sum_{i=1}^{3} U_{\alpha i} \nu_i + \dots \right] Z^\mu$$

$$\theta_1 = m_D \left( M_R \mu^{-1} M_R^T \right)^{-1} \propto \frac{v_{\mathrm{EW}} \Lambda(\mu)}{\Lambda(M_R)^2}, \qquad ||m_\nu|| \propto \theta_1 v_{\mathrm{EW}}$$

$$\theta_2 = \theta_1 M_R \mu^{-1} \propto \frac{v_{\mathrm{EW}}}{\Lambda(M_R)} \gg \theta_1. \qquad U^2_{\mathrm{mix}} \propto |\theta_2|^2$$

The matrix $\mathcal{U}$ is determined by the mass matrix pattern of the sterile sector $\mathcal{X}$ and has nontrivial features with $\mathsf{p} \neq \mathsf{q}$. For $\mathsf{p} = \mathsf{q}$ it looks like a block rotation with $\pi/4$ angle.

# Grand Unification Theory - source of Left right symmetry

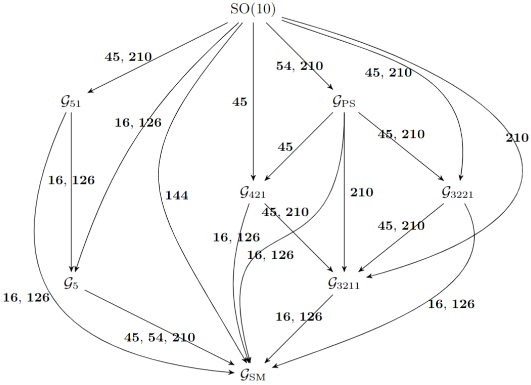
**High-energy source of LRSM:**
Gauge group of LR-minimal symmetric model $\mathcal{G}_{3221}$ appears in SO(10) GU-model (or $E_6$ GUT from triangularity $\mathcal{G}_{333} = SU(3)_c \times SU(3)_L \times SU(3)_R$ gauged model).

**Notation:**

| | | |
|---|---|---|
| $\mathcal{G}_{51}$ | = | $SU(5)\times U(1)$ |
| $\mathcal{G}_{5}$ | = | $SU(5)$ |
| $\mathcal{G}_{PS}$ | = | $SU(4)_C\times SU(2)_L\times SU(2)_R$ |
| $\mathcal{G}_{421}$ | = | $SU(4)_C\times SU(2)_L\times U(1)_{B-L}$ |
| $\mathcal{G}_{3221}$ | = | $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1$ |
| $\mathcal{G}_{3211}$ | = | $SU(3)_C\times SU(2)_L\times U(1)_R\times U(1)_{}$ |

Possible symmetry breaking chains of SO(10) theory to $\mathcal{G}_{SM}$.

[Fig. from M.Pernow,"Models of SO(10) Grand Unified Theories 2021, Doctoral thesis]



# Model (D)LRSM: Field content for ISS(3,4)

**LRSM-gauge group:** $\mathcal{G}_{LR} = SU(3)_c \times SU(2)_R \times SU(2)_L \times U(1)_{B-L}$
**(D)** means using of Higgs **D**oublets $\chi_L$ and $\chi_R$ in addition to bi-doublet $\Phi$.

| Field | $SU(3)_c$ | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ |
|---|---|---|---|---|
| $\chi_L = \begin{pmatrix} \delta_L^+ \\ \delta_L^0 \end{pmatrix}$ | **1** | **2** | **1** | $2$ |
| $\chi_R = \begin{pmatrix} \delta_R^+ \\ \delta_R^0 \end{pmatrix}$ | **1** | **1** | **2** | $2$ |
| $\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}$ | **1** | **2** | **2** | $0$ |
| $L_{\alpha_L} = \begin{pmatrix} \nu_{\alpha_L} \\ l_{\alpha_L} \end{pmatrix}$ | **1** | **2** | **1** | $-1$ |
| $L_{\alpha_R} = \begin{pmatrix} \nu_{\alpha_R} \\ l_{\alpha_R} \end{pmatrix}$ | **1** | **1** | **2** | $-1$ |
| $Q_{a_L} = \begin{pmatrix} u_{a_L} \\ d_{a_L} \end{pmatrix}$ | **3** | **2** | **1** | $\frac{1}{3}$ |
| $Q_{a_R} = \begin{pmatrix} u_{a_R} \\ d_{a_R} \end{pmatrix}$ | **3** | **1** | **2** | $\frac{1}{3}$ |
| $(S_R)_j \quad (S_L)_j \equiv (S_R^c)_j$ <br> $j = \overline{1,4}$ | **1** | **1** | **1** | $0$ |

# (D)LRSM vacuum and symmetry breaking

$$\langle\Phi\rangle = \begin{pmatrix} \kappa_1 & 0 \\ 0 & \kappa_2 e^{i\varepsilon_2} \end{pmatrix} \qquad \langle\chi_L\rangle = \begin{pmatrix} 0 \\ \frac{v_L}{\sqrt{2}} \end{pmatrix} \qquad \langle\chi_R\rangle = \begin{pmatrix} 0 \\ \frac{v_R}{\sqrt{2}} \end{pmatrix}$$

$$SU(2)_R \times SU(2)_L \times U(1)_{B-L} \overset{\langle\chi_R\rangle}{\rightarrow} SU(2)_L \times U(1)_Y \xrightarrow[\langle\Phi\rangle]{\langle\chi_L\rangle} U(1)_Q$$

Quantum numbers: $T_R^3 + \frac{B-L}{2} = \frac{Y}{2} \qquad T_L^3 + \frac{Y}{2} = Q$

Charged gauge bosons $W_\mu - W'_\mu$ mixed with angle $\xi$

$$\tan|2\xi| = \frac{4\kappa_1\kappa_2}{v_R^2} \qquad \sin\xi \simeq \frac{2\kappa_1\kappa_2}{v_R^2}$$

Neutral gauge bosons mixing

$$\tan|2\zeta| \approx \frac{4g_{B-L}\left(k_1^2+k_2^2\right)\cos\left(2\theta_W\right)}{g v_R^2 \cos^3\theta_W \tan\left(2\theta_W\right)}$$

{arXiv:2508.01220 [hep-ph]}

{Another LRSM version is model with Higgs triplets $\Delta_L$, $\Delta_R$ and bi-doublet $\Phi$. For more details you are welcome to Elena Fedotova's talk ''Lepton number violating processes ...'' (May 19th) }

# Yukawa interaction

**Neutrino - Bi-doublet $\&$ Neutrino - Doublet interaction:**

$$\mathcal{L}_{\rm Yukawa} \supset Y\overline{L_L}\Phi L_R + \widetilde{Y}\overline{L_R}\widetilde{\Phi}L_L + Y_L\overline{L_L}\widetilde{\chi}_L S_R + Y_R\overline{L_R}\widetilde{\chi}_R S_R^c + \text{H.c.},$$

**Soft-breaking term of $\Delta L = 2$ violation:**

$\mathcal{L} \supset -\frac{1}{2}\mu\overline{S_R^c}S_R, \quad \mu \ll v_{\rm EW}$ $\{$See e.g. [Phys. Rev. D 83, 093009]$\}$

**Discrete $LR$-symmetry** $\mathcal{P}: \Phi \leftrightarrow \Phi^\dagger$, $L_L \leftrightarrow L_R$ leads to

$$Y = Y^\dagger \qquad \widetilde{Y} = \widetilde{Y}^\dagger \qquad \mu = \mu^\dagger$$

**Mass matrices for neutral and charged leptons**

$$-\mathcal{L}_{\rm masses} \supset m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L) + m_l(\overline{l_L}l_R + \overline{l_R}l_L) + M_R\overline{\nu_R}S_R^c + \frac{1}{2}\mu\overline{S_R}S_R^c$$

$$m_l = \frac{1}{\sqrt{2}}\left(\widetilde{Y}\kappa_1 + Y\kappa_2\right) \qquad m_D = \frac{1}{\sqrt{2}}\left(Y\kappa_1 + \widetilde{Y}\kappa_2\right) \qquad M_R = Y_R\frac{v_R}{\sqrt{2}}$$

# Benchmark simple scenario

## Benchmark

- $\boxed{v_L = 0}$ ($\delta_L^0$ from $\chi_L$ is inert scalar field)
- $\boxed{\kappa_2 = 0} \longrightarrow \kappa_1 = \sqrt{\kappa_1^2 + \kappa_2^2 + v_L^2} = v_{\rm EW}$
- $\boxed{g_L = g_R \equiv g}$ - so called Manifest LRSM condition

Simplified mass matrix for charged leptons is $m_l = \widetilde{Y}\frac{v_{\rm EW}}{\sqrt{2}}$ and for neutrino in basis $(\nu_L^c,\, \nu_R,\, S_R)$ is

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & Y v_{\rm EW} & \overset{\sim v_L}{\boxed{0}} \\ Y^T v_{\rm EW} & 0 & Y_R v_R \\ \underset{\sim v_L}{\boxed{0}} & Y_R^T v_R & \mu \end{pmatrix}$$

**ISS-(D)LRSM scales:**

$$\Lambda(M_R) = v_R$$
$$\Lambda(\mu) = \mathcal{O}\left(10 \text{ keV}\right)$$
$$1 \leq Y_{\alpha i} < 10^{-6}$$
$$(Y_R)_{ab} \simeq 1$$

## Masses of gauge bosons:

$$m^2(W) \simeq \frac{g^2 v_{\rm EW}^2}{4}, \qquad m^2(Z) = \frac{m^2(W)}{\cos^2\theta_W}$$

$$m^2(W') \approx \frac{g^2 v_R^2}{4}, \qquad m^2(Z') = m^2(W')\frac{\cos^2\theta_W}{\cos 2\theta_W} - m^2(W)\frac{(\tan(2\theta_W)+4)\tan^2\theta_W}{2}$$

# Dark matter candidate

**From ISS:** the new scale $\Lambda(\mu)$ of soft $\Delta L = 2$ violation appearing in the theory generates small neutrino masses at a relatively low scale $\Lambda(M_R) \sim 1 - 100$ TeV and under natural assumptions about the Yukawa coupling $Y \sim 1 - 10^{-3}$.

**From LRSM:** New heavy vector and Higgs bosons lie in a region potentially accessible to experiments (compared to High energy seesaw type I, which requires a scale $M_R \sim M_{\rm GUT} \simeq 10^{14}$ GeV).

**From cosmology:** Warm dark matter with a mass of several keV solves some problems of gravitational structure formation within the Cold Dark Matter paradigm: "Too big to fail", "Missing Satellites Problem".

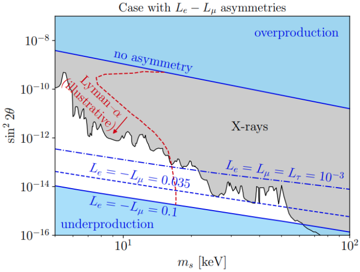
**Key idea:**

*TO IDENTIFY* cosmology Warm Dark Matter mass *WITH* $\Delta L = 2$-vialoation scale in ISS models $M_{\rm DM} \simeq \Lambda(\mu) \sim \mathcal{O}\,(10\text{ keV})$ and $\Lambda(\mu)$ in LRSM fraemwork.

{See also Abada, Lucente, Nucl. Phys. B 885 (2014) 651 }

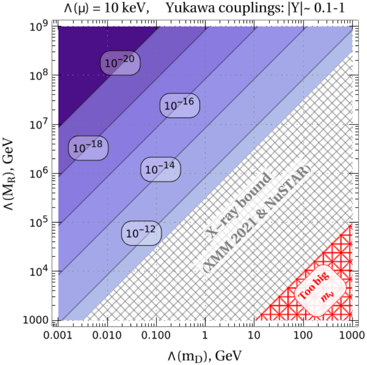
# X-ray bounds on DM mixing

$$\Gamma_{N_1\to 3\nu} = \frac{G_F^2 M_{\rm DM}^5}{96\pi^3}\mathbf{U}^2_{\rm DM},\quad \Gamma_{N\to\gamma\nu} = \frac{27\alpha_{\rm EM}}{4\pi}\Gamma_{N_1\to 3\nu},\quad \mathbf{U}^2_{\rm DM} \equiv \sum_{\alpha=e,\mu,\tau}|\mathbf{U}_{\alpha 4}|^2$$




Akita, Hamaguchi, Ovchynnikov 2507.20659 [hep-ph

$\sin^2 2\theta \equiv 4\mathbf{U}^2{}_{\rm DM}$ $\quad m_s \equiv M_{\rm DM}$

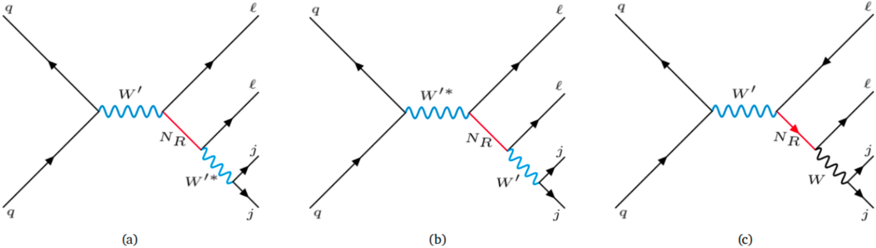



$$\mathbf{U}^2_{\rm DM} \approx Y^2_{\rm DM}\left(\frac{v_{\rm EW}}{v_R}\right)^2 \in [10^{-14}, 10^{-11}] \qquad m_\nu \approx Y^2\left(\frac{v_{\rm EW}}{v_R}\right)^2 \in [10^{-3}\text{ eV},\ 10^{-2}\text{ eV}]$$

For $\mu = 10$ keV, $v_R = 10$ TeV: $\quad 10^{-8} < \frac{Y_{\rm DM}}{Y} < 10^{-2}, \qquad 0.01 < Y < 0.1$

# Possible collider signatures [Phys. Rev. D 105, 115007]

{Fig. from [Arun et al. Phys. Rev. D 105, 115007]}




(a): $M_{W'} > M_N$ (b): $M_{W'} < M_N$ (c): $M_{W'} > M_N$

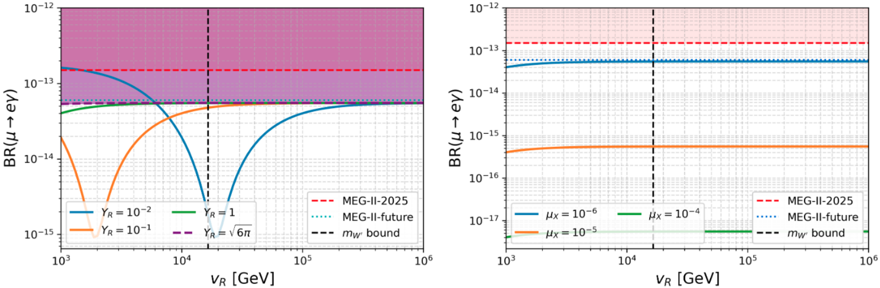
**KS-process (Keung-Senjanovic')**: {Phys. Rev. D 97, 115018}
Canonical KS-process is suppressed $\sigma_{\text{same}}/\sigma_{\text{oposite}} \sim \mu/v_R \simeq 10^{-9}$

- (Same sign): $pp \to W_R(W_R^*) \to l + N \to l + l + W_R \to l\ l\ q\ q$
- (Oposite-sign): $pp \to W_R(W_R^*) \to l + N' \to l + \bar{l} + W \to l\ \bar{l}\ q\ q'$ (Arun et al.)

**LFV:**

$$\mathrm{Br}(\mu \to e\gamma) < 1.5 \times 10^{-13} \qquad \text{[MEG-II (2025)]}$$

$$\mathrm{Br}(\mu \to e\gamma) < 6 \times 10^{-14} \qquad \text{[MEG-II future]}$$




{Fig. from arXiv:2508.01220 [hep-ph]}

# Conclusions

**High-energy motivation:**

- Low-scale phenomenology without unnatural small Yukawa interraction and/or extreme fine-tuning;
- Naturalness of small neutrino masses.
- Connection to perspective GUT-models such as $SO(10)$, $E_6$.
- Connection of model scale with cosmology warm dark matter mass

**Low-energy signatures:**

- Heavy Higgs scalars and heavy vector bosons $W'$ and $Z'$ with masses $\propto v_R \simeq 1 - 10$ TeV.
- $\mu \to e\gamma$ with $\mu \simeq 10$ keV and $v_R = 10$ TeV with $Y_R \sim 1$ is close to projected sensitivity of MEG-II.
- Oposite sign leptons and jet from boosted $W$: $pp \to W_R(W_R^*) \to l\ \bar{l}\ q\ q'$ - inverse seesaw probing
- X-ray DM bounds: if DM is light neutrino $N_{\rm light}$ and $v_R = 10$ TeV, then an additional hierarchy of Yukawa constants is required $Y_{\rm DM} \sim 0.001$ and $Y = 0.1$ (for dominant Yukawa constants in $m_\nu$).

# Thank you for your attention

**Acknowledgments:** This study was conducted within the scientific program of the National Center for Physics and Mathematics, section 5 «Particle Physics and Cosmology». Stage 2026-2027.

# BACKUP: Higgs potential

$$
\begin{aligned}
V(\chi_L, \chi_R, \Phi) \quad = \quad & -\mu_1^2 \mathrm{Tr}\,\Phi^\dagger\Phi - \mu_2^2\left(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R\right) \\
& +\lambda_1\left(\mathrm{Tr}\,\Phi^\dagger\Phi\right)^2 + \lambda_2\,\mathrm{Tr}\,\Phi^\dagger\Phi\Phi^\dagger\Phi \\
& +\frac{1}{2}\lambda_3\left(\mathrm{Tr}\,\Phi^\dagger\tilde{\Phi} + \mathrm{Tr}\,\tilde{\Phi}^\dagger\Phi\right)^2 + \frac{1}{2}\lambda_4\left(\mathrm{Tr}\,\Phi^\dagger\tilde{\Phi} - \mathrm{Tr}\,\tilde{\Phi}^\dagger\Phi\right)^2 \\
& +\lambda_5\,\mathrm{Tr}\,\Phi^\dagger\Phi\tilde{\Phi}^\dagger\tilde{\Phi} + \frac{1}{2}\lambda_6\left[\mathrm{Tr}\,\Phi^\dagger\tilde{\Phi}\Phi^\dagger\tilde{\Phi} + \text{ H.c.}\right] \\
& +\rho_1\left(\left(\chi_L^\dagger\chi_L\right)^2 + \left(\chi_R^\dagger\chi_R\right)^2\right) + \rho_2\chi_L^\dagger\chi_L\chi_R^\dagger\chi_R \\
& +\alpha_1\,\mathrm{Tr}\,\Phi^\dagger\Phi\left(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R\right) + \alpha_2\left(\chi_L^\dagger\Phi\Phi^\dagger\chi_L + \chi_R^\dagger\Phi^\dagger\Phi\chi_R\right) \\
& +\alpha_3\left(\chi_L^\dagger\tilde{\Phi}\tilde{\Phi}^\dagger\chi_L + \chi_R^\dagger\tilde{\Phi}^\dagger\tilde{\Phi}\chi_R\right)
\end{aligned}
$$

For details see also: {arXiv:2508.01220 [hep-ph]}