On α_s determination from KEDR and BESIII e^+e^- to hadrons data below thresholds of charm quark production

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based on arXiV:2603.39803 and work in progress

Quarks-2026 , Petrozavodsk, 19 May 2026

How to define QCD coupling constant α_s ?

The QCD Lagrangian is :

$$L_{QCD} = \sum_f \overline{\Psi}_f \big(i\gamma^\mu \partial_\mu - g \frac{\lambda^a}{2} \gamma^\mu A^a_\mu - m_f \big) \Psi_f - \frac{1}{4} F^a_{mu\nu} F^a_{\mu\nu} \qquad (1)$$

QCD "coupling constant"is $\alpha_s = \frac{g^2}{4\pi}$ in analogy with the **experimentally distinguished on-shell scheme QED fine coupling constant** $\alpha = \frac{e^2}{4\pi}$**=1/137.035999** where **e** is the MEASURABLE electromagnetic CHARGE of LEPTONS e^-, μ^-, τ^-.

But free **colored** quarks, gluons are not observed !!! Thus we are unable in QCD to 1) define physical on-shell scheme ; 2) measure (not extract, but measure) color charge **g** and thus theoretical parameter α_s.

In general α_s depends on how it is defined and what is the scale where it is defined even in the commonly used in QCD not on-shell , but $\overline{MS}$-scheme (the ingredient of the QCD analogy of the system of units SI).

How to define QCD coupling constant α_s ?

To define the expression for α_s notion of the renormalization group β-function and its theoretical expression are used . Within dimensional regularization with $D-4=2\epsilon$

$$\alpha_s^{bare}(\mu^2) = \mu^{2\epsilon} Z_g(\alpha_s(\mu^2))\alpha_s(\mu^2) \qquad (2)$$

$$\beta(\alpha_s) = lim_{\epsilon\to 0}\frac{1}{\pi}\mu^2\frac{\partial\alpha_s}{\partial\mu^2}|_{\alpha_s^{bare}\ \ fixed} = -\sum_{n\geq 0}\beta_n\left(\frac{\alpha_s}{\pi}\right)^{n+2}$$

Next step - to introduce renormalization invariant (!) but scheme-dependent set of parameters parameter Λ_{QCD} , namely $\Lambda^{(f)}_{\overline{MS}}$

$$ln\frac{\mu_2^2}{\mu_1^2} = \int_{\alpha_s(\mu_1^2)/\pi}^{\alpha_s(\mu_2^2)/\pi}\frac{dx}{\beta(x)} \ , \quad ln\frac{\mu^2}{\Lambda^2_{\overline{MS}}} = \int_{\delta}^{\alpha_s(\mu^2)/\pi}\frac{dx}{\beta(x)} \qquad (3)$$

Set of parameters: $\Lambda^{(f=3)}_{\overline{MS}}$, $\Lambda^{(f=4)}_{\overline{MS}}$, $\Lambda^{(f=5)}_{\overline{MS}}$; for different quark flavours

$$\beta_0 = 2.75 - 0.16667f \ ; \quad \beta_1 = 6.375 - 0.79167f \qquad (4)$$

$$\beta_2^{\overline{MS}} = 22.320 - 4.3689f + 0.094039f^2$$

$$\beta_3^{\overline{MS}} = 114.23 - 27.134f + 1.5824f^2 + 0.0058567f^3$$

$$\beta_4^{\overline{MS}} = 524.558 - 181.80f + 17.156f^2 - 0.22586f^3 - 0.0017993f^4 \ .$$

Define $a_s = \alpha_s/\pi$ as the solution of second equation in different orders of PT (next-to-leading order (NLO) ; next-to-next-to-leading order (NNLO or N^2LO) and further on by iterations in $1/(\beta_0 L)$=$1/(\beta_0 Ln(\mu^2/\Lambda^2_{\overline{MS}}))$

$$\begin{aligned} a_s^{NLO}(\mu^2) &= \frac{1}{\beta_0 L} - \frac{\beta_1 lnL}{\beta_0^3 L^2} \\ a_s^{NNLO}(\mu^2) &= a_s^{NLO}(\mu^2) + \Delta_{NNLO} \\ a_s^{N^3LO}(\mu^2) &= a_s^{NNLO}(\mu^2) + \Delta_{N^3LO} \\ a_s^{N^4LO}(\mu^2) &= a_s^{N^3LO}(\mu^2) + \Delta_{N^4LO} \quad . \end{aligned} \tag{5}$$

$$\begin{aligned} \Delta_{NNLO} &= \frac{\beta_1^2(L^2 - L - 1) + \beta_0\beta_2}{\beta_0^5 L^3} \quad , \\ \Delta_{N^3LO} &= \frac{\beta_1^3(-2L^3 + 5L^2 + 4L - 1) - 6\beta_0\beta_2\beta_1 L + \beta_0\beta_3}{2\beta_0^7 L^4} \\ \Delta_{N^4LO} &= \frac{18\beta_0\beta_2\beta_1^3(2L^2 - L - 1) + \beta_1^4(6L^4 - 26L^3 - 9L^2 + 24L + 7)}{6\beta_0^9 L^4} \\ &- \frac{\beta_0^3\beta_1\beta_3(12L + 1) - 10\beta_0^3\beta_2^2 - 2\beta_0^4\beta_4}{\beta_0^9 L^4} \end{aligned} \tag{6}$$

α_s may be defined both in the Eucledian an and Minkowski regions .

Extract α_s from comparison of QCD theoretical predictions for different observable processes at different energies with the available experimental data ; If different extractions do agree= Test of validity of QCD, of perturbative calculations; of experiment; If not agree- why ? Where are

the problems ? Not perfect calculations ? Not perfect experiment ? Or new Physical or theoretical not yet detected effects ?

Experiments for Deep Inelastic scattering (previously Protvino (ν N), SLAC (e N), CERN (μ N); Tevatron (ν N) DESY (e p) ; at present JLAB (eN) ; hopefully Brookhafen in future ; DIS is Eucledian processes $\alpha_s(Q_E^2)$

e^+e^- annihilation to hadrons process in the time-like Minkowski region : experimental data and $\alpha_s(s)$ extraction from data of SPEAR (Cornell, USA), DORIS (Germany), CESR (Cornell,USA), PETRA (Germany), PEP(USA), TRISTAN (Japan), SLC(USA), LEP (France/Switzerland)

Question: whether the analysis of the intermediate energy data of VEPP-4M (Novosibirsk) and BEPC(Bejing) can add any new information on the value and behavior of α_s ??? Whether the data of these machines is more precise ? Is is possible to get impression on the new theoretical effects by the analysis of these data ?
Hopefully yes .

Example of comparison of α_s values in 1994

A.L.Kataev; QCD summary talk at second RADCOR-1994; USA ; hep-ph/9410308; Essential difference with PDG-2024= both experiment and theory became more precise and we are now interested in 4-th significant digit : PDG-2024 World average is $\alpha_s(M_Z) = 0.1180 \pm 0.0009$. Possible new theory uncertainties may exist.

running and pole masses result in the appearance of an additional contribution, which has the following approximate form:

$$m_q(m_q^p) \approx m_q^p - \frac{2}{3\beta_0}\Lambda_{QCD} \qquad (7)$$

where β_0 is the first coefficient of the QCD β-function, defined as $\beta(\alpha_s) = \mu^2\partial a/\partial\mu^2$ ($a = \alpha_s/\pi$), and Λ_{QCD} is the QCD scale parameter in some non-fixed scheme. Therefore, the pole mass is not defined to an accuracy better than Λ_{QCD} within perturbation theory [30]. The indirect indication of this problem comes from detailed studies [31] of the effects of the QCD corrections to $\Gamma(H^0 \to b\bar{b}) = \Gamma_{Hb\bar{b}}$ [21] in the case when the value of m_b^p is considered as the input parameter and the RG-imroved version of the relation of Eq. (6) is used to sum up the RG- controllable $\log(M_H/m_b^p)$-terms and to present the final expression for $\Gamma_{Hb\bar{b}}$ in terms of $m_b(M_H)$.

Indeed, it was observed [31] that for reasonably large values of the Higgs boson mass M_H the corresponding NNLO corrections to the ratio $R_{Hb\bar{b}} = \Gamma_{Hb\bar{b}}/\Gamma_0^{(b)}$ (where $\Gamma_0^{(b)} = 3\sqrt{2}/(8\pi)G_F M_H(m_b^p)^2$) are larger than the NLO ones. This observation was further considered as an indication of the asymptotic explosion of the corresponding NNLO approximation, which is related to the one of Eq. (6). The latter feature might indicate the importance of the careful treating of the renormalon contributions to Eq. (6) responsible for the asymptotic structure of the corresponding perturbative relation. Now we can convince ourselves that in order to avoid the problem of the study of the asymptotic behaviour of the perturbative series related to Eq. (6) it might be better to use, in the phenomenological considerations, the value of $m_b(\mu \approx M_\Upsilon/2 \approx m_b^p)$, evolve it using the RG equation to any high-energy scale, and then to study the scheme dependence of the corresponding results. This point of view is in agreement with the one of Marciano [32] and with the studies of the dependence of the results of calculations of the QCD corrections to the ρ-parameter on the definition of the top-quark mass [33]. An analogous observation came previously from the results of calculations of the $O(m_b^2/M_Z^2)$ corrections to $\Gamma(Z^0 \to \text{hadrons})$ [34]. Note that a definite attempt to estimate the renormalon-type contributions in Eq. (7) was recently made [35]. This analysis was based on the following values of the c-quark and b-quark running masses in the $\overline{MS}$ scheme: $m_c(m_c^p) = 1.23^{+0.02}_{-0.04} \pm 0.03$ GeV, $m_b(m_b^p) = 4.23^{+0.03}_{-0.04} \pm 0.02$ GeV. The numerical values of the renormalon contributions Δm_q to Eq. (7) were estimated to be $\Delta m_c \approx 30 \pm 20$ MeV and $\Delta m_b \approx 70$ MeV [35]. However, we think that this phenomenological analysis is only the first step in the direction of future, more

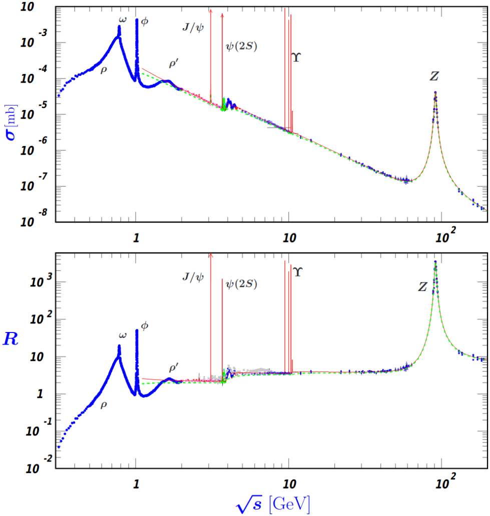
Process	Q [GeV]	$\alpha_s(Q)$	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s(M_{Z^0})$ exp.	$\Delta\alpha_s(M_{Z^0})$ theor.	Theory
DIS [e,μ; Bj-SR]	1.58	$0.375^{+0.062}_{-0.081}$	$0.122^{+0.005}_{-0.009}$	–	–	NNLO
DIS [ν; GLS-SR]	1.73	0.32 ± 0.05	0.115 ± 0.006	0.005	0.003	NNLO
R_τ [LEP]	1.78	0.360 ± 0.040	0.122 ± 0.005	0.002	0.004	NNLO
DIS [ν; F_2 *and* F_3]	5.0	$0.193^{+0.019}_{-0.018}$	0.111 ± 0.006	0.004	0.004	NLO
DIS [μ; F_2]	7.1	0.180 ± 0.014	0.113 ± 0.005	0.003	0.004	NLO
$Q\overline{Q}$ states	5.0	0.188 ± 0.018	0.110 ± 0.006	0.000	0.006	LGT
$J/\Psi + \Upsilon$ decays	10.0	$0.167^{+0.015}_{-0.011}$	$0.113^{+0.007}_{-0.005}$	0.001	$^{+0.007}_{-0.005}$	NLO
e^+e^- [σ_{had}]	34.0	$0.146^{+0.031}_{-0.026}$	$0.124^{+0.021}_{-0.019}$	$^{+0.021}_{-0.019}$	–	NNLO
e^+e^- [ev. shapes]	35.0	0.14 ± 0.02	0.119 ± 0.014	–	–	NLO
e^+e^- [ev. shapes]	58.0	0.132 ± 0.008	0.123 ± 0.007	0.003	0.007	resum
$p\bar{p} \to b\bar{b}X$	20.0	$0.138^{+0.028}_{-0.019}$	$0.109^{+0.016}_{-0.012}$	$^{+0.012}_{-0.007}$	$^{+0.011}_{-0.010}$	NLO
$p\bar{p},\ pp \to \gamma X$	24.2	$0.137^{+0.017}_{-0.014}$	$0.112^{+0.012}_{-0.008}$	0.006	$^{+0.010}_{-0.005}$	NLO
$p\bar{p} \to W$ *jets*	80.6	0.123 ± 0.025	0.121 ± 0.024	0.017	0.016	NLO
$e^+e^- \to Z^0$:						
$\Gamma(Z^0 \to had.)$	91.2	0.126 ± 0.007	0.126 ± 0.007	0.006	$^{+0.003}_{-0.004}$	NNLO
had. event shapes	91.2	0.119 ± 0.006	0.119 ± 0.006	0.001	0.006	NLO
had. event shapes	91.2	0.123 ± 0.006	0.123 ± 0.006	0.001	0.006	resum.

Table 1. World summary of measurements of α_s. Abbreviations: DIS = deep-inelastic scattering; GLS-SR = Gross-Llewellyn-Smith sum rules; Bj-SR = Bjorken sum rules; LGT = lattice gauge theory; resum. = resummed NLO. Reference: S. Bethke, *Proc. of the QCD'94*, Montpellier, France, July, 1994; Aachen preprint PITHA-94-30. The comments on the first result and its uncertainties are presented in Section 4.

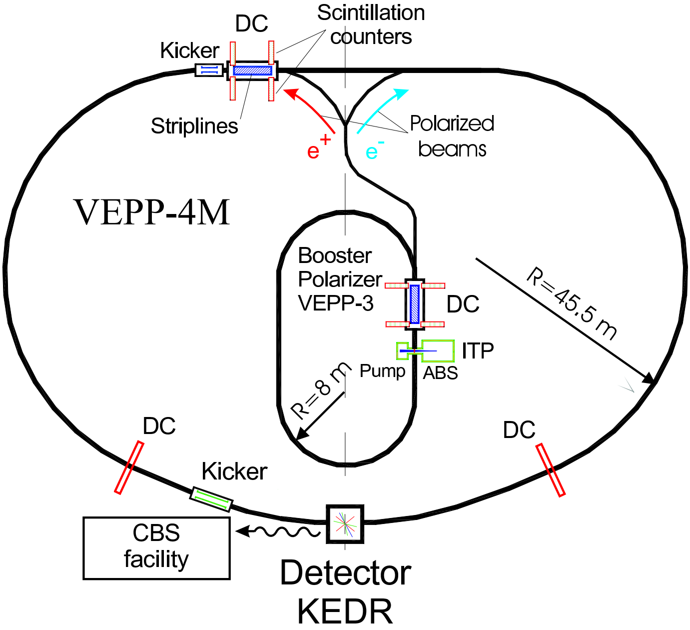
Of course, the application of the $\overline{MS}$ scheme represents the example of the phe- … d experimentalists. Indeed, it does … due to the existence of the scale- … osium by Brodsky … However, it

Experimental motivation: from PDG-2021

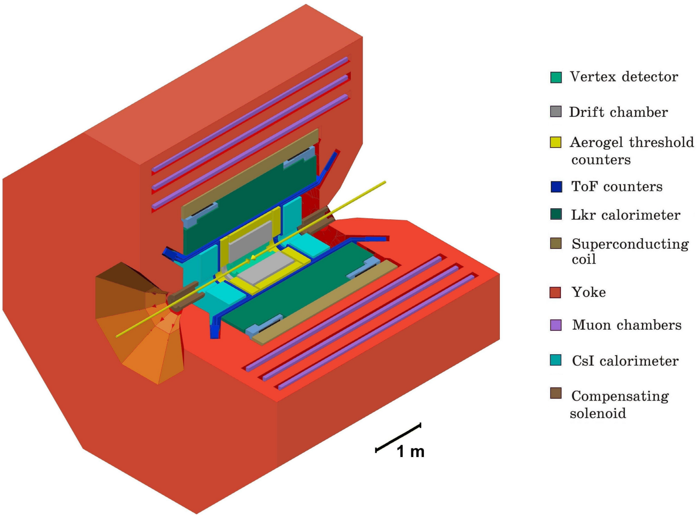
$$R(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma, Z^0 \to hadrons)}{\sigma_0 = 4\pi^2\alpha^2/3s = \sigma_0(e^+e^- \to \mu^+\mu^-)} \quad (7)$$



Experimental motivation



Experimental motivation



Experimental motivation

BEPCII @ IHEP

$E_{\mathrm{cm}} = 2 - 4.95$ GeV

2004, started construction

2009-2024, BESIII Physics run

Design Luminosity:

$\mathcal{L}_D = 1 \times 10^{33}\mathrm{cm}^{-2}s^{-1}$@$E_{\mathrm{cm}}$= 3.773 GeV

Peak luminosity:

2016 achieved $1.0 \times \mathcal{L}_D$

2023 achieved $1.1 \times \mathcal{L}_D$

Jul. 1, 2024 – Aug. 31, 2028:

BEPCII upgrade → BEPCII-U

January 2025, restart

- Luminosity $\times$ 3 @E_{cm}= 4.7 GeV
- Beam energy up to 2.8 GeV (2028)

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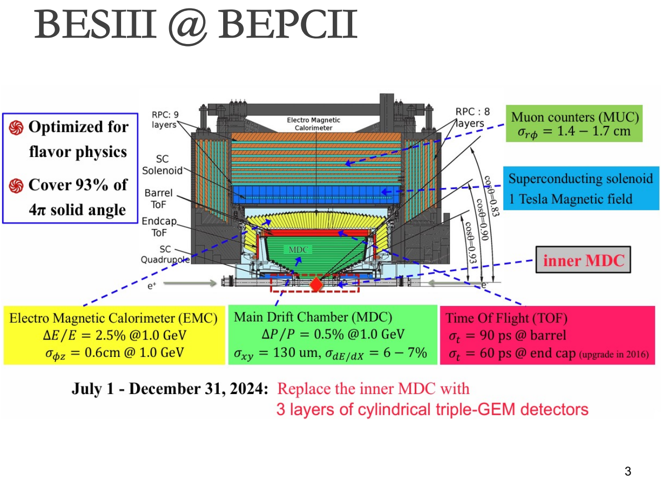
From Bai-Cian Ke BESIII experimental related talk at NP RAS Division Session of 2026 (Novosibirsk, March 2026)

Experimental motivation



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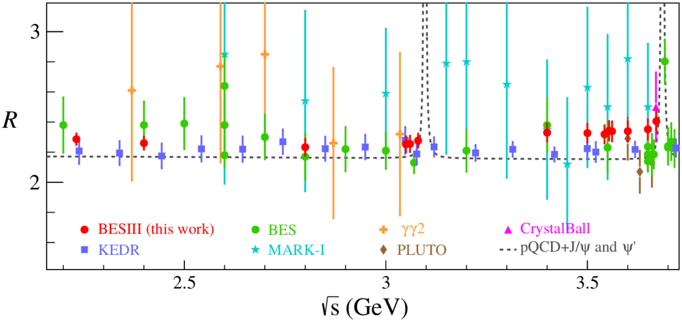
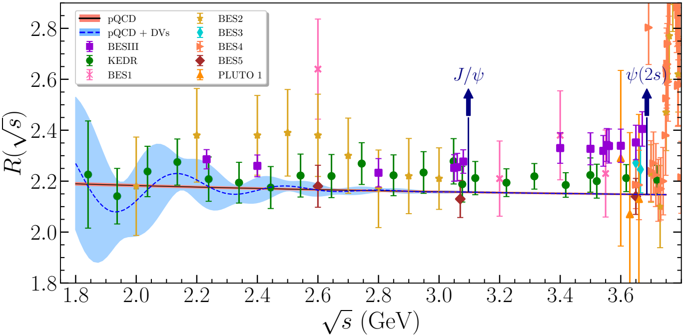
Experimental motivation



From M. Ablikim et al. (BESIII Collaboration) Phys. Rev. Lett. 128, 062004 (2022)

New BESIII data (190 data points) between 2.0 GeV and 4.95 Gev should appear soon (Guangshan Huang- private communication to A.K.)
New KEDR data data, but in higher energy region from 4.56 GeV to 6.96 GeV (17 points) are in the process of preparations for final publication ; T.Kharlamova, talk at NP Division Session of RAS (10.03.2026,Novosibirsk)

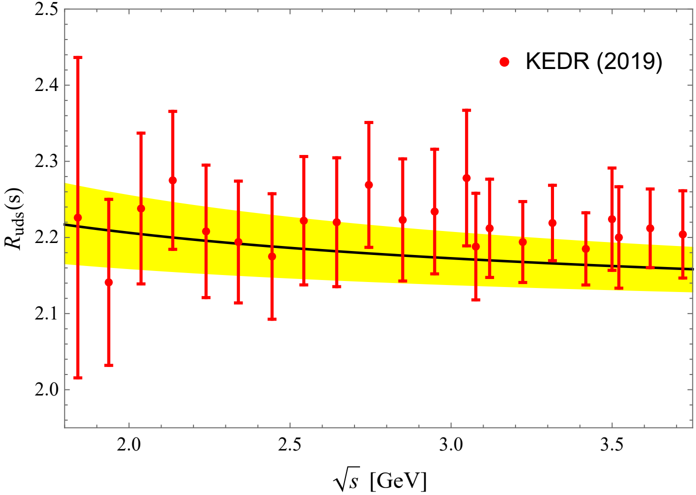
Existing QCD analysis of BESIII and KEDR data



D.Boito and M.Caram, Phys. Rev D 112 (2025) 9, 094052

N^3LO QCD analysis ; But the value of $\alpha_s(M_Z)$ fixed at PDG world average value and then evolved to energies of KEDR and BESIII data; Aim- stuidy of the possibility to detect duality violation effects (possibility and significance of existing in theory in case of hadronic τ-decays: hot discussions between Barcelona+Boito group (Boito) and Valencia group (A.Pich))). Tension between KEDR and BESIII data (in favour of KEDR data)

Existing QCD analysis of KEDR data



Jian-Ming Shen et al. JHEP07(2023)109; PMC (principle of maxiomal conformality) related extraction of $\alpha_s(M_Z)$. Theory uncertainties are essentially underestimated: Kataev and Mikhailov (2015 - 2025)

Perturbative QCD available results

Aim of our study: to make independent fits QCD of KEDR and BESIII data with the aim **to extract** $\alpha_s(M_Z)$ **values**.

$$R^{th}(s) = 3\sum Q_f^2\Bigg[F_0(\frac{m_f^2}{s}) + F_1(\frac{m_f^2}{s})\frac{\alpha_s(s)}{\pi} + F_2(\frac{m_f^2}{s})\left(\frac{\alpha_s(s)}{\pi}\right)^2 \qquad (8)$$
$$+ \quad F_3(\frac{m_f^2}{s})\left(\frac{\alpha_s(s)}{\pi}\right)^3 + F_4(0)\left(\frac{\alpha_s(s)}{\pi}\right)^4 + F_5^{est}(0)\left(\frac{\alpha_s(s)}{\pi}\right)^5 + \ldots\Bigg]$$
$$+\left(\sum_f Q_f\right)^2\Bigg[\Phi_3(\frac{m_f^2}{s})\left(\frac{\alpha_s(s)}{\pi}\right)^3 + \Phi_4(0)\left(\frac{\alpha_s(s)}{\pi}\right)^4 + \Phi_5^{est}(0)\left(\frac{\alpha_s(s)}{\pi}\right)^5$$

Non-perturbative effects, considered in lower e^+e^- energies of $\sqrt{s} \leq 2\ GeV$ (preregative of SND and CMD Novosibirsk Collaborations working at VEP-2000) and considered previously **WITHOUT HIGH-ORDER PT QCD EFFECTS** (Eidelman,Kurdadze,Vainshten (79) and Grozin,Pinelis (86-88)) are neglected.

R-ratio QCD

$$F_0(\frac{m_f^2}{s}) = v_f(3 - v_f^2)/2; with \quad v_f = \sqrt{1 - 4\frac{m_f^2}{s}} \quad , \quad F(0) = 1 \qquad (9)$$

$$F_1(\frac{m_f^2}{s}) \approx (Shwinger) \approx \frac{4}{3}\left[\frac{\pi^2}{2v_f} - \frac{1}{4}(3+v_f)\left(\frac{\pi^2}{2} - \frac{3}{4}\right)\right]F_0(v_f) \ , \ F_1(0) = 1 \qquad (10)$$

$F_2(\frac{m_f^2}{s})$ is known up to $O(\frac{m_f^2}{s})$ expansion thanks to Gorishny,Kataev,Larin (86) and Chetyrkin,Kuhn,Steinhauser up to higher order terms (1995-1997). $F_3(\frac{m_f^2}{s})$ up to $O(\frac{m^4}{s^2})$ terms- Chetyrkin,Harlander,Kuhn(2000)

$$F_2(0) = 1.9857 - 0.1152f; Chetyrkin, Kataev, Tkachov(79)$$
$$F_3(0) = -6.63694 - 1.20013f - 0.00518f^2; Gorishny, Kataev, Larin(88; 91)$$
$$F_4(0) = -156.61 + 18.77f - 0.7974f^2 + 0.0215f^3,$$
$$F_5(0) \approx -505; -134; 168; Baikov, Chetyrkin, Kuhn(2008) + Rittinger(2012)$$

There are the number of high-order estimates ; should be studied with care ; aim for DIRECT analytical or numerical calculations

Fit procedure; Todyshev+Kataev arXiV: 2603.39803

Modifications of the fitting program, previously used in the analysis of KEDR collaboration data (Phys.Lett.B 753 (2016) 533; arXiV: 1510.02667 [hep-ex])

$$\chi_0^2 = \sum_i \sum_j \Big(R^{exp}(s_i) - R^{th}(s_i) \Big) C_{ij}^{-1} \Big(R^{exp}(s_j) - R^{th}(s_j) \Big) \qquad (11)$$

where $R^{exp}(s_i)$ and $R^{th}(s_i)$ are the measured and theoretically defined values of the R ratio , fixed at the concrete energy points, and C_{ij}^{-1} are the coefficients of the inverse covariance matrix. New definition of χ^2 following combined fits of e^+e^- to hadrons previous data Marshall (1988-1989)

$$\chi_1^2 = \frac{(\nu-1)^2}{\nu^2\sigma_0^2} + \frac{1}{\nu^2} \sum_i \sum_j \Big(\nu R^{exp}(s_i) - R^{th}(s_i) \Big) C_{ij}^{-1} \Big(\nu R^{exp}(s_j) - R^{th}(s_j) \Big) \qquad (12)$$

where ν is a free normalization parameter, which reflects, to a certain extent, how much, on average, the experimental data differ from the theoretical dependence over the entire energy range under consideration.

Results of combined fits of KEDR and truncated BESIII data: Kataev,Todyshev arXiV: 2603.39803

R(s) approximation	$O(\alpha_s^2)$	$O(\alpha_s^3)$	$O(\alpha_s^4)$	$O(\alpha_s^5)$ estimate
$(\chi_1^2(BESIII) + \chi_1^2(KEDR))/ndf$	11.445/25	10.958/25	9.433/25	10.419/25
$\Lambda_{\overline{MS}}^{(f=3)}$, MeV	374^{+101}_{-113}	420^{+131}_{-136}	618^{+68}_{-162}	518
ν (BES)	0.966 ± 0.019	0.969 ± 0.020	0.964 ± 0.013	0.955
ν (KEDR)	0.990 ± 0.012	0.992 ± 0.013	0.9946 ± 0.013	0.990
$\alpha_s(m_\tau)$	$0.3187^{+0.0564}_{-0.0556}$	$0.3667^{+0.0836}_{-0.0725}$	$0.5201^{+0.0769}_{-0.1305}$	0.4203
$\alpha_s(M_Z)$ (NLO) $\Lambda_{\overline{MS}}^{(f=5)}$, MeV	$0.1176^{+0.0052}_{-0.0071}$ 254^{+96}_{-94}			
$\alpha_s(M_Z)$ $(NNLO)$ $\Lambda_{\overline{MS}}^{(f=5)}$, MeV		$0.1215^{+0.0063}_{-0.0080}$ 254^{+96}_{-94}		
$\alpha_s(M_Z)$ (N^3LO) $\Lambda_{\overline{MS}}^{(f=5)}$, MeV			$0.1310^{+0.0027}_{-0.0075}$ 401^{+53}_{-122}	
$\alpha_s(M_Z)$ (N^4LO) $\Lambda_{\overline{MS}}^{(f=5)}$, MeV				0.1265 325

Why truncated ? - There is indeed tension between KEDR and whole set of BESIII data (known to BESIII collaboration members (2022) and Boito,Caram (2025)) 7 points excluded- see below.

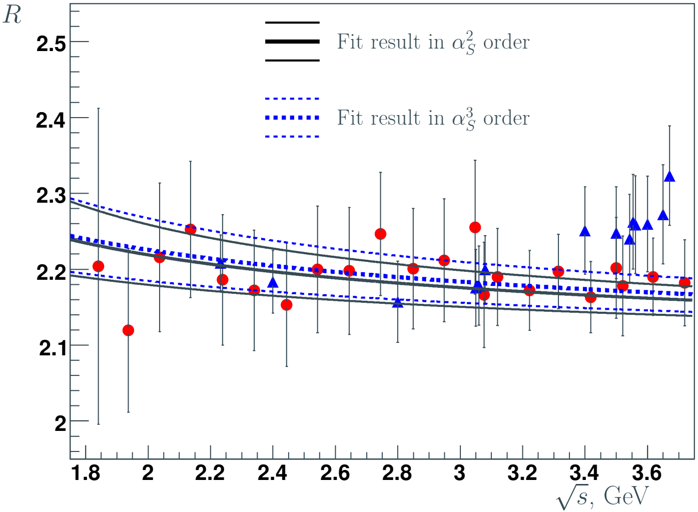
Physical outcomes: agreement with world average within systematical uncertainties; however growth of central values of $\alpha_s(M_Z)$!!!

Confirmation of troubles of BESIII data

R(s) approximation	$O(\alpha_s^2)$	$O(\alpha_s^3)$	$O(\alpha_s^4)$	$O(\alpha_s^5)$ estima
$\chi_0^2(BESIII)/ndf$	53.723/13	53.603/13	53.484/13	53.386/13
$\Lambda^{(f=3)}_{\overline{\rm MS}}$, MeV	184^{+65}_{-63}	607^{+71}_{-66}	573^{+92}_{-78}	229
$\chi_1^2(BESIII)/ndf$	51.183/12	51.176/12	51.166/12	53.386/12
$\Lambda^{(f=3)}_{\overline{\rm MS}}$, MeV	19^{+102}_{-18}	18^{+104}_{-17}	20^{+118}_{-19}	20
ν	0.96 ± 0.02	0.96 ± 0.02	0.96 ± 0.02	0.96

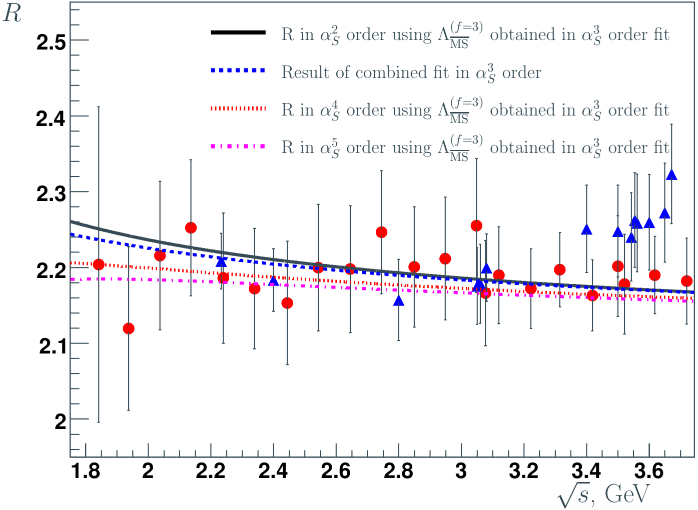
R(s) approximation	$O(\alpha_s^2)$	$O(\alpha_s^3)$	$O(\alpha_s^4)$
$\chi_0^2(BESIII)/ndf$ (6 points)	8.128/5	7.377/5	12.596/5
$\Lambda^{(f=3)}_{\overline{\rm MS}}$, MeV	527^{+59}_{-67}	607^{+84}_{-89}	573^{+67}_{-86}
$\alpha_s(m_\tau)$	$0.4087^{+0.0425}_{-0.0418}$	$0.488^{+0.0781}_{-0.0665}$	$0.478^{+0.0654}_{-0.0678}$
$\alpha_s(M_Z)$ (NLO) $\Lambda^{(f=5)}_{\overline{\rm MS}}$, MeV	$0.1253^{+0.0026}_{-0.0031}$ 333^{+44}_{-49}		
$\alpha_s(M_Z)$ $(NNLO)$ $\Lambda^{(f=5)}_{\overline{\rm MS}}$, MeV		$0.1306^{+0.0034}_{-0.0040}$ 393^{+65}_{-68}	
$\alpha_s(M_Z)$ (N^3LO)			$0.1291^{+0.0029}_{-0.0041}$ 367^{+52}_{-65}

Comparison of PT QCD results with data at NLO and NNLO



The result of the joint fit of the KEDR and truncated BESIII experimental data.

$R(s)$ in different orders of pQCD and troubles of N^3LO PT QCD approxiamtion



$R(s)$ curves obtained in different orders of perturbative QCD with $\Lambda_{\overline{MS}}^{(3)}$ fixed by the result of joint fit of KEDR and the truncated BESIII data.

Why growth of α_s ?

$$R_{uds}(s) = 3\sum_{q=u,d,s} Q_q^2\left[1 + r_1 a_s(s) + \sum_{k\geq 2} r_k(f) a_s(s)^k\right]$$

$$D(Q^2) = 3\sum_{q=u,d,s} Q_q^2\left[1 + d_1 a_s(Q^2) + \sum_{k\geq 2} d_k(f) a_s(Q^2)^k\right]; r_k(f) = d_k(f) - \Delta_k(f) \quad .$$

$$\Delta_1 = \Delta_2 = 0; \Delta_3(f) = \frac{\pi^2\beta_0^2}{3} d_1; \Delta_4(f) = \pi^2\left[\beta_0^2 d_2(f) + \frac{5}{6}\beta_0\beta_1 d_1\right]$$

$$\Delta_5(f) = \pi^2\left[2\beta_0^2 d_3(f) + \frac{7}{3}\beta_0\beta_1 d_2(f) + \frac{1}{2}\beta_1^2 d_1 + \beta_0\beta_2 d_1\right] - \frac{\pi^4}{5}\beta_0^4 d_1$$

$$R(s) = 2\Big[1 + a_s + \underline{1.6398} a_s^2 + \left(\underline{6.3710} - \underline{\underline{16.5550}}\right) a_s^3$$

$$+ \left(\underline{49.076} - \underline{\underline{155.9555}}\right) a_s^4 + \left(\underline{\sim 275} - \underline{\underline{779.581}}\right) a_s^5 + O(a_s^6)\Big] \quad .$$

Different structure of the asymptotic PT series in Eucledian and Minkowski regions Kataev,Starshenko (1995); In Eucledian- n! growth- In Minkowski- it is modified by the analytical continuation effects;

$$R(s)|_{s=m_\tau^2} = 2\left[1 + 0.1164 + 0.0221 - 0.0161 - 0.0195 - 0.0107 + \ldots\right] \quad (13)$$

Massive dependence effects- Kataev,Todyshev -preliminary

In principle these effects can be summed up within APT techinque of Shirkov-Solovtsov and related π^2 resummations (Krasnikov,Pivovarov) Low energy experiments in e^+e^- machines data already are sensitive to these effects; As previouslu R_τ-data of LEP. Though starnge quark mass dependence should be considerd

R(s) approximation	$O(\alpha_s^2)$	$O(\alpha_s^3)$	$O(\alpha_s^4)$	$O(\alpha_s^5)$ estimate
$(\chi_1^2(BESIII) + \chi_1^2(KEDR))/ndf$ $m_s = 0.0935$, GeV	11.524/25	11.129/25	9.315/25	10.099/25
$\Lambda_{\overline{\rm MS}}^{(f=3)}$, MeV	366^{+100}_{-112}	168^{+125}_{-131}	638^{+62}_{-160}	545
ν (BES)	0.966 ± 0.019	0.968 ± 0.020	0.966 ± 0.013	0.957
ν (KEDR)	0.990 ± 0.012	0.992 ± 0.013	0.997 ± 0.011	0.992
$\alpha_s(m_\tau)$	$0.3148^{+0.0554}_{-0.0548}$	$0.3506^{+0.0772}_{-0.0689}$	$0.5407^{+0.0753}_{-0.0136}$	0.4386
$\alpha_s(M_Z)$ (NLO) $\Lambda_{\overline{\rm MS}}^{(f=5)}$, MeV	$0.1172^{+0.0053}_{-0.0071}$ 216^{+72}_{-76}			
$\alpha_s(M_Z)$ $(NNLO)$ $\Lambda_{\overline{\rm MS}}^{(f=5)}$, MeV		$0.1216^{+0.0063}_{-0.0081}$ 241^{+91}_{-90}		
$\alpha_s(M_Z)$ (N^3LO) $\Lambda_{\overline{\rm MS}}^{(f=5)}$, MeV			$0.1319^{+0.0025}_{-0.0072}$ 417^{+49}_{-121}	
$\alpha_s(M_Z)$ (N^4LO) $\Lambda_{\overline{\rm MS}}^{(f=5)}$				0.1278 346

Conclusion

We have demonstrated that the direct fits of KEDR and BESIII collaboration data for the e^+e^- hadrons R ratio allow one to get reasonable values of $\alpha_s(M_Z)$.

The new values of $\alpha_s(M_Z)$, extracted by us while applying the NLO and NNLO approximations

A certain tendency of growth of $\alpha_s(M_Z)$ values starting from NLO to N^3LO orders of applied perturbative expansions is detected.

Experimental data in the Minkowski region are precise enough to feel effects of transformation from Euclidean to Minkowski regions !

Possible further studies

- New Data of BESIII -170 data points (work in progress)
- New Data of KEDR (work in progress)
- The Data of SND (exist, but not anbalysed)
- Super c- τ-factory (?)

- Effects of masses of s- and c-quarks
- Duality violation effects
- Application of APT (Shirkov-Solovstsov) or CIPT; Bakulev,Mikhailov,Stefanis formula exist
- New QCD tasks for VEPP-... related experimental machines
- Holographical QCD vs PT QCD related definitions of α_s
- New high-order calculations α_s^5

QCD : NLO = Chetyrkin,Kataev,Tkachov (1979); NNLO Gorishny,Kataev,Larin (1987-1988;1991); N^3LO Baikov,Chetyrkin,Kuhn (2008)

Who is next ?