On deformations of the Yang—Mills theory cutoff + background field

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The presentation is based on recent work: see PDMI Preprint — 05/2026

http://www.pdmi.ras.ru/preprint/2026/26-05.html

QUARKS — 2026

Content

1) **Background field method**

2) **Regularization**

3) **The main aim**

4) **Results**

5) **Discussions**

Background field method

Scalar models:

which one?

$$\int \mathscr{D}a \, \exp\Big(- S_{\mathrm{cl}}[B + a] + (J, a) \Big)$$

Way to get generating functional:
B.DeWitt (1967)
G.'t Hooft (1976)
L.Abbott (1982)
I.Aref'eva, A.Slavnov, L.Faddeev (1974)
I.Jack, H.Osborn (1982)

Gauge theories:

$$\int \mathscr{D}a \, \exp\Big(- S_{\mathrm{cl}}[B + a] + \cancel{(J, a)} \Big) \times \det\Big(\delta G / \delta\alpha \,|_{\alpha=0} \Big) \delta\big(G(\phi) \big)$$

But G may depend on B without the combination $B + a$.

$\downarrow$

Problems may occur after regularization.

Further we follow Faddeev's plan see arXiv 2013

Our case

We use the gauge condition $\left(\partial_{x^\mu}\delta^{ac} + f^{adc}e^d_\mu(x)\right)a^c_\mu(x) = 0.$ **Here** $e^a_\mu \sim B^a_\mu$ **!**

$\mathfrak{D} = \partial + e$
$D = \partial + B$

Under integration: $S_{\mathrm{cl}}[B+ga] + S_{\mathrm{gf}}[a,e] + S_{\mathrm{g}}[B,a,c,\bar{c},e]\,.$

$\xi = 1$

Yang, Mills (1954)
Faddeev, Popov (1967)
Slavnov, Faddeev (1978)
…

$$S_{\mathrm{cl}} = \frac{1}{4g^2}\int \mathrm{d}x\, F^2 \qquad S_{\mathrm{gf}} = \frac{1}{2}\int \mathrm{d}x\,\left(\mathfrak{D}a\right)^2 \qquad S_{\mathrm{g}} = \int \mathrm{d}x\left(\bar{c}\left(-\mathfrak{D}D\right)c + g(\mathfrak{D}\bar{c})(ac)\right)$$

Let W **denote regularized strongly connected action on the diagonal** $e^a_\mu = B^a_\mu\,.$

Then $W = W[B,e,reg(B)]\,|_{e=B}$, **and we get:**

$$\frac{\mathrm{d}W}{\mathrm{d}B} = \frac{\partial W}{\partial B} + \frac{\partial W}{\partial e}\bigg|_{e=B} + \frac{\partial reg(B)}{\partial B}\frac{\partial W}{\partial reg(B)}\,.$$

density from equation **dependence on gauge** **dependence on reg.**

for scalar models

it is easy to satisfy

$$\frac{\mathrm{d}W}{\mathrm{d}B} = \frac{\partial W}{\partial B}$$

see dim. reg. or

cutoff by Iv.-Kharuk

Regularizations

1) dimensional regularization — Bollini, Giambiagi (1972) 't Hooft (1972) …

2) higher covariant derivatives — Slavnov (1971) Bakeyev, Slavnov (1996) Stepanyanz (2020) …

3) implicit — Brizola, Battistel, Sampaio, Nemes, Cherchiglia (recent) …

4) Feynman and Pauli—Vallars — Feynman (1948) Pauli, Villars (1949) Bogolyubov, Shirkov (1980) …

5) cutoff — Polchinski, Shatashvili, Oleszczuk, Liao, Cynolter, Lendvai, Bagaev …

….

Our choice!
See Iv., Kharuk, Akacevich, Korenev (2019-present)

the main idea

a) strong deformation: $a^b_\mu(x) \to a^{\Lambda,b}_\mu(x) = \int_{\mathrm{B}_{1/2}} \mathrm{d}^4y\, \omega(|y|) a^b_\mu(x+y/\Lambda) \longleftrightarrow$ applying of $\Omega\big(-\partial^2/\Lambda^2\big)$

b) weak deformation: applying of $\Omega\big(-D^2/\Lambda^2\big) \longleftarrow$ invariant under gauge tr. of B^a_μ.

The model is nonlocal (quasi-local). See, for example, Efimov's works.

Strong case

not depend on B^a_μ

ln det

two loops

Action on diagonal: $$W^{\rm sc}_{\rm ren}[B,B,reg] = \frac{1}{4g^2_{\rm ren}} W_{-1}[B] + W^\Lambda_0[B] + g^2_{\rm ren} W^\Lambda_1[B] + \mathcal{O}\left(g^4_{\rm ren}\right)$$

1) $W^\Lambda_0 \implies$ **coefficient** $a_0(\Lambda) = \dfrac{11 c_2 L}{24\pi^2} + \tilde{a}_0$ ⟵ **standard value** Gross, Wilczek (1973) Politzer (1973)

counter-vertex $\sim \Lambda^2 S_2$ ⟵ **power singularity (mass or measure?)**

2) $W^\Lambda_1 \implies$ **coefficient** $a_1(\Lambda) = \dfrac{c_2^2 L}{(4\pi)^4}\left(26 + 2^4 17 \rho_2/3 - 2^7 \rho_4\right) + \tilde{a}_1$ ⟵ **second value is different** Caswell (1974) Jones (1974) Abbott (1981)

counter-vertex $-\left(\dfrac{5Lc_2}{48\pi^2} + \tilde{b}_1\right) S_{\rm gf}$ ⟵ **standard logarithmic part** Jack, Osborn (1982)

counter-vertex $\sim \Lambda^2 (L + a) S_2$ ⟵ **power singularity**

lin. comb. $\left(L + \tilde{d}_i\right) \times W^i_{-1}[B]$ ⟵ **split action**

violation of gauge invariance

First variation: $$\left(\partial_s W^{\rm sc}_{\rm ren}[B, B + s\varepsilon, reg]\right)\Big|^{\rm 1st\ loop}_{s=0} \implies \Gamma_1[\varepsilon]\left(\frac{c_2 L}{2\pi^2} + \tilde{b}_0\right)$$ ⟵ **standard value**

All results are consistent with Ivanov, Kharuk (2020, 2022)

Weak case

Gauge invariance for B_μ^a is restored! See Ivanov (2022)

depends on B_μ^a

Action on diagonal: $W^{sc}_{ren}[B,B,reg(B)] = \frac{1}{4g^2_{ren}} W_{-1}[B] + W^\Lambda_0[B] + g^2_{ren} W^\Lambda_1[B] + \mathcal{O}\left(g^4_{ren}\right)$

1) $W^\Lambda_0 \implies$ **coefficient** $a_0(\Lambda) = \frac{11 c_2 L}{24\pi^2} + \tilde{a}_0$ $\longleftarrow$ **standard value** Gross, Wilczek (1973) Politzer (1973)

2) $W^\Lambda_1 \implies$ **coefficient** $a_1(\Lambda) = \frac{c_2^2 L}{(4\pi)^4}\left(175/9 + 2^4 7\rho_2/3 - 2^7\rho_4\right) + \tilde{a}_1$ $\longleftarrow$ **this value is different** Caswell (1974) Jones (1974) Abbott (1981)

counter-vertex $-\left(\frac{5Lc_2}{48\pi^2} + \tilde{b}_1\right) S_{gf}$ $\longleftarrow$ **standard logarithmic part** Jack, Osborn (1982)

counter-vertex $\sim \Lambda^2 S_2$ $\longleftarrow$ **power singularity (mass or measure?)**

First variations: $\left(\partial_s W^{sc}_{ren}[B, B+s\varepsilon, reg(B)]\right)\Big|^{\text{1st loop}}_{s=0} \implies \Gamma_1[\varepsilon]\left(\frac{c_2 L}{2\pi^2} + \tilde{b}_0\right)$ $\longleftarrow$ **standard value**

$\left(\partial_s W^{sc}_{ren}[B, B, reg(B+s\varepsilon)]\right)\Big|^{\text{1st loop}}_{s=0} \implies 0$ $\longleftarrow$ **standard value**

All results are consistent with Ivanov, Kharuk (2020, 2022)

But what in the next loops?

Master-integrals

Cutoff regularization			Dimensional regularization	
Int.$\times(4\pi)^4$	Main part	Correction	Int.$\times(4\pi)^4\sigma^{2\varepsilon}$	Singularity
$\hat{\mathrm{I}}_1$	$2L^2+L(2+4\rho_1)$	$L(4\rho_2)$	$-(\mathrm{I}_4+d\mathrm{I}_3)/(2c_2^2)$	$2/\varepsilon^2+1/\varepsilon$
$\hat{\mathrm{I}}_2$	$4L^2+L(8\rho_1)$	$L(8\rho_2)$	$-d\mathrm{I}_3/c_2^2$	$4/\varepsilon^2$
$\hat{\mathrm{I}}_3$	0	$L(24\rho_3\rho_5-8\rho_4)$	$-d(\mathrm{I}_1+\mathrm{I}_2)/(4c_2^2)$	0
$\hat{\mathrm{I}}_4$	$2L^2+L(2+4\rho_1)$	0	$-d(\mathrm{I}_1-\mathrm{I}_7)/(2c_2^2)$	$2/\varepsilon^2+1/\varepsilon$
$\hat{\mathrm{I}}_5$	$2L^2+L(2+4\rho_1)$	0	$-2(\mathrm{I}_6-\mathrm{I}_5)/c_2^2$	$2/\varepsilon^2+1/\varepsilon$
$\hat{\mathrm{I}}_6$	$2L^2+L(1+4\rho_1)$	0	$2\mathrm{I}_5/c_2^2$	$2/\varepsilon^2+1/(2\varepsilon)$
$\hat{\mathrm{I}}_7$	$8L$	0	$8d\mathrm{I}_7/c_2^2=8d\mathrm{I}_8/c_2^2$	$4/\varepsilon$

$$2\mathcal{J}_1\times(4\pi)^4\sigma^{2\varepsilon}\overset{\text{s.p.}}{=}\Big(2d\mathrm{I}_1+d\mathrm{I}_2+\mathrm{I}_4-4\mathrm{I}_5+4\mathrm{I}_7\Big)+\varepsilon\Big(2\mathrm{I}_1+\mathrm{I}_2\Big)$$

$$\Big(-2\hat{\mathrm{I}}_1+\hat{\mathrm{I}}_2-4\hat{\mathrm{I}}_3-2\hat{\mathrm{I}}_4-2\hat{\mathrm{I}}_6+\hat{\mathrm{I}}_7/4\Big)\times c_2^2,$$

linear dependence

correction

inside

Comparison of two-loop parts

	Cutoff reg.		Dimensional reg.	
Diagram	Main part	Correction	Main part	Correction
$2\mathcal{J}_1$	$-8L^2 + L(-8 + 16\rho_1)$	$L(32\rho_4)$	$-8/\varepsilon^2 - 4/\varepsilon$	$-1/\varepsilon$
$2\mathcal{J}_2$	$16L^2 + L(16 + 32\rho_1)$	$L(-64\rho_4)$	$16/\varepsilon^2 + 8/\varepsilon$	$-2/\varepsilon$
$2\mathcal{J}_3$	$4L^2 + L(2 + 8\rho_1)$	$L(8\rho_2 - 32\rho_4)$	$4/\varepsilon^2 + 1/\varepsilon$	$1/\varepsilon$
$2\mathcal{J}_4$	$-L$	$L(4\rho_2 - 8\rho_4)$	$-1/(2\varepsilon)$	$3/(4\varepsilon)$
$\mathbb{H}_0^{\mathrm{sc}}(\Gamma_3^2)$	$12L^2 + L(9 + 24\rho_1)$	$L(12\rho_2 - 72\rho_4)$	$12/\varepsilon^2 + 9/(2\varepsilon)$	$-5/(4\varepsilon)$
$\mathbb{H}_0^{\mathrm{sc}}(\Omega_3^2)$	L	$L(8\rho_4)$	$1/(2\varepsilon)$	$1/(4\varepsilon)$
$-\frac{1}{2}\mathbb{H}_0^{\mathrm{sc}}(\Gamma_4)$	$-12L^2 + L(-24\rho_1)$	0	$-12/\varepsilon^2$	0
$-\frac{\mathrm{sum}}{2}$	$-5L$	$L(-6\rho_2 + 32\rho_4)$	$-5/(2\varepsilon)$	$1/(2\varepsilon)$
$-\mathcal{J}_7$	0	$L(5/36 - 10\rho_2/3)$	0	$-5/(6\varepsilon)$
Answer	$L(-175/36 - 28\rho_2/3 + 32\rho_4)$		$-17/(6\varepsilon)$	

Discussions

1) Three-loop is quite real task (for weak case).

2) What is the sense of deformed coefficients?

3) Explicit connection to Tkachov's ideas.

4) What is the sense of "mass" counter-terms?

5) Deformed coefficients and quasi-local vertices: resummation?

6) What about renormalizability?

Many thanks!