Image of a wormhole with an arbitrary throat profile

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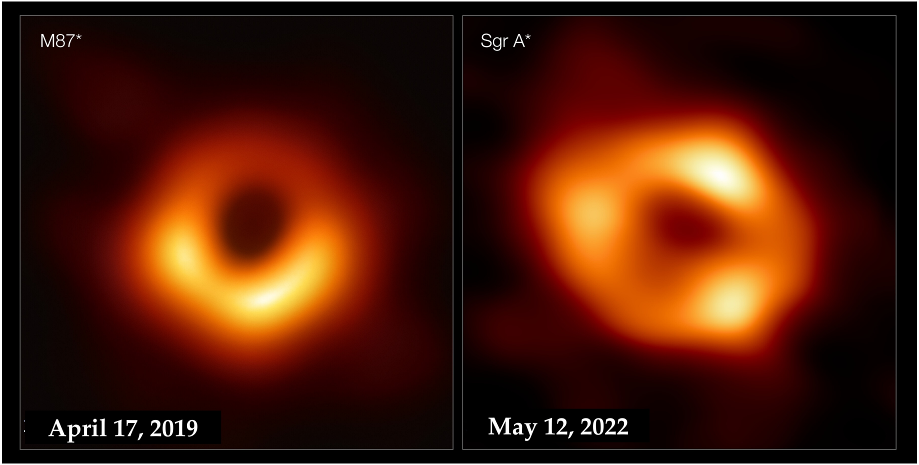
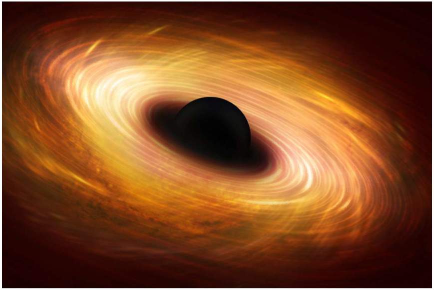
Outline

First images of supermassive compact objects

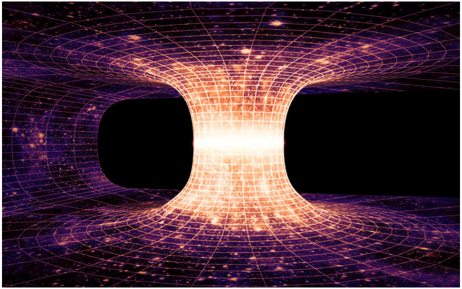


EHT Collaboration

Black holes and wormholes

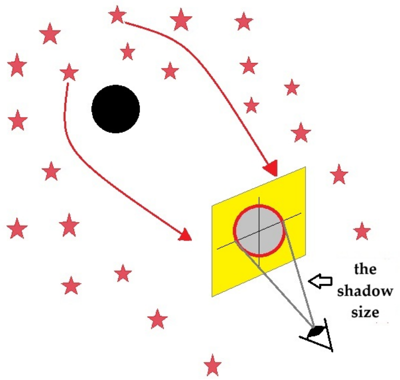


Black hole

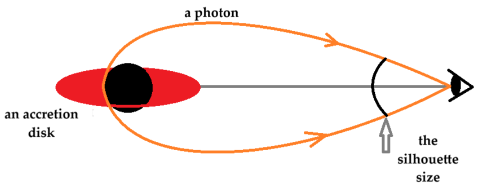


Wormhole

How to "see" a compact object?



Formation of the shadow image



Formation of the silhouette image

The **shadow boundary** is formed by photons that move on circular orbits (discrete captured and bending photons). The **silhouette boundary** is formed by photons that are emitted from the throat or event horizon

General static, spherically symmetric wormhole

$$ds^2 = -N^2(u)\,dt^2 + du^2 + r^2(u)\left(d\theta^2 + \sin^2\theta\,d\phi^2\right), \quad (1)$$

- u is a proper radial coordinate, $-\infty < u < +\infty$
- Throat is located at $u_{\rm th} = 0$: $r'(0) = 0, \qquad r''(0) \geq 0$
- Asymptotic flatness on the observer's side ($u \to +\infty$):

$$\lim_{u\to+\infty} N^2(u) = 1 - \frac{2m}{u} + O(u^{-2}), \qquad \lim_{u\to+\infty} r^2(u) = u^2\left(1 + O(u^{-1})\right)$$

- $N(u)$ is finite everywhere

Geodesic equations

$$\begin{aligned}
\frac{dt}{d\lambda} &= \frac{E}{N^2(u)}, \\
\frac{du}{d\lambda} &= \pm E\sqrt{\frac{1}{N^2(u)} - \frac{k^2}{r^2(u)} - \frac{1}{\varepsilon^2}} = \pm E\sqrt{U(u)}, \\
\frac{d\theta}{d\lambda} &= \pm\frac{E}{r^2(u)}\sqrt{k^2 - \frac{l^2}{\sin^2\theta}} = \pm\frac{E}{r^2(u)}\sqrt{\Theta(\theta)}, \\
\frac{d\phi}{d\lambda} &= E\frac{l}{r^2(u)\sin^2\theta}.
\end{aligned}$$

Here $U(u)$ and $\Theta(\theta)$ are the effective potentials, and the impact parameters

$$\varepsilon = \frac{E}{\mu}, \qquad l = \frac{L}{E}, \qquad k = \frac{\sqrt{K}}{E},$$

where λ is an affine parameter along the geodesic, E is the total energy of the particle, K is the square of the total angular momentum, L is the azimuthal angular momentum, μ is the rest mass (for photons $\mu = 0$, $1/\varepsilon = 0$).

Particle motion in the equatorial plane $\theta = \pi/2$ ($k^2 = l^2$)

Photon circular orbit ($\mu = 0$): $U(u_{\rm ph}) = 0$, $U'(u_{\rm ph}) = 0 \;\Rightarrow$

$$l_{\rm ph}^2 = \frac{r^2}{N^2}, \qquad \left(\frac{r'}{r} - \frac{N'}{N}\right)_{u_{\rm ph}} = 0$$

Massive particle stable circular orbits: $U(u) = 0$, $U'(u) = 0$, $U''(u) \leq 0 \;\Rightarrow$

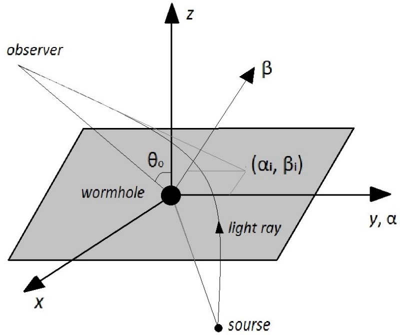
$$l_{\text{orbit}}^2 = \frac{r^3 N'}{N^3 r'}, \qquad \varepsilon_{\text{orbit}}^2 = \frac{N^3 r'}{N r' - r N'}.$$

The innermost stable circular orbit (ISCO): $U''(u_{\rm isco}) = 0$:

$$\left[N'\left(\frac{r''}{r'} - \frac{3r'}{r}\right) + \frac{3N'^2}{N} - N''\right]\Bigg|_{u=u_{\rm isco}} = 0.$$

Orbits at the throat ($u = 0$)

The function $N(u)$ must be symmetric with respect to $u_{\rm th} = 0$: $N'(0) = 0$



In the observer's sky, the coordinates of an incoming light ray are[a]:

$$\alpha_i = -r_o^2 \sin\theta_o \frac{d\phi}{dr}\Big|_{r=r_o}, \qquad \beta_i = -r_o^2 \frac{d\theta}{dr}\Big|_{r=r_o}$$

For $r_o \gg m$, $\theta_o = \pi/2$, the boundary of the shadow (or throat silhouette) is a circle with a radius $\alpha = |l|$:

- **Shadow radius:**

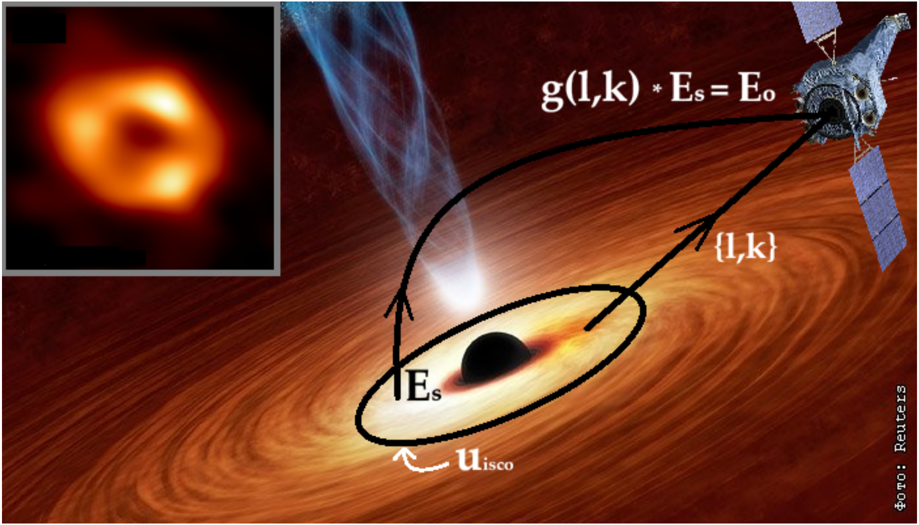
$$\alpha_{\rm sh} = |l_{\rm ph}| = \left|\frac{r(u_{\rm ph})}{N(u_{\rm ph})}\right|$$

- Integral equation for **the silhouette radius** $\alpha_{\rm sil} = |l_{\rm th}|$:

$$\int_0^\infty \frac{du}{r^2(u)\sqrt{N^2(u) - \frac{\alpha_{\rm sil}^2}{r^2(u)}}} = \frac{\pi}{\alpha_{\rm sil}}.$$

[a] Vazquez S. E. Strong field gravitational lensing by a Kerr black hole / S. E. Vazquez, E. P. Esteban // Nuovo Cimento B. – 2004. – Vol. 119, № 5. – P. 489–519.

Constructing the image of a thin accretion disk



Massless wormhole ($N = 1$)

- Circular orbits exist only at the throat $u = 0$,
- They are unstable: $U''(0) > 0$
- No stable orbits of massive particle $\Rightarrow$ *no accretion disk*

Long–throat wormhole model

$$ds^2 = -N^2(u)\,dt^2 + du^2 + r^2(u)\left(d\theta^2 + \sin^2\theta\,d\phi^2\right),$$

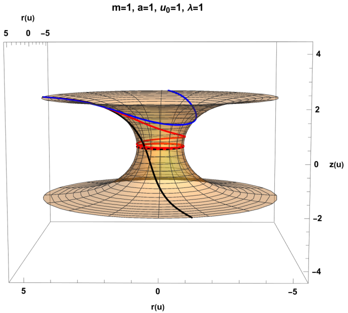
$$N(u) = \exp\Big[m\Big(\arctan\sqrt{u^2+u_0^2} - \frac{\pi}{2}\Big)\Big],$$

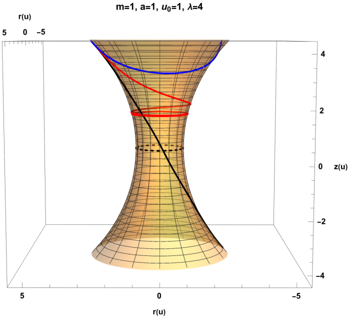
$$r(u) = u\coth\Big(\frac{u}{\lambda}\Big) - \lambda + a.$$

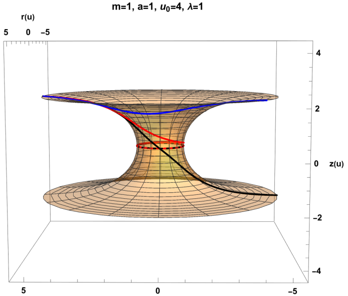
- m – ADM mass
- u_0 – regulate the depth of the gravitational well (with increasing u_0 the depth decreases)
- $a = r(0)$ – throat radius
- λ – throat length (for $\lambda \gg 1$ the throat becomes nearly cylindrical)

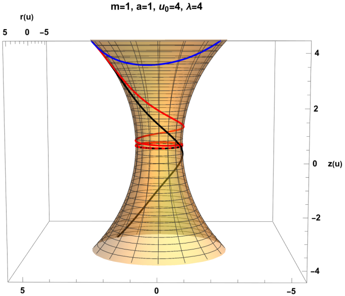
Functions $N(u)$, $r(u)$ satisfy asymptotic flatness, $r'(0) = 0$, and $N'(0) = 0$.

Embedding diagrams

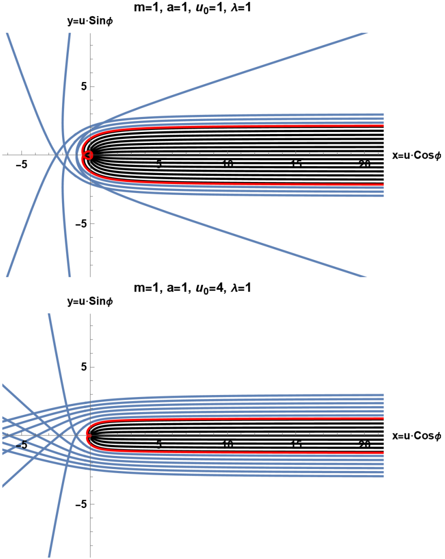






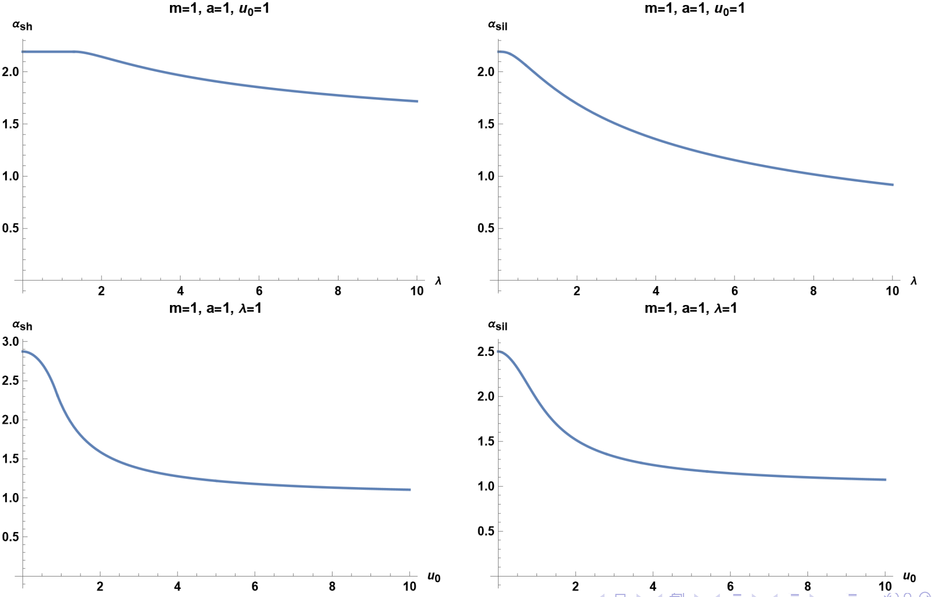


Photon trajectories $u(\phi)$

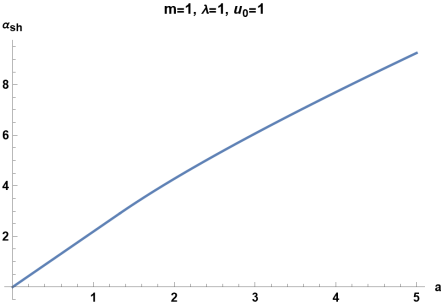


Black: $|l| > |l_{\rm ph}|$; Blue: $|l| < |l_{\rm ph}|$; Red: $|l| = |l_{\rm ph}|$.

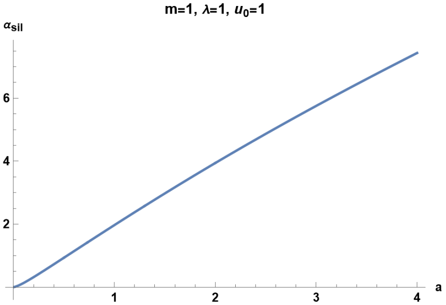
Shadow (left) and throat silhouette (right)



Dependence on a & comparison with Schwarzschild



$\alpha_{\rm sh}(a)$



$\alpha_{\rm sil}(a)$

- Both radii grow almost linearly with the throat radius a.
- Schwarzschild black hole: $\alpha_{\rm sh}^{\rm Schw} = 3\sqrt{3}m \approx 5.196m, \quad \alpha_{\rm sil}^{\rm Schw} \approx 4.457m.$
- For the wormhole with $m = 1$, $\lambda = 1$, $u_0 = 1$:

$$\alpha_{\rm sh}^{\rm wh} = \alpha_{\rm sh}^{\rm Schw} \text{ for } a \approx 2.5, \qquad \alpha_{\rm sil}^{\rm wh} = \alpha_{\rm sil}^{\rm Schw} \text{ for } a \approx 2.27$$

Accretion disk images

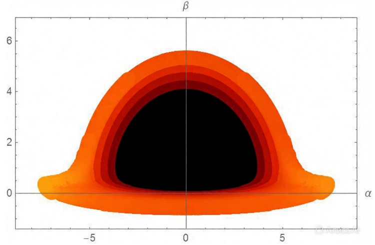
- Photons emitted in the equatorial plane $\theta_s = \pi/2$, $0 \le u_s \le u_{\rm ISCO}$.
- Parameters l, k obtained from equations of motion.
- Energy shift factor $g = 1/\varepsilon(l,k)$[1] (gravitational redshift and Doppler effect).
- Colour scale: dark red ($g \approx 0$) $\to$ bright yellow ($g \gtrsim 1$).
- Observer: $u_o = 10\,000\,m$, $\theta_o = 84.24^\circ$ (Sgr A* orientation).
- Wormhole parameters: $m_{\rm wh} = m_{\rm Schw}, a = r_{\rm h} = 2m$

Size of the image ($\Delta\alpha$, $\Delta\beta$) and energy shift g for the horizon/throat and ISCO

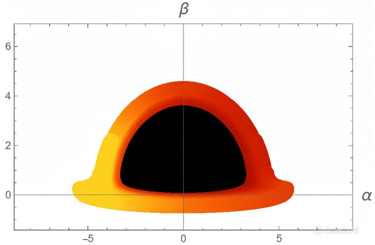
Object	Region	$\Delta\alpha$	$\Delta\beta$	$g_{\min}$	$g_{\max}$
Schwarzschild	horizon	7.70	4.36	≈ 0	≈ 0
	ISCO	14.82	5.98	0.58	0.74
WH $\lambda = 1, u_0 = 1$	throat	7.36	4.12	0.12	0.37
	ISCO	10.84	4.76	0.37	1.45
WH $\lambda = 1, u_0 = 0.5$	throat	8.08	4.54	0.06	0.10
	ISCO	12.32	5.19	0.41	1.44
WH $\lambda = 4, u_0 = 1$	throat	5.32	3.02	0.12	0.18
	ISCO	9.94	4.12	0.53	1.31

[1] Dokuchaev V. To see the invisible: Image of the event horizon within the black hole shadow, *International Journal of Modern Physics D*, 2019, vol. 28, no. 13, p. 1941005.

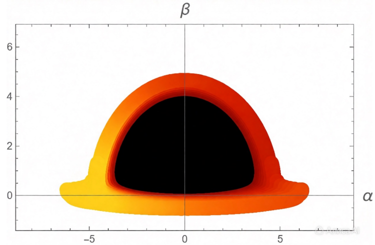
Accretion disk images



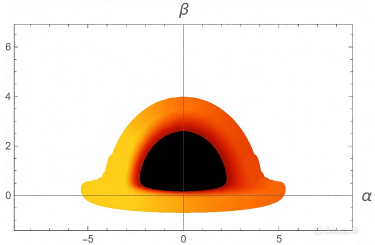
Schwarzschild black hole



Wormhole $\lambda = 1, u_0 = 1$



Wormhole $\lambda = 1, u_0 = 0.5$



Wormhole $\lambda = 4, u_0 = 1$

Conclusion

Results:

1. General expressions for particle orbits, the shadow radius ($\alpha_{\rm sh} \to |l_{\rm ph}|$), and the throat silhouette have been obtained for a static, spherically symmetric wormhole.
2. It has been shown that particle orbits can be located exactly at the throat only for symmetric wormholes.
3. It has been shown that no accretion disk can form around a massless wormhole ($N = 1$).
4. For the specific wormhole metric with a long throat, it has been shown that
   - its shadow and silhouette sizes can coincide with those of a Schwarzschild black hole;
   - the images of an accreting wormhole differ significantly from the image of an accreting Schwarzschild black hole.

Acknowledgments:
This work was supported by the Theoretical Physics and Mathematics Advancement Foundation "BASIS" grant No. 24-1-1-39-2.

Thank you for your attention!