

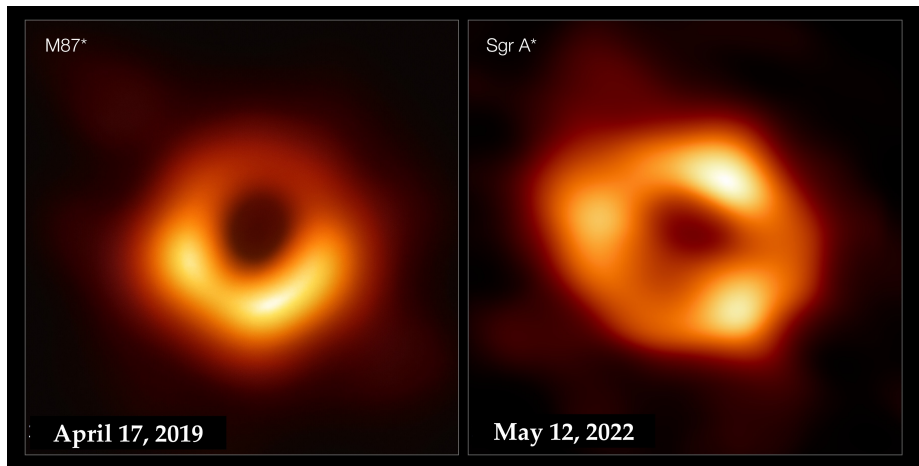
Image of a wormhole with an arbitrary throat profile

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First images of supermassive compact objects

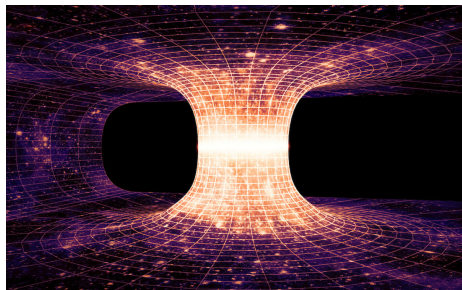


EHT Collaboration

Black holes and wormholes

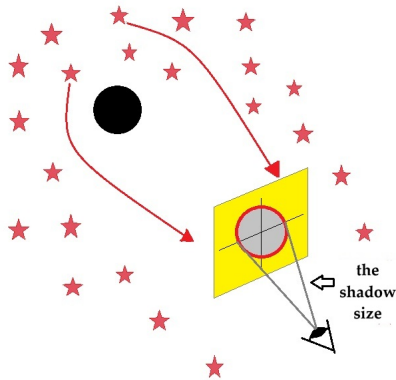


Black hole

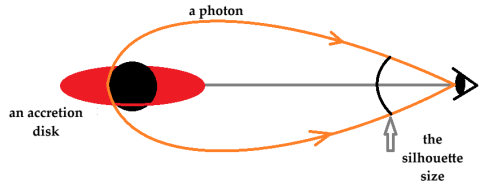


Wormhole

How to “see” a compact object?



Formation of the shadow image



Formation of the silhouette image

The **shadow boundary** is formed by photons that move on circular orbits (discrete captured and bending photons). The **silhouette boundary** is formed by photons that are emitted from the throat or event horizon

General static, spherically symmetric wormhole

$$ds^2 = -N^2(u) dt^2 + du^2 + r^2(u) (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

- u is a proper radial coordinate, $-\infty < u < +\infty$
- Throat is located at $u_{\text{th}} = 0$: $r'(0) = 0$, $r''(0) \geq 0$
- Asymptotic flatness on the observer's side ($u \rightarrow +\infty$):

$$\lim_{u \rightarrow +\infty} N^2(u) = 1 - \frac{2m}{u} + O(u^{-2}), \quad \lim_{u \rightarrow +\infty} r^2(u) = u^2(1 + O(u^{-1}))$$

- $N(u)$ is finite everywhere

Geodesic equations

$$\begin{aligned}\frac{dt}{d\lambda} &= \frac{E}{N^2(u)}, \\ \frac{du}{d\lambda} &= \pm E \sqrt{\frac{1}{N^2(u)} - \frac{k^2}{r^2(u)} - \frac{1}{\varepsilon^2}} = \pm E \sqrt{U(u)}, \\ \frac{d\theta}{d\lambda} &= \pm \frac{E}{r^2(u)} \sqrt{k^2 - \frac{l^2}{\sin^2 \theta}} = \pm \frac{E}{r^2(u)} \sqrt{\Theta(\theta)}, \\ \frac{d\phi}{d\lambda} &= E \frac{l}{r^2(u) \sin^2 \theta}.\end{aligned}$$

Here $U(u)$ and $\Theta(\theta)$ are the effective potentials, and the impact parameters

$$\varepsilon = \frac{E}{\mu}, \quad l = \frac{L}{E}, \quad k = \frac{\sqrt{K}}{E},$$

where λ is an affine parameter along the geodesic, E is the total energy of the particle, K is the square of the total angular momentum, L is the azimuthal angular momentum, μ is the rest mass (for photons $\mu = 0$, $1/\varepsilon = 0$).

Particle motion in the equatorial plane $\theta = \pi/2$ ($k^2 = l^2$)

Photon circular orbit ($\mu = 0$): $U(u_{\text{ph}}) = 0, U'(u_{\text{ph}}) = 0 \Rightarrow$

$$l_{\text{ph}}^2 = \frac{r^2}{N^2}, \quad \left(\frac{r'}{r} - \frac{N'}{N} \right)_{u_{\text{ph}}} = 0$$

Massive particle stable circular orbits: $U(u) = 0, U'(u) = 0, U''(u) \leq 0 \Rightarrow$

$$l_{\text{orbit}}^2 = \frac{r^3 N'}{N^3 r'}, \quad \varepsilon_{\text{orbit}}^2 = \frac{N^3 r'}{N r' - r N'}$$

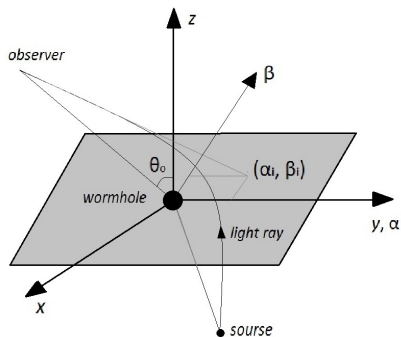
The innermost stable circular orbit (ISCO): $U''(u_{\text{isco}}) = 0$:

$$\left[N' \left(\frac{r''}{r'} - \frac{3r'}{r} \right) + \frac{3N'^2}{N} - N'' \right] \Big|_{u=u_{\text{isco}}} = 0.$$

Orbits at the throat ($u = 0$)

The function $N(u)$ must be symmetric with respect to $u_{\text{th}} = 0$: $N'(0) = 0$

Shadow and throat silhouette



In the observer's sky, the coordinates of an incoming light ray are^a:

$$\alpha_i = -r_o^2 \sin \theta_o \frac{d\phi}{dr} \Big|_{r=r_o}, \quad \beta_i = -r_o^2 \frac{d\theta}{dr} \Big|_{r=r_o}$$

For $r_o \gg m$, $\theta_o = \pi/2$, the boundary of the shadow (or throat silhouette) is a circle with a radius $\alpha = |l|$:

- **Shadow radius:**

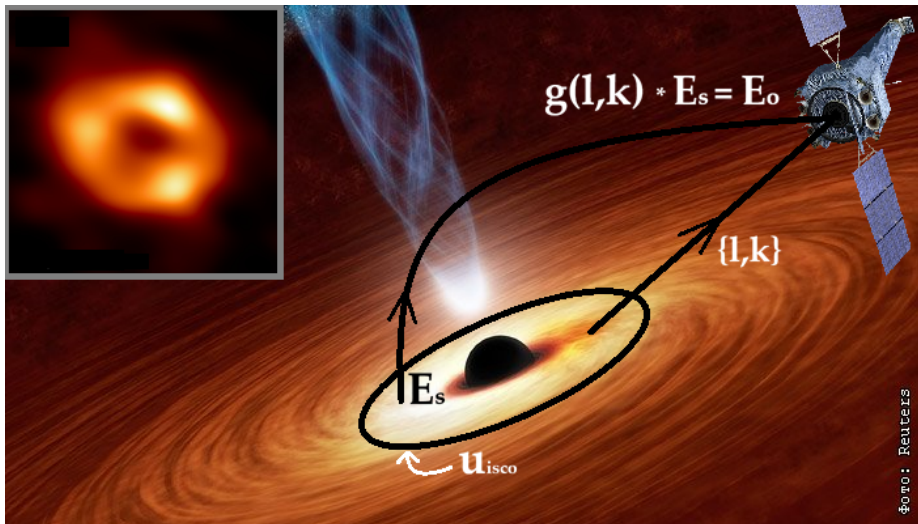
$$\alpha_{\text{sh}} = |l_{\text{ph}}| = \left| \frac{r(u_{\text{ph}})}{N(u_{\text{ph}})} \right|$$

- Integral equation for **the silhouette radius** $\alpha_{\text{sil}} = |l_{\text{th}}|$:

$$\int_0^\infty \frac{du}{r^2(u) \sqrt{N^2(u) - \frac{\alpha_{\text{sil}}^2}{r^2(u)}}} = \frac{\pi}{\alpha_{\text{sil}}}$$

^aVazquez S. E. Strong field gravitational lensing by a Kerr black hole / S. E. Vazquez, E. P. Esteban // Nuovo Cimento B. – 2004. – Vol. 119, № 5. – P. 489–519.

Constructing the image of a thin accretion disk



Massless wormhole ($N = 1$)

- Circular orbits exist only at the throat $u = 0$,
- They are unstable: $U''(0) > 0$
- No stable orbits of massive particle \Rightarrow *no accretion disk*

Long-throat wormhole model

$$ds^2 = -N^2(u) dt^2 + du^2 + r^2(u) (d\theta^2 + \sin^2 \theta d\phi^2),$$

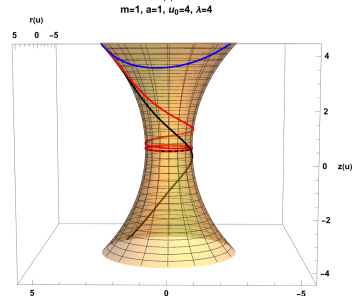
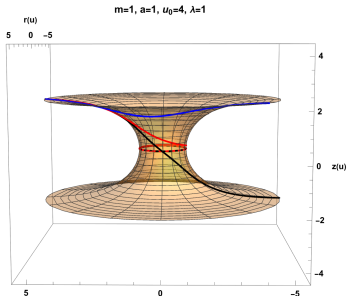
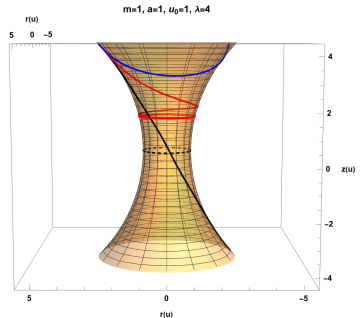
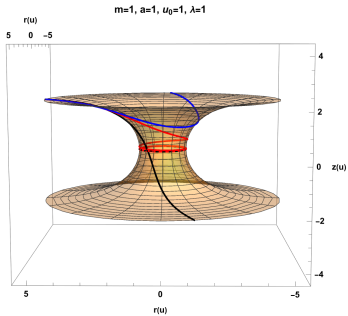
$$N(u) = \exp\left[m\left(\arctan \sqrt{u^2 + u_0^2} - \frac{\pi}{2}\right)\right],$$

$$r(u) = u \coth\left(\frac{u}{\lambda}\right) - \lambda + a.$$

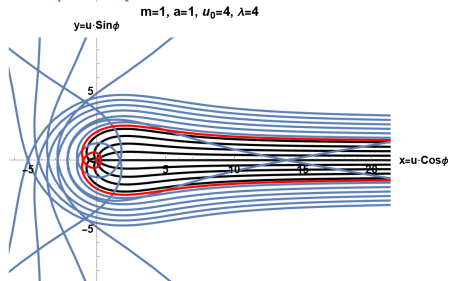
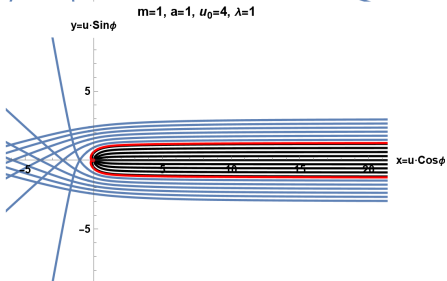
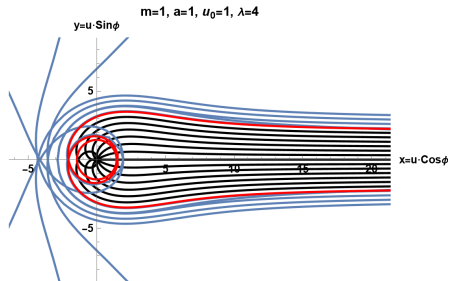
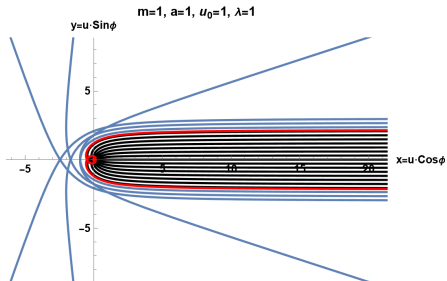
- m – ADM mass
- u_0 – regulate the depth of the gravitational well
(with increasing u_0 the depth decreases)
- $a = r(0)$ – throat radius
- λ – throat length (for $\lambda \gg 1$ the throat becomes nearly cylindrical)

Functions $N(u)$, $r(u)$ satisfy asymptotic flatness, $r'(0) = 0$, and $N'(0) = 0$.

Embedding diagrams



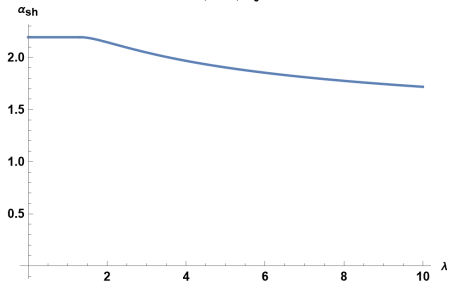
Photon trajectories $u(\phi)$



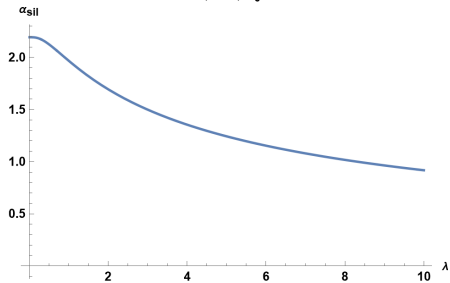
Black: $|l| > |l_{\text{ph}}|$; Blue: $|l| < |l_{\text{ph}}|$; Red: $|l| = |l_{\text{ph}}|$

Shadow (left) and throat silhouette (right)

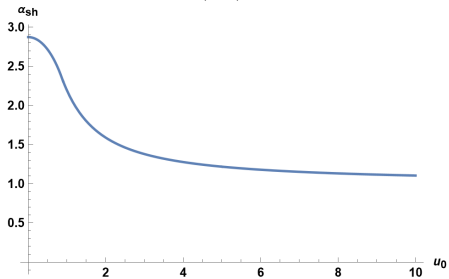
$m=1, a=1, u_0=1$



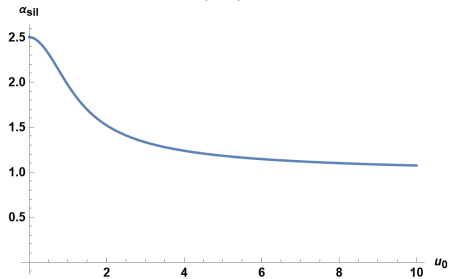
$m=1, a=1, u_0=1$



$m=1, a=1, \lambda=1$

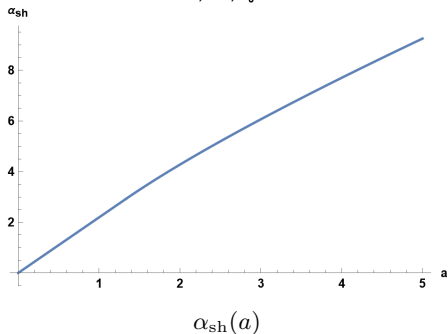


$m=1, a=1, \lambda=1$

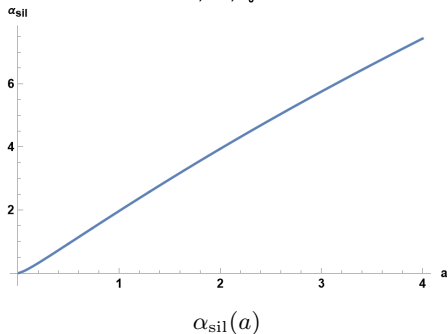


Dependence on a & comparison with Schwarzschild

$m=1, \lambda=1, u_0=1$



$m=1, \lambda=1, u_0=1$



- Both radii grow almost linearly with the throat radius a .
- Schwarzschild black hole: $\alpha_{sh}^{Schw} = 3\sqrt{3}m \approx 5.196m$, $\alpha_{sil}^{Schw} \approx 4.457m$.
- For the wormhole with $m = 1, \lambda = 1, u_0 = 1$:

$$\alpha_{sh}^{wh} = \alpha_{sh}^{Schw} \text{ for } a \approx 2.5, \quad \alpha_{sil}^{wh} = \alpha_{sil}^{Schw} \text{ for } a \approx 2.27$$

Accretion disk images

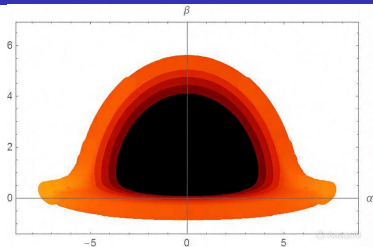
- Photons emitted in the equatorial plane $\theta_s = \pi/2$, $0 \leq u_s \leq u_{\text{ISCO}}$.
- Parameters l, k obtained from equations of motion.
- Energy shift factor $g = 1/\varepsilon(l, k)^1$ (gravitational redshift and Doppler effect).
- Colour scale: dark red ($g \approx 0$) \rightarrow bright yellow ($g \gtrsim 1$).
- Observer: $u_o = 10\,000 m$, $\theta_o = 84.24^\circ$ (Sgr A* orientation).
- Wormhole parameters: $m_{\text{wh}} = m_{\text{Schw}}$, $a = r_h = 2m$

Size of the image ($\Delta\alpha$, $\Delta\beta$) and energy shift g for the horizon/throat and ISCO

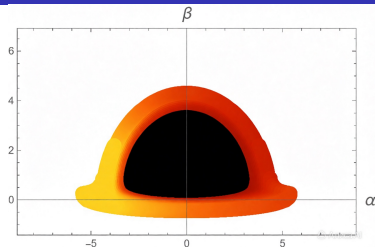
Object	Region	$\Delta\alpha$	$\Delta\beta$	g_{min}	g_{max}
Schwarzschild	horizon	7.70	4.36	≈ 0	≈ 0
	ISCO	14.82	5.98	0.58	0.74
WH $\lambda = 1, u_0 = 1$	throat	7.36	4.12	0.12	0.37
	ISCO	10.84	4.76	0.37	1.45
WH $\lambda = 1, u_0 = 0.5$	throat	8.08	4.54	0.06	0.10
	ISCO	12.32	5.19	0.41	1.44
WH $\lambda = 4, u_0 = 1$	throat	5.32	3.02	0.12	0.18
	ISCO	9.94	4.12	0.53	1.31

¹Dokuchaev V. To see the invisible: Image of the event horizon within the black hole shadow, *International Journal of Modern Physics D*, 2019, vol. 28, no. 13, p. 1941005.

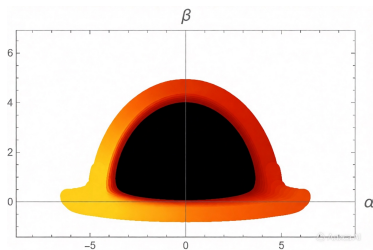
Accretion disk images



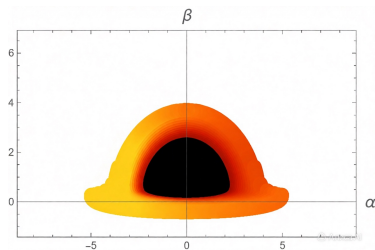
Schwarzschild black hole



Wormhole $\lambda = 1, u_0 = 1$



Wormhole $\lambda = 1, u_0 = 0.5$



Wormhole $\lambda = 4, u_0 = 1$

Results:

- 1 General expressions for particle orbits, the shadow radius ($\alpha_{\text{sh}} \rightarrow |l_{\text{ph}}|$), and the throat silhouette have been obtained for a static, spherically symmetric wormhole.
- 2 It has been shown that particle orbits can be located exactly at the throat only for symmetric wormholes.
- 3 It has been shown that no accretion disk can form around a massless wormhole ($N = 1$).
- 4 For the specific wormhole metric with a long throat, it has been shown that
 - its shadow and silhouette sizes can coincide with those of a Schwarzschild black hole;
 - the images of an accreting wormhole differ significantly from the image of an accreting Schwarzschild black hole.

Acknowledgments:

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Thank you for your attention!