

Color superconductivity in general dimension via holography

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Introduction

model setup

CSC phase with $N_c \geq 2$

Conclusion

introduction

- ▶ Holographic model for the color superconductivity (CSC) phase in QCD is one of the interesting problem in high energy physics
- ▶ The previous holographic model for the CSC phase always use AdS_6 to study this
- ▶ The main problem in holographic model for CSC phase is the number of color $N_c > 1$. If we use the Einstein-Maxwell gravity we only have CSC phase with $N_c = 1$ (Kazuo 2019).
- ▶ In this project, we generalize the conception of holo CSC phase for the d-dimension instead of 6d to study generally and we also use the arbitrary $SU(N_c)$. And we will try to use Einstein-Maxwell gravity for d-dimension

Holographic dictionary

- ▶ gravity in d-dimension AdS \leftrightarrow QFT in (d-1)-dimension boundary
- ▶ The Cooper pair (diquark in QCD) \leftrightarrow the scalar field ψ and q is its U(1) charge correspond to the charge (in analogy to QCD CSC the baryon number) of the Cooper pair operator and this relate the color number as $q = \frac{2}{N_c}$
- ▶ The U(1) gauge field A_μ in the bulk \leftrightarrow the current in analogy to current baryon number in QCD CSC
- ▶ The temperature \leftrightarrow the Hawking temperature of RNAdS planar black hole
- ▶ The confinement phase \leftrightarrow the AdS soliton in d-dimension
- ▶ The confined SU_{N_c} scale \leftrightarrow the compact extra dimension y with the radius R_y

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Einstein-Maxwell action

The action of the d -dimension Einstein-Maxwell gravity for the CSC phase transition is given by

$$S = \int d^d x \sqrt{-g} \left(\mathcal{R} + \frac{(d-1)(d-2)}{L^2} - \frac{1}{4} F^2 - |(\partial_\mu - iqA_\mu)\psi|^2 - m^2 |\psi|^2 \right) \quad (1)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, the cosmological constant Λ is determined by $\Lambda = -\frac{(d-1)(d-2)}{2L^2}$. And we set $L^2 = 1$

The ansatz in deconfinement phase

The ansatz for the vector field and scalar fields read

$$A_\mu dx^\mu = \phi(r)dt, \psi = \psi(r) \quad (2)$$

The RN planar black hole solution whose the metric is given by the following ansatz:

$$ds_{BH}^2 = r^2(-f(r)dt^2 + h_{ij}dx^i dx^j + dy^2) + \frac{dr^2}{r^2 f(r)} \quad (3)$$

where $h_{ij}dx^i dx^j = dx_1^2 + \dots + dx_{d-3}^2$ is the line element of the $(d-3)$ -dimension hypersurface and dy^2 is the compact coordinate with the radius R_y

Equation of motion

In the deconfinement phase

$$\phi''(r) + \frac{d-2}{r}\phi'(r) - \frac{2q^2\psi^2(r)}{r^2 f(r)}\phi(r) = 0 \quad (4)$$

$$\psi''(r) + \left[\frac{f'(r)}{f(r)} + \frac{d}{r} \right] \psi'(r) + \frac{1}{r^2 f(r)} \left[\frac{q^2 \phi^2(r)}{r^2 f(r)} - m^2 \right] \psi(r) = 0$$

The blackening function of this AdS black hole is

$$f(r) = 1 - \left(1 + \frac{(d-3)\mu^2}{8r_+^2} \right) \left(\frac{r_+}{r} \right)^{d-1} + \frac{(d-3)\mu^2 r_+^d}{8r^{d+2}} \quad (5)$$

The Hawking temperature

The temperature of physics system is dual to the Hawking temperature and

$$T = \frac{r_+^2 f'(r_+)}{4\pi} = \frac{1}{4\pi} \left((d-1)r_+ - \frac{3(d-3)\mu^2}{8r_+} \right), \quad (6)$$

the condition $T \geq 0$ we have the constraint of μ

$$\frac{\mu^2}{r_+^2} \leq \frac{8(d-1)}{3(d-3)} \quad (7)$$

Near boundary conditon

Near the boundary ($r \rightarrow \infty$) we have the form of the matter fields:

$$\begin{aligned}\phi(r) &= \mu - \frac{\bar{d}}{r^{d-3}} \\ \psi(r) &= \frac{J_C}{r^{\Delta_-}} + \frac{C}{r^{\Delta_+}}\end{aligned}\tag{8}$$

where μ , \bar{d} , J_C and C are regarded as the chemical potential, charge density, source, and the condensates value (VEV) of the Cooper pair operator dual to ψ , like the diquark Cooper pair in QCD, respectively, and we can see that one constraint $d > 3$.

The conformal dimension Δ_{\pm} in this case read

$$\Delta_{\pm} = \frac{1}{2} \left((d-1) \pm \sqrt{(d-1)^2 + 4m^2} \right) \quad (9)$$

And the BF bound is

$$m^2 \geq -\frac{(d-1)^2}{4} \quad (10)$$

We choose $m^2 = 2 - d$ to have $\Delta_- = 1$ and we also obtain $\Delta_+ = d - 2$.
Hence

$$\psi(r) = \frac{J_C}{r} + \frac{C}{r^{d-2}} \quad (11)$$

Confinement phase

In the confinement phase, the gravity dual is the d-dimension AdS soliton solution and it is described by

$$ds_{sol}^2 = r^2(-dt^2 + h_{ij}dx^i dx^j + f(r)dy^2) + \frac{dr^2}{r^2 f(r)}, \quad (12)$$

with

$$f(r) = 1 - \left(\frac{r_0}{r}\right)^{d-1}, \quad (13)$$

and $r_0 = \frac{2}{(d-1)R_y}$.

The equation of motion in the confinement phase

$$\begin{aligned}\phi''(r) + \left[\frac{d-2}{r} + \frac{f'(r)}{f(r)} \right] \phi'(r) - \frac{2q^2\psi^2(r)}{r^2 f(r)} \phi(r) &= 0 \\ \psi''(r) + \left[\frac{d}{r} + \frac{f'(r)}{f(r)} \right] \psi'(r) + \frac{1}{r^2 f(r)} \left[\frac{q^2\phi^2(r)}{r^2} - m^2 \right] \psi(r) &= 0\end{aligned}\tag{14}$$

The boundary condition at r_0

$$\begin{aligned}\phi'(r_0) &= \frac{2q^2\psi^2(r_0)}{(d-1)r_0} \phi(r_0) \\ \psi'(r_0) &= -\frac{1}{(d-1)r_0} \left(\frac{q^2\phi^2(r_0)}{r_0^2} - m^2 \right) \psi(r_0)\end{aligned}\tag{15}$$

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Near the critical chemical potential

At the critical point, $\psi = 0$ hence in near critical chemical potential (it means $\mu > \mu_c$ but near μ_c) the back reaction of the scalar field is negligible. We obtain the bulk configuration is determined by:

$$S = \int d^d x \sqrt{-g} \left(\mathcal{R} - 2\Lambda - \frac{1}{4} F^2 \right) \quad (16)$$

Hence, the solution of the gauge field in this case is given by

$$\phi(r) = \mu \left(1 - \left(\frac{r_+}{r} \right)^{d-3} \right) \quad (17)$$

And, the solution of the gauge field with confinement case:

$$\psi(r) = \mu \quad (18)$$

The free energy and the Hawking-Page phase transition

If $d > 4$ and even from Olea (2011) we obtain

$$\begin{aligned}\Omega_{BH2k} &= Vol(\Gamma_{d-2})[(r^2 f)'r^{d-2}|_{r_+}^\infty - (r^2 f)'(r^2 f)^{k-1}|_{r_+}^\infty - r^{d-2}\phi\phi'|_{r_+}^\infty] \\ &= [-r_+^{d-1} - \frac{(d-3)(7-d)}{8}\mu^2 r_+^{d-3}]Vol(\Gamma_{d-2}),\end{aligned}\tag{19}$$

with AdS black hole and

$$\Omega_{soliton2k} = -r_0^{d-1}Vol(\Gamma_{d-2}),\tag{20}$$

with AdS soliton ($d = 2k$).

The Hawking-Page phase transition occurs when

$$\Omega_{BH} = \Omega_{soliton}\tag{21}$$

with $d = 6$ we have $\mu_{cd6} = 1.73$

The phase diagram in $d = 6$

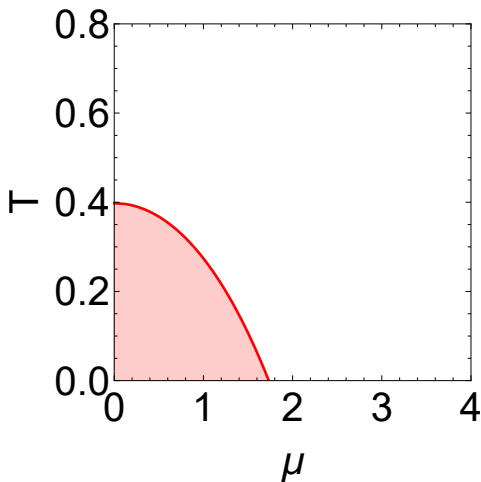


Figure: Phase diagram with $d=6$ The chemical potential for c-d transition is 1.73

In $d = 4$ case we have:

$$\Omega_{4dbh} = -r_+^3 \left(1 + \frac{\mu^2}{8r_+^2} \right) X R_y, \quad (22)$$

And

$$\Omega_{4dsoliton} = -r_0^3 X R_y \quad (23)$$

and with $d = 8$ we have no confinement-deconfinement phase transition

The phase diagram in $d = 4$

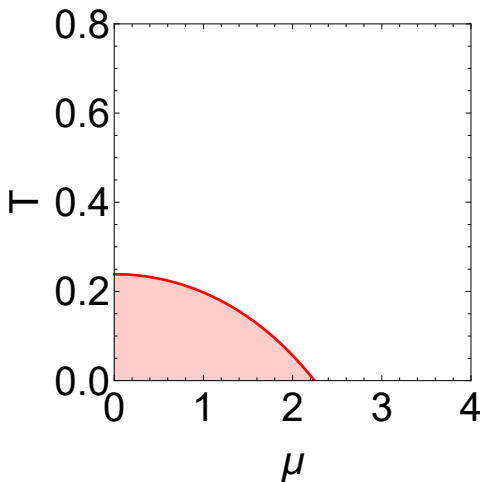


Figure: Phase diagram with $d=4$ The chemical potential for c-d transition is 2.22

The condition

From the equation of motion, we introduce the effective mass

$$m_{eff}^2 = m^2 - \Delta m^2 = m^2 - \frac{q^2 \phi^2(r)}{r^2 f(r)} \quad (24)$$

To the Cooper pair condensed state appear, we must to have the instability of the bulk scalar field (with the effective mass). And this correspond to the BF bound is broken, we have a condition

$$m^2 - \frac{q^2 \phi^2(r)}{r^2 f(r)} < \frac{-(d-1)^2}{4} \quad (25)$$

We obtain

$$\frac{q^2 \phi^2(r)}{r^2 f(r)} > \frac{(d-3)^2}{4} \quad (26)$$

Plug the value of $\phi(r)$ and $f(r)$ we obtain

$$\frac{q^2 \hat{\mu}^2 z^2 (1 - z^{d-3})^2}{1 - \left(1 + \frac{(d-3)\hat{\mu}^2}{8}\right) z^{d-1} + \frac{(d-3)\hat{\mu}^2 z^{d+2}}{8}} = q^2 F(\hat{\mu}, z, d) > \frac{(d-3)^2}{4} \quad (27)$$

with $\hat{\mu} = \frac{\mu}{r_+}$, and $z = \frac{r_{\pm}}{r}$

And we obtain

$$N_c < \frac{4\sqrt{F_{max}(\hat{\mu}, z)}}{d-3} \quad (28)$$

- ▶ We see that with $d = 5$ the Einstein-Maxwell gravity can give $N_c = 2$ CSC phase, because with $d = 5$, $N_c < 2.47$
- ▶ With $d = 4$, $N_c < 3.77$, the Einstein-Maxwell gravity in $4d$ can describe the CSC phase with maximum number of color $N_c = 3$
- ▶ With $d = 3$, there is no CSC phase

Confinement case

From the BF bound breaking condition (with $r_0 = 1$) we have

$$q^2 \phi^2(r) > \frac{(d-3)^2}{4} \quad (29)$$

After some manipulation, we obtain

$$N_c < \frac{4\mu_{cd}}{d-3} \quad (30)$$

Here μ_{cd} is the critical chemical potential for the confinement-deconfinement phase transition.

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- ▶ We built the holographic model for CSC phase by the Einstein-Maxwell gravity in d -dimension
- ▶ With $d = 5$ we can obtain the color superconductivity with $N_c = 2$ in the without confinement case.
- ▶ With $d = 4$ we can use the Einstein-Maxwell gravity describe CSC phase with $N_c = 3$
- ▶ with $d \geq 6$ we only study CSC with $N_c = 1$ and with $d \geq 8$ we have no confinement-deconfinement phase transition
- ▶ There is no CSC phase in $d = 3$ for all N_c
- ▶ In the future we will study the p-wave, d-wave CSC phase and generalize the CFL phase. Moreover we will try to study the holographic entanglement entropy in CSC phase

Thank you for attention!