Searching for heavy neutrinos in $e^+e^- \to W^+W^-$: it is all about unitarity

G. A. Chachava, S. I. Godunov

based on
arXiv:2604.04254

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$e^+e^- \to W^+W^-$ in the Standard Model

At the dawn of the Standard Model it was quickly realized that there is a strong compensation between diagrams for this process. Selected papers:

- A. I. Vainshtein and I. B. Khriplovich. Yad. Fiz., 13:198–211, 1971.
- Steven Weinberg. Phys. Rev. Lett., 27:1688–1691, 1971.
- E. B. Bogomolnyi. Yad. Fiz., 18:574–582, 1973.
- L. B. Okun. In *Proceedings of the First ITEP Winter School*, Moscow, 1973. ATOMIZDAT. In Russian.

Now it is a textbook material:

- Lev B. Okun. *Leptons and Quarks: Special Edition Commemorating the Discovery of the Higgs Boson*. North-Holland, Amsterdam, Netherlands, 1982.
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There are various plans to build new lepton colliders in the future, and it is a good reason for us to revisit this beautiful physics:

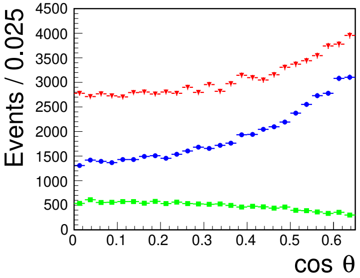
Project	ILC	CLIC	CEPC	FCC-ee	Muon collider
$\sqrt{s}$, GeV	250–1000	380–3000	240	90–350	3000–10000 TeV

HNL contribution

Heavy Neutral Leptons (HNL) are expected to give a small contribution to the total cross section for a wide parameters regions at these energies due to small mixing parameters. *In the region* $\sqrt{s} \gg M_W$ *bounds are relatively weak.*

A. Drutskoy, E. Vasenin, Phys. Rev. D, 111(1):015025, 2025

It was suggested that differential cross section in the central region is sensitive to the HNL contributions due to the shape of the SM cross section. It was demonstrated that future experiments will allow a significant improvement in the HNL parameters bounds (or discover HNL).



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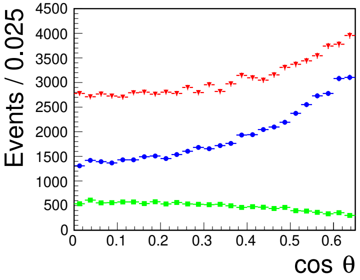

Monte Carlo generators require model input for the detector simulation. However, many popular models are implemented with some approximations.

We need to be extremely careful
since there is a very strong compensation in this process!

Within the framework of the see-saw type-I model, heavy states mix with the massless neutrinos, so after mass matrix diagonalization we get three light and $\mathcal{N}$ heavy neutrinos:

$$\nu_{Le} = P_L \left(\sum_{i=1}^{3} U_{ei} \nu_i + \sum_{I=1}^{\mathcal{N}} V_{eI} N_I \right).$$

Let us consider constraints on mixing parameters and masses:

1. See-saw relation from mass matrix diagonalization:

$$m_{\text{eff}}^{\nu} + \sum_{i=1}^{\mathcal{N}} V_{eI}^2 M_I = 0,$$

where $m_{\text{eff}}^{\nu} = \sum_{i=1}^{3} U_{ei}^2 m_i$.

2. Searches for neutrinoless double beta decay (GERDA, KamLAND-Zen)

$$|m_{\text{eff}}| = \left| m_{\text{eff}}^{\nu} + \sum_{I=1}^{\mathcal{N}} V_{eI}^2 M_I f_I \right| < 0.180 \text{ eV},$$

3. Global fit including direct measurement, oscillations and cosmological data (NuFIT): $|m_{\text{eff}}^{\nu}| \lesssim 0.1$ eV.
4. There are bounds from the electroweak precision tests (EWPO), see this paper:

$$\sum_{I=1}^{\mathcal{N}} |V_{eI}|^2 \equiv |V_{eN}|^2 < 1.25 \cdot 10^{-3}.$$

Exact vs linearized mixing

The incorporation of heavy neutrinos into the Standard Model can be carried out in two ways — *exactly* or within a *linear* approximation. In the first approach, heavy neutrinos are introduced in a fully unitary manner, satisfying the relation:

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{I=1}^{\mathcal{N}} |V_{eI}|^2 = 1.$$

It should be noted that under this construction the matrix U ceases to be strictly unitary, which in turn modifies the coupling constants of the light neutrinos.

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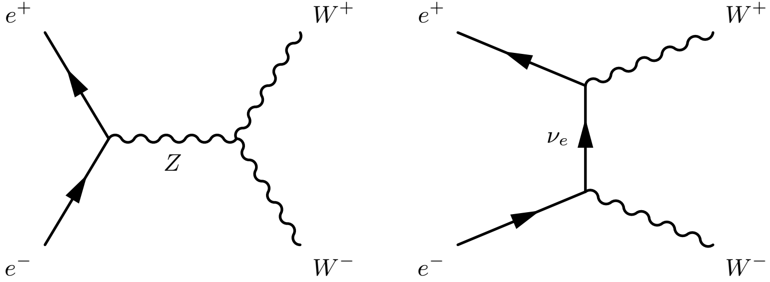
The second approach introduces heavy neutrinos with mixing parameters V_{eI} while keeping the matrix U unitary:

$$\sum_{i=1}^{3} |U_{ei}|^2 + \sum_{I=1}^{\mathcal{N}} |V_{eI}|^2 = 1 + \sum_{I=1}^{\mathcal{N}} |V_{eI}|^2 = 1 + \mathcal{O}\big(|V_{eN}|^2\big).$$

Though strictly speaking it is a dangerous thing to do, since the neutrinos kinetic term would not be in the canonical form either in the flavor or mass basis, for many processes it is a reasonable approximation since the value of $|V_{eN}|^2$ is small. This latter method is the one implemented in the `HeavyN` model, and it provides adequate results for a broad class of processes. However, it turns out that in certain cases such an approximate treatment leads to physically inconsistent results (i.e. violation of S-matrix unitarity), in which case the fully unitary approach must be employed. An important example of this is the process $e^+e^- \to W^+W^-$.

Toy Model

Let us consider a simplified model – a gauge theory *with only an $SU(2)$ symmetry* group containing gauge bosons W and Z of equal mass $M_Z = M_W \equiv M$, and a massless electron. Due to the fact that heavy neutrinos contribute only to diagrams with left-handed leptons and right-handed anti-leptons, we also consider all electrons to be left-handed particles and positrons to be right-handed particles throughout the paper.



Despite its simplicity, this model retains all the essential features present in the Standard Model. It could be understood in the following way:

- diagram with a Higgs boson compensate all effects from electron mass
- γ and Z diagrams interfere negatively, neutralizing the effects of electroweak mixing ($\bar{g} \to g$).

Massless neutrinos in the Toy Model

Upon squaring the amplitude, summing over the polarizations of W-bosons, one obtains the following differential cross section in the limit $\sqrt{s} \gg 2M$:

$$\frac{\mathrm{d}\sigma^{\mathrm{sim}}}{\mathrm{d}\cos\theta} = \frac{g^4(1+\cos\theta)}{512\pi s(1-\cos\theta)} \cdot \left(9 - 2\cos\theta + 9\cos^2\theta\right), \quad \Rightarrow \quad A_s + A_t \sim \mathrm{const}$$

where "sim" stands for the simplest model under consideration. As evident from the result, the cross section scales with energy as $\frac{1}{s}$. However, when considering the s- and t-channel contributions separately, one finds leading terms that grow with energy as s:

$$\frac{\mathrm{d}\sigma_s^{\mathrm{sim}}}{\mathrm{d}\cos\theta} = \frac{g^4 s}{512\pi M^4} \cdot \left(1 - \cos^2\theta\right), \quad \Rightarrow \quad A_s \propto s$$

$$\frac{\mathrm{d}\sigma_t^{\mathrm{sim}}}{\mathrm{d}\cos\theta} = \frac{g^4 s}{512\pi M^4} \cdot \left(1 - \cos^2\theta\right). \quad \Rightarrow \quad A_t \propto s$$

The cancellation of these energy-growing terms arises because the s and t-channel amplitudes enter the total amplitude with opposite signs, resulting in their mutual subtraction. Consequently, the unitarity-consistent asymptotic behavior $\sigma \propto \frac{1}{s}$ is restored.

Amplitudes for the exact mixing

Let us introduce one heavy neutrino.

Notations

- A_s — s-channel amplitude,
- A_t — t-channel amplitude for massless neutrino (without mixing matrix elements),
- A_N — t-channel amplitude for heavy neutrino (without mixing matrix elements).

Under the assumption of unitary mixing between light and heavy neutrino states, we obtain:

$$A = A_s + \overbrace{\left(|U_{e1}|^2 + |U_{e2}|^2 + |U_{e3}|^2\right)}^{1-|V_{eN}|^2} A_t + |V_{eN}|^2 A_N = \underbrace{A_s + A_t}_{\sim\text{const}} + |V_{eN}|^2 (A_N - A_t).$$

The sum of the first two amplitudes remains constant at large s, while the behavior of $A_N - A_t$ is determined by the difference between the neutrino propagators:

$$\frac{\hat{p}_- - \hat{k}_1}{(p_- - k_1)^2 - M_N^2} - \frac{\hat{p}_- - \hat{k}_1}{(p_- - k_1)^2} = \frac{\hat{p}_- - \hat{k}_1}{(p_- - k_1)^2} \cdot \underbrace{\frac{M_N^2}{(p_- - k_1)^2 - M_N^2}}_{\text{additional } 1/s \text{ suppression}}.$$

Thus, although each individual amplitude, A_t and A_N, grows proportional to s, their difference remains constant, leading to the $\sigma \sim \frac{1}{s}$ dependence of the cross section.

Amplitudes for the linearized mixing

If the heavy neutrino is simply added without adjusting the coupling of the light neutrino, the resulting expression

$$A = \underbrace{A_s + A_t}_{\sim\text{const}} + |V_{eN}|^2 \underbrace{A_N}_{\sim s},$$

in addition to sum $A_s + A_t$ that behaves like a constant at large s, contains a term $|V_{eN}|^2 A_N$ that grows with energy proportional to s, thereby violating the unitarity bound on the cross section:

$$\sigma \sim s.$$

It is therefore crucial that heavy neutrino states be incorporated in a unitary manner to preserve the asymptotic behavior $\sigma \propto \frac{1}{s}$ at high energies.

Results for the Toy Model

Analytical results are not very compact even in the Toy Model, however we present leading terms:

- For the exactly unitary mixing we get (“un” stands for unitary mixing)

$$\frac{\mathrm{d}\sigma^{\mathrm{un}}}{\mathrm{d}\cos\theta} = \frac{g^4|V_{eN}|^4 M_N^4}{128\pi M^4} \cdot \frac{s}{(s + 2M_N^2 - s\cos\theta)^2} \cdot (1-\cos^2\theta),$$

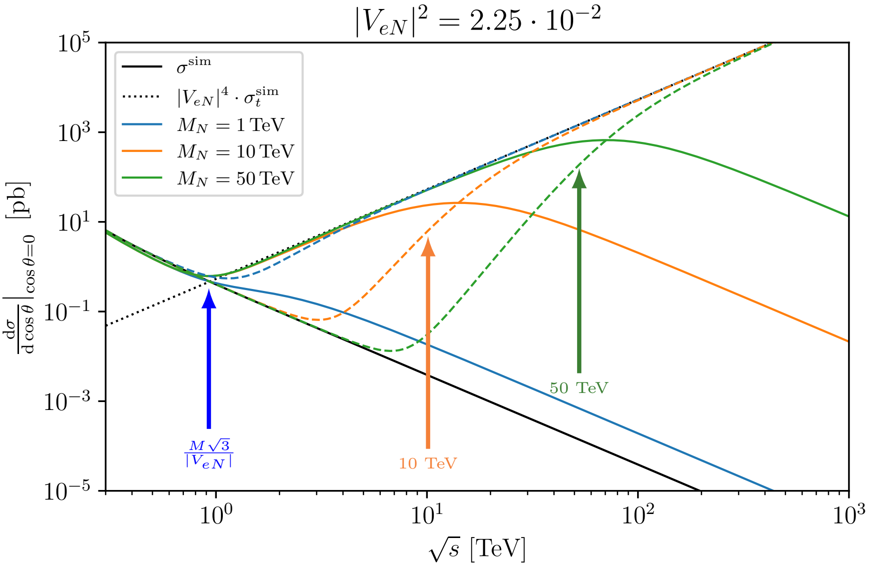
  valid in the limit $\sqrt{s} \gtrsim \frac{M\sqrt{3}}{|V_{eN}|}$.

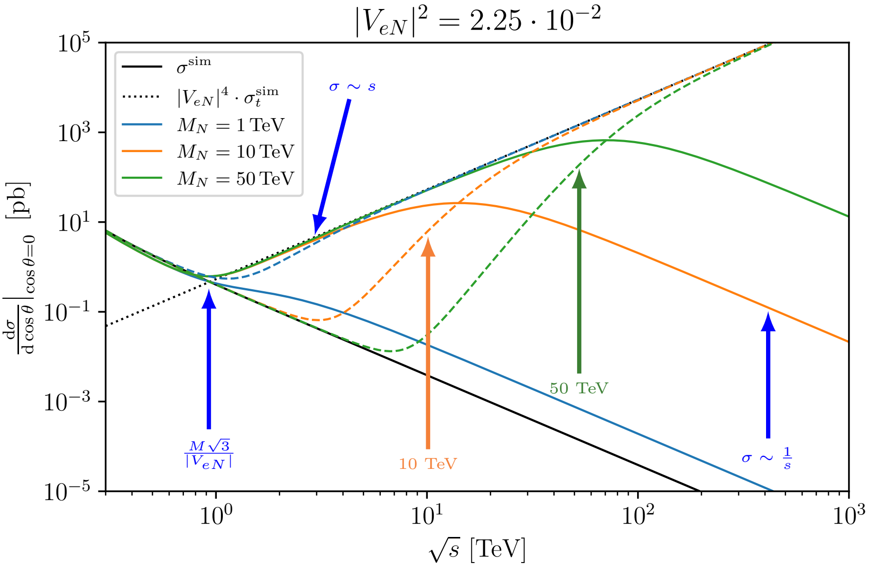
- For the linearized mixing the differential cross section is given by (“lin” stands for linearized mixing):

$$\frac{\mathrm{d}\sigma^{\mathrm{lin}}}{\mathrm{d}\cos\theta} = \frac{g^4|V_{eN}|^4}{512\pi M^4} \cdot \frac{s^3}{(s + 2M_N^2 - s\cos\theta)^2} \cdot (1-\cos^2\theta)(1-\cos\theta)^2,$$

  which is valid in the limit $\sqrt{s} \gtrsim \max\left(\sqrt{\frac{MM_N}{|V_{eN}|}}, \frac{M\sqrt{3}}{|V_{eN}|}\right)$.

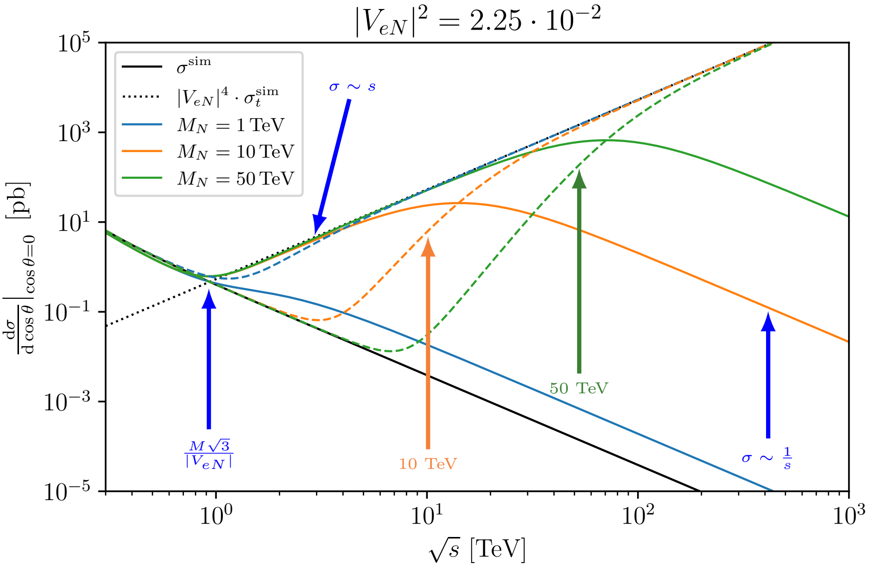
Results for the Toy Model





$|V_{eN}|^2 = 2.25 \cdot 10^{-2}$
σ^{sim}
$|V_{eN}|^4 \cdot \sigma_t^{\text{sim}}$
$M_N = 1$ TeV
$M_N = 10$ TeV
$M_N = 50$ TeV
$\sigma \sim s$
$\frac{M\sqrt{3}}{|V_{eN}|}$
10 TeV
50 TeV
$\sigma \sim \frac{1}{s}$
$\left.\frac{d\sigma}{d\cos\theta}\right|_{\cos\theta=0}$ [pb]
$\sqrt{s}$ [TeV]
10^5
10^3
10^1
10^{-1}
10^{-3}
10^{-5}
10^0
10^1
10^2
10^3

Results for the Toy Model



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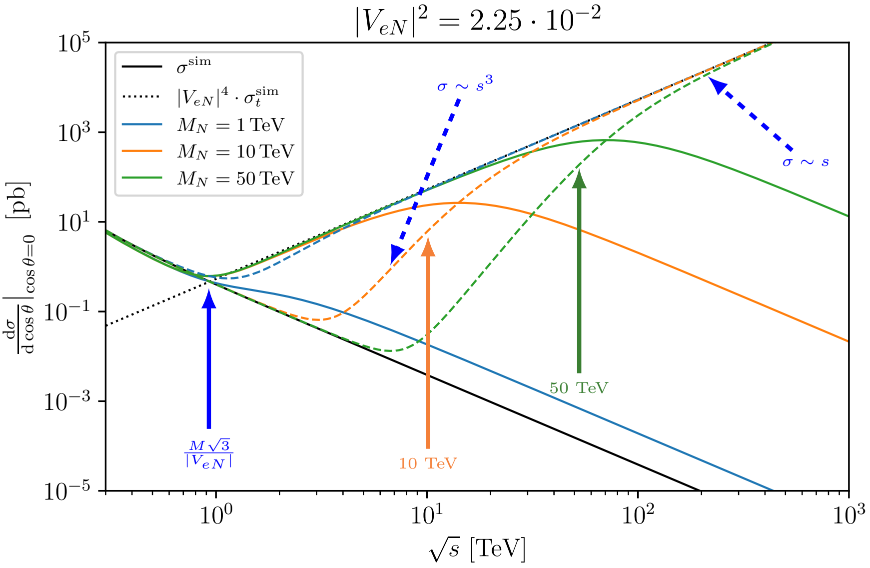
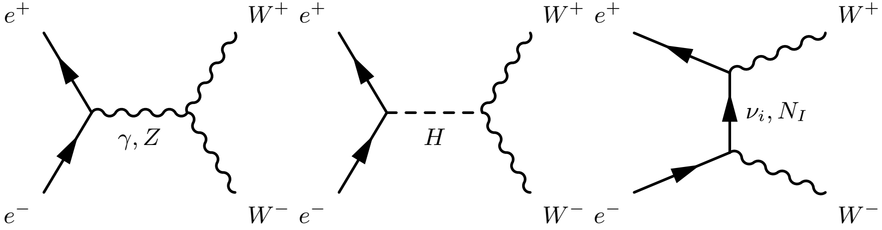
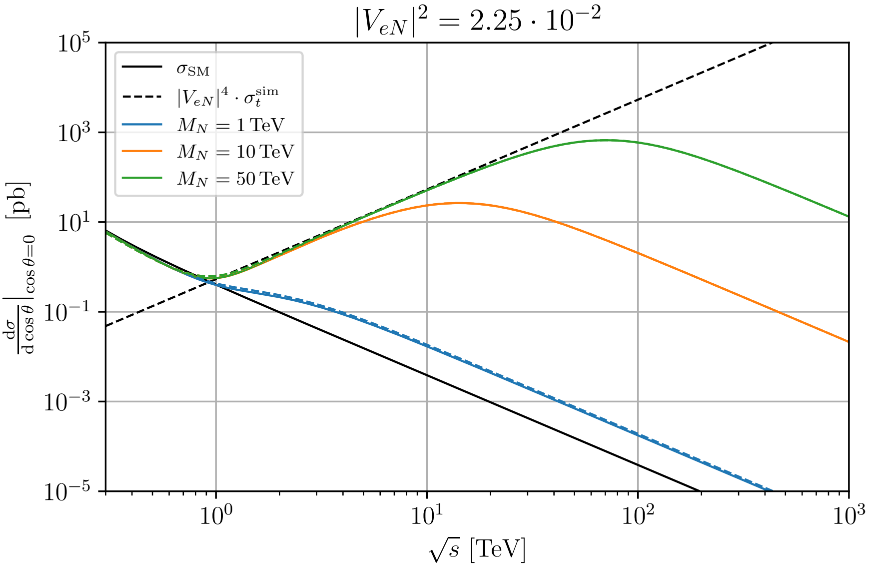


In the SM see-saw type-I extension with $\mathcal{N}$ heavy neutrinos, the process $e^+e^- \to W^+W^-$ is represented by $\mathcal{N}+6$ Feynman diagrams: three corresponding to the s-channel and $\mathcal{N}+3$ to the t-channel.



Comparison to the Toy Model



HeavyNU

HeavyNU[1] is

- template for **FeynRules** model file implementing 3 heavy neutrinos
- Python script for arbitrary mixing implementation
- **FeynRules** model file for default mixing scheme described below
- Wolfram Mathematica file for **UFO** generation (suitable for **MadGraph**, **Wizard**, etc)

Takehiko Asaka and Takanao Tsuyuki, *Phys. Rev. D*, 92(9):094012, 2015.

If there is only one heavy neutrino then $|V_{eN}|^2 = m_{\text{eff}}/M_1 \ll 1$ — too small for the collider experiments. However, if three neutrino are introduced, all constraints could be lifted except ones from EWPO. In this scheme we get only one neutrino with sizable mixing parameter ($\sim V_{eN}$), though this does not mean that other neutrinos do not contribute to the process. It was studied for e^-e^- collisions, but e^+e^- could be very different, so we want to check this.

Default mixing scheme

$$M_1 \ll M_2 \ll M_3$$

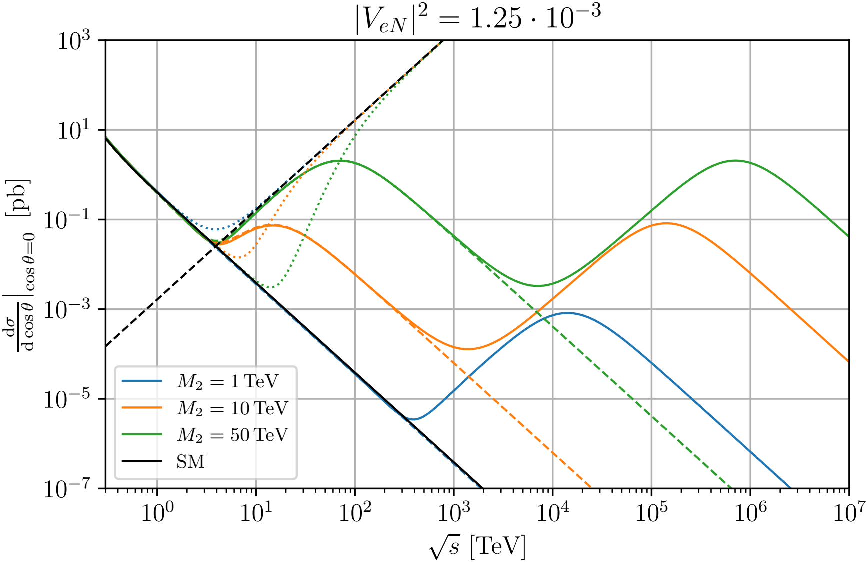
$$m_{\nu_1} = m, \quad m_{\nu_2} = m_{\nu_3} = 0,$$

$$|V_{e1}|^2 = \frac{M_1}{M_2}|V_{eN}|^2, \quad |V_{e2}|^2 \equiv |V_{eN}|^2, \quad |V_{e3}|^2 = \frac{M_2}{M_3}|V_{eN}|^2$$

We take m_{ν_1} non-zero to satisfy the see-saw relation for our mixing parameters.

[1] It is called **HeavyNU** since it is based on **HeavyN**, and "**U**" stands for unitary neutrino mixing.

Three heavy neutrinos



$$\frac{M_1}{M_2} = \frac{M_2}{M_3} = 10^{-4}$$

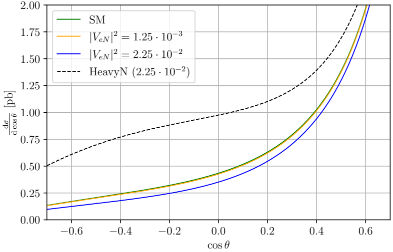
Amplification range $\frac{M\sqrt{3}}{|V_{eN}|} \lesssim \sqrt{s} \lesssim M_N$

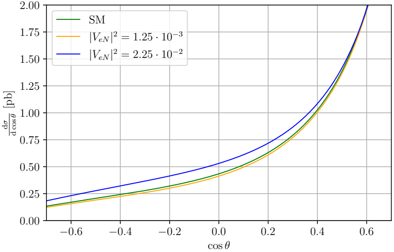
$$\frac{M\sqrt{3}}{|V_{eN}|} = \begin{cases} 3.940 \text{ TeV for } |V_{eN}|^2 = 1.25 \cdot 10^{-3} \Rightarrow \text{No amplification for } M_N = 0.5 \text{ TeV and } M_N = 3 \text{ TeV} \\ 0.930 \text{ TeV for } |V_{eN}|^2 = 2.25 \cdot 10^{-2} \Rightarrow \text{No amplification for } M_N = 0.5 \text{ TeV} \end{cases}$$

$M_N = 0.5$ TeV

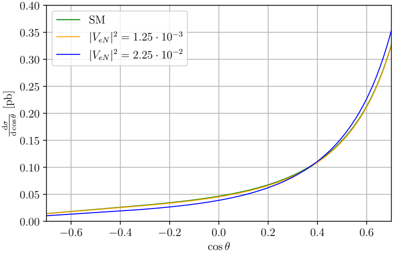
$M_N = 3$ TeV

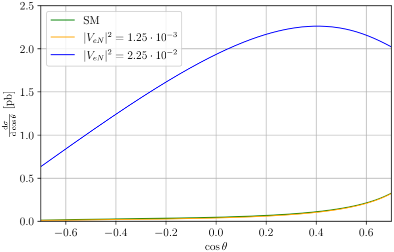
$\sqrt{s} = 1$ TeV



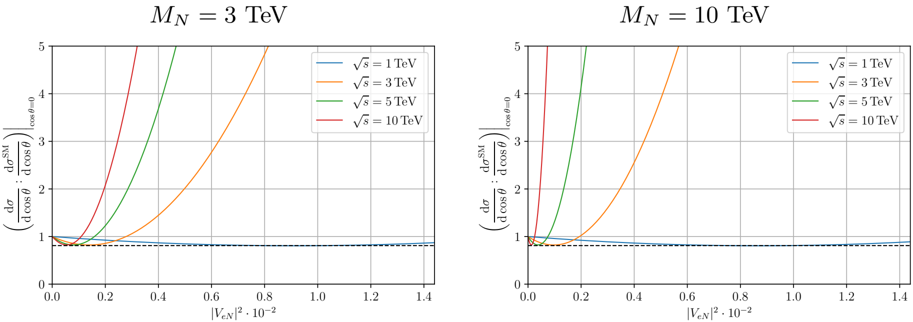


$\sqrt{s} = 3$ TeV





Suppression at smaller energies



For the Toy Model in the limit $\sqrt{s} \gg M_W$ we get

$$|V_{eN}|^2_{\text{min}} = \frac{M_W^2\left(s + 2M_N^2\right)}{2M_N^2 s} \quad \Rightarrow \quad \left(\frac{d\sigma^{\text{un}}}{d\cos\theta} : \frac{d\sigma^{\text{sim}}}{d\cos\theta}\right)\Bigg|_{\cos\theta=0} = \frac{8}{9}.$$

The minimal value of the cross section in this Standard Model extension can be found in the same limit $\sqrt{s} \gg M_W$ using FeynCalc:

$$\left(|V_{eN}|^2\right)^{\mathcal{N}}_{\text{min}} = \frac{M_W^2\left(s + 2M_N^2\right)}{2M_N^2 s \cdot \cos^2\theta_W},$$

$$\left(\frac{d\sigma^{\mathcal{N}}}{d\cos\theta} : \frac{d\sigma^{\text{SM}}}{d\cos\theta}\right)\Bigg|_{\cos\theta=0} = \frac{8 - 16\sin^2\theta_W + 8\sin^4\theta_W}{9 - 16\sin^2\theta_W + 8\sin^4\theta_W} \approx 0.82.$$

Recent paper

Shortly before our paper the following preprint was published:

E. Gabrielli, A. Lind, L. Marzola, K. Müürsepp, and E. Nardi. Testing the unitarity of the light neutrino mixing matrix. arXiv:2603.12385.

Key points and differences:

- It was indicated that if the PMNS matrix is not unitary there should be linear growth of the cross section with s. It is possible to look for it in future experiments.
- The limit $M_N \to \infty$ was considered, so there is no dependence on M_N anywhere, while in our paper we have found the conditions for M_N and mixing parameters in order to get sizable corrections to the SM predictions.
- We also emphasize that the minimal value of the ratio to the SM differential cross section does not depend on both $\sqrt{s}$ and M_N in the relevant parameters region.
- The comparison with the linearized mixing is one of the main points in our paper, and it could not be done in the limit $M_N \to \infty$ since all effects would be at $\sqrt{s} \to \infty$.
- We provide `FeynRules` model file for MC event generators.

Conclusions

- The process $e^+e^- \to W^+W^-$ is studied within a see-saw type-I framework.
- An analytic calculation for a single heavy neutrino was performed in a simplified toy model, whose results reproduce the key features of the corresponding extension of the Standard Model.
- We have also obtained results for the case of three heavy neutrinos for which we provide `FeynRules` model files.
- Furthermore, we have highlighted the main differences between the exact unitary mixing of light and heavy neutrinos and its linearized approximation, the latter becoming inapplicable once the center-of-mass energy $\sqrt{s}$ reaches the point where SM cross section crosses the asymptotic regime.
- It is also important that the cross section can be suppressed by up to 18 % in exact unitary mixing approach, while for the linearized mixing the cross section is always greater than in the SM. This implies that precise measurements of the differential cross section in the central angular region could also reveal the presence of heavy neutrinos.

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Thank you for your attention!