False vacuum decay around black holes

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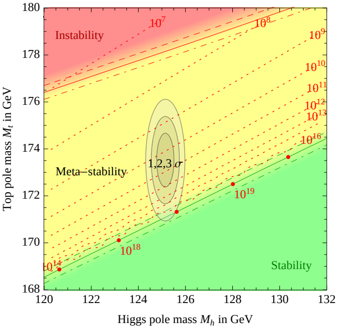
Motivation

According to experimental data and calculations within the Standard Model the electroweak vacuum is metastable and can decay due to quantum tunneling.

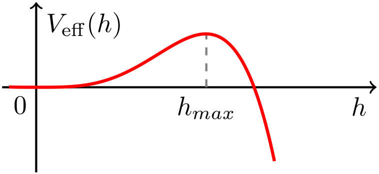


D. Buttazzo et al (2013)

$$V_{\text{eff}}(h) = \frac{1}{4}\lambda_{\text{eff}}(h)h^4$$

$$\lambda_{\text{eff}}(h) \simeq -b\ln\left(\frac{h^2}{h_{max}^2\sqrt{e}}\right)\ ,$$

$$h_{max} \simeq 5\times 10^{10}\,\text{GeV}$$



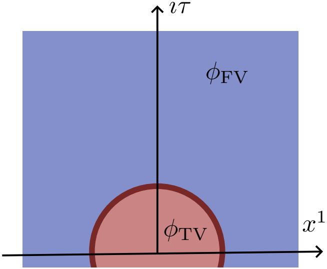
Instantons

The standard approach is based on semiclassical solutions in Euclidean time – instantons.

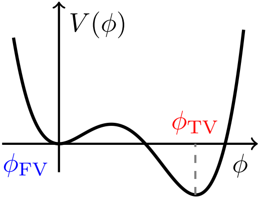
$$S = \frac{1}{\lambda} \int d^4x \left(\frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\partial_x \phi)^2 - V(\phi) \right)$$

The probability of false vacuum decay:

$$\mathcal{P}_{\text{decay}} \sim \int \mathcal{D}\phi \, e^{-S_E[\phi]} \xrightarrow[\text{saddle point, } \lambda \ll 1]{} \boxed{\mathcal{P}_{\text{decay}} \sim e^{-S_E[\phi_s]}}$$



An instanton

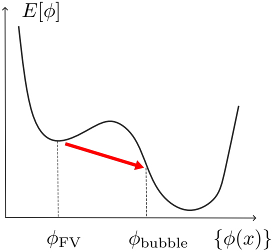


M. B. Voloshin et al (1974)

S. R. Coleman (1977)

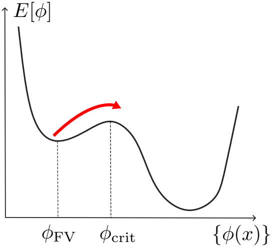
Tunneling at $T = 0$ and $T \neq 0$

The energy barrier between two vacua



$$T = 0$$

$$\mathcal{P}_{\mathrm{decay}} \sim e^{-S_0}$$



$$T > T_{\mathrm{crit}}$$

$$\mathcal{P}_{\mathrm{decay}} \sim e^{-E_{\mathrm{crit}}/T}$$

– the decay probability can be dramatically increased in a hot thermal bath!

Black holes as nucleation seeds

At zero temperature, the Higgs vacuum decay time is much longer than the age of the Universe. But the decay time can be decreased in the thermal bath.

Black hole temperature

$$T_{BH} = \frac{M_{Pl}^2}{8\pi M_{BH}}$$

Conjecture: small black holes have high temperatures $\Longrightarrow$ significantly increase the decay probability.

P. Burda et al (2016)

Calculations from first principles in the (1+1)-dimensional model do not support this assumption. The (1+3)-dimensional case, however, requires numerical computation.

A. Shkerin, S. Sibiryakov (2021)

- We consider a model with potential:

$$V(\phi) = \frac{1}{2}m^2\phi^2 - \frac{1}{2}\mathrm{g}m\phi^3\,.$$

- Asssumnptions:
  1. $E_{\text{field}} \ll M_{BH}$ – the metric is static ($E_{\text{field}} \sim m/\mathrm{g}^2 \ll 2r_h/M_{Pl}^2$).

  2. $\phi(x)$ is spherically symmetric.
- The action with Schwarzschild metric:

$$S = \int \sqrt{-g}d^4x\left(-\frac{1}{2}\phi\Box\phi - V\left(\phi\right)\right)$$

$$ds^2 = f\left(r\right)dt^2 - \frac{dr^2}{f\left(r\right)} - r^2 d\Omega^2\,, \qquad f\left(r\right) = 1 - \frac{r_h}{r}$$

Initial and Final states

- The initial field state at $t_i \to -\infty$ for evaporating BH in empty space is Unruh vacuum. This state is described by the density matrix:

  $$\hat{\rho}_{\rm out} \sim e^{-\hat{H}_{\rm out}/T_{BH}} ,$$

  where T_{BH} is the temperature of outgoing radiation.
- In the case of non-zero incoming flux at temperature T_∞, it is described by the thermal density matrix:

  $$\hat{\rho}_{\rm in} \sim e^{-\hat{H}_{\rm in}/T_\infty} .$$

- The total initial state is the product

  $$\hat{\rho} = \hat{\rho}_{\rm in} \otimes \hat{\rho}_{\rm out}$$

  In the limit $T_\infty \to 0$, $\hat{\rho}_{\rm in} \to |0\rangle_{\rm in}\langle 0|_{\rm in}$, where $|0\rangle_{\rm in}$ is the vacuum state for incoming waves.
- The final state after vacuum decay can be any configuration state $|\phi_{\rm f}\rangle$ if a configuration $\phi_{\rm f}(\mathbf{x})$ includes an expanding true vacuum bubble.

- We calculate the decay probability with the path integral:

$$\mathcal{P}_{\rm decay} \sim \int_{\phi_{\rm f}\in {\rm TV}} \mathcal{D}\phi_{\rm f}\, \langle \phi_{\rm f}|\hat{\mathcal{S}}\hat{\rho}\hat{\mathcal{S}}^{\dagger}|\phi_{\rm f}\rangle\,,$$

  where $\hat{\mathcal{S}}$ is the S-matrix and the integration is taken over all configurations $\phi_{\rm f}$ in the basin of attraction of the true vacuum.

- It is convenient to introduce a new field φ and the tortoise coordinate x:

$$\varphi = {\rm g}\phi r\,, \quad dr/dx = f(r)\,, \quad x = r + r_h \log\left(r/r_h - 1\right)\,.$$

  Tnen the action takes the form:

$$S = \frac{4\pi}{{\rm g}^2}\int dt dx\left[-\frac{1}{2}\varphi(\partial_t^2\varphi - \partial_x^2\varphi) - \frac{1}{2}U_{\rm eff}(x)\varphi^2 + \frac{f(r)}{2r}m\varphi^3\right]\,,$$

  where

$$U_{\rm eff}(x) = \frac{f(r)f'(r)}{r} + m^2 f(r)\,.$$

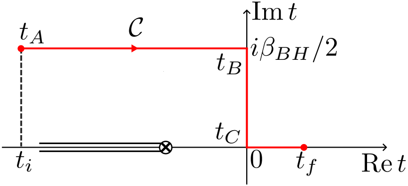
- The problem can be considered as a general (1+1)-dimensional problem with three independent parameters r_h, T_{BH} and T_∞.

Semiclassical method

- The decay probability can be estimated in the saddle-point approximation $\mathcal{P}_{\text{decay}} \sim e^{-F}$ if $\mathrm{g} \ll 1$.
- The path integral is saturated by the saddle solution $\varphi_{\text{cl}}(t,x)$:

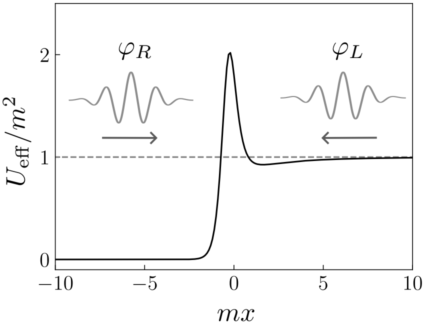
$$\partial_t^2 \varphi_{\text{cl}} - \partial_x^2 \varphi_{\text{cl}} + U_{\text{eff}}(x)\varphi_{\text{cl}} - \frac{3f(r)}{2r} m\varphi^2 = 0\,.$$

This solution is defined on the complex contour $\{t_{\text{i}} t_A t_B t_C t_{\text{f}}\}$ shown, where $t_{\text{f}} - t_{\text{i}} \to \infty$.



S. Yu. Khlebnikov et al (1990)

V. A. Rubakov, M. E. Shaposhnikov (1996)



The semiclassical method

- At $t = t_{\rm i} \to -\infty$, the field $\varphi_{\rm cl}$ is close to the false vacuum, and all perturbations above the false vacuum are located in the regions $x \to \pm\infty$, where the equation is linear. Thus, we can consider only the linearized equation:

$$\partial_t^2 \varphi_{\rm cl} - \partial_x^2 \varphi_{\rm cl} + U_{\rm eff}(x) \varphi_{\rm cl} = 0\,.$$

- The solution to this equation can be linearly decomposed into right- and left-moving modes:

$$\varphi_{\rm cl}(t_{\rm i}, x) = \varphi_R(t_{\rm i}, x) + \varphi_L(t_{\rm i}, x) =$$

$$= \sum_{I=R,L} \int \frac{d\omega}{\sqrt{4\pi\omega}} \left[a_{I,\omega} f_{I,\omega}(x) e^{-i\omega t_{\rm i}} + b_{I,\omega} f^*_{I,\omega}(x) e^{i\omega t_{\rm i}} \right] ,$$

where $a_{I,\omega}$ and $b_{I,\omega}$ are amplitudes – semiclassical analogues of annihilation and creation operators. $f_{I,\omega}(x)$ are right- and left-moving modes satisfying:

$$- f''_{I,\omega}(x) + U_{\rm eff}(x) f_{I,\omega}(x) = \omega^2 f_{I,\omega}(x)\,.$$

Asymptotic expressions:

$$f_{R,\omega} = \begin{cases} \alpha_\omega e^{i\omega x} + \beta_\omega e^{-i\omega x}\,, & x \to -\infty \\ \gamma_\omega e^{ikx}\,, & x \to +\infty \end{cases}\,, \qquad \omega > 0\,,$$

$$f_{L,\omega} = \begin{cases} \tilde{\beta}_\omega e^{-i\omega x}\,, & x \to -\infty \\ \tilde{\gamma}_\omega e^{ikx} + \tilde{\delta}_\omega e^{-ikx}\,, & x \to +\infty \end{cases}\,, \qquad \omega > m\,.$$

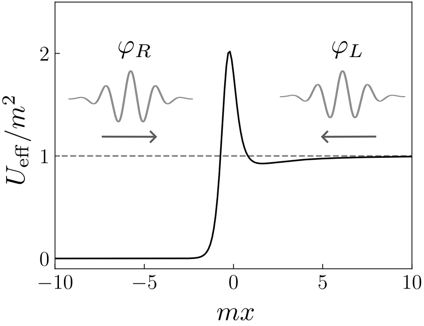
The initial conditions are also obtained from the path integral in the saddle-point approximation. It is convenient to impose them on the coefficients $a_{I,\omega}$ and $b_{I,\omega}$:

$$a_{L,\omega} = e^{-\beta_\infty \omega}(b_{L,\omega})^*\,,$$

$$a_{R,\omega} = e^{-\beta_{\rm BH} \omega}(b_{R,\omega})^*\,,$$

where β_∞ and β_{BH} are the inverse temperatures.

$T = T_{\rm BH}$ – **Hartle-Hawking** vacuum
$T = 0$ – **Unruh** vacuum.



- The field $\varphi_{\rm cl}$ at $t_{\rm f} \to +\infty$ should belong to the basin of attraction of the true vacuum:

$$\mathrm{Im}\,\varphi_{\rm cl}(t_{\rm f}, x) = \mathrm{Im}\,\partial_t \varphi_{\rm cl}(t_{\rm f}, x) = 0\,.$$

- The decay probability in the saddle-point approximation is exponentially suppressed, $\mathcal{P} \sim e^{-F[\varphi_{\rm cl}]}$, where the suppression F is a functional of the form:

$$\begin{aligned} & F = 2\mathrm{Im}\,S = \\ & = 2\mathrm{Im}\left[\frac{4\pi}{\mathrm{g}^2}\int_{\mathcal{C}} dt \int_{-\infty}^{\infty} dx\, r^2 f(r)\left(\frac{1}{2}\varphi_{\rm cl} V'_{\rm int}\left(\frac{\varphi_{\rm cl}}{r}\right) - V_{\rm int}\left(\frac{\varphi_{\rm cl}}{r}\right)\right)\right], \end{aligned} \tag{1}$$

where $V_{\rm int}(\phi)$ is the non-quadratic part of the potential.

Problem

The equation:

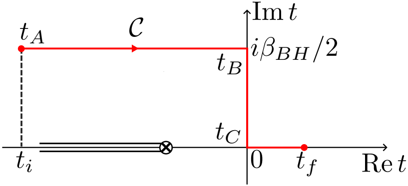
$$\partial_t^2\varphi_{\rm cl} - \partial_x^2\varphi_{\rm cl} + U_{\rm eff}(x)\varphi_{\rm cl} - \frac{3f(r)}{2r}m\varphi^2 = 0\,.$$

The boundary conditions:

$$a_{L,\omega} = e^{-\beta_\infty\omega}(b_{L,\omega})^*\,,$$

$$a_{R,\omega} = e^{-\beta_{\rm BH}\omega}(b_{R,\omega})^*\,,$$

$$\mathrm{Im}\,\varphi_{\rm cl}(t_{\rm f},x) = \mathrm{Im}\,\partial_t\varphi_{\rm cl}(t_{\rm f},x) = 0\,.$$



With the solution, we can calculate the suppression

$$F = 2\mathrm{Im}S\,.$$

The probability in the saddle point approximation

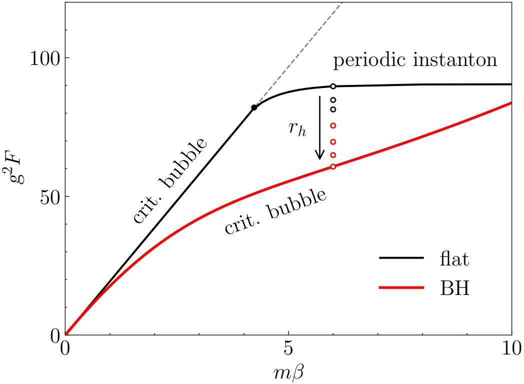
$$\mathcal{P} \sim e^{-F}$$

Numerical implementation

- Introduce $N_t \times N_x$ lattice $\{t_i, x_j\}$ that covers the domain $-L_1 \leq x \leq L_2$ and the contour $\mathcal{C}$ with finite t_i and t_f.
  We use the standard second order discretization of the problem
  $\Longrightarrow$ we obtain a system of nonlinear algebraic equations.
- Newton-Raphson method: given an initial guess $\varphi^{(0)}$, substituting $\varphi_i = \varphi_i^{(0)} + u_i$ into the discretized equations yields a system of linearized equations. Once the system is solved for $u^{(0)}$, we set $\varphi_i^{(1)} = \varphi_i^{(0)} + u^{(0)}$, $\varphi_i = \varphi_i^{(1)} + u_i$ and iterate until the configuration $\varphi^{(n)}$ converges with the required accuracy.
- We can use the solution for parameters β_{BH}, β_∞, r_h as a seed to find the solution for parameters $\beta_{BH} + \Delta\beta_{BH}$, $\beta_\infty + \Delta\beta_\infty$, $r_h + \Delta r_h$, where $\Delta\beta_{BH}$, $\Delta\beta_\infty$, Δr_h are small.
- Typical values of the parameters are $L_1, L_2 = 50 \div 100$, $(t_\mathrm{f} - t_\mathrm{i}) = 25 \div 30$, $\Delta x = 0.05 \div 0.1$, $\Delta t = 0.025 \div 0.05$.
  The numerical errors are under control. Typical relative numerical errors $\delta F/F$ are: $\delta F/F < 0.2\%$ for Δx, $\delta F/F < 0.07\%$ for Δt, $\delta F/F < 0.2\%$ for L_1, L_2. Energy is conserved with $\delta E/E < 0.2\%$. The linearization at $t = t_\mathrm{i}$ is accurate to $|E_\mathrm{lin} - E|/E < 0.3\%$.

Thermal transitions

Simple case $T_{BH} = T_\infty \implies$ the solution is real and only Euclidean part contributes to the suppression. That is **periodic instanton**.



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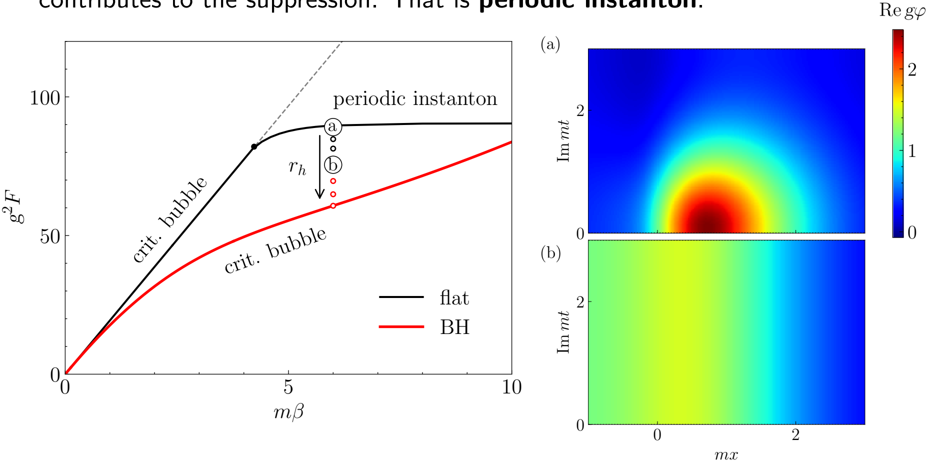
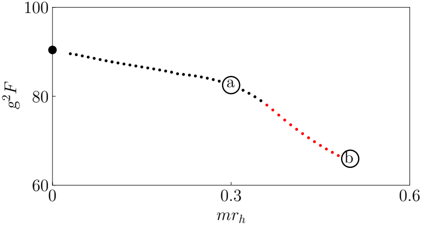
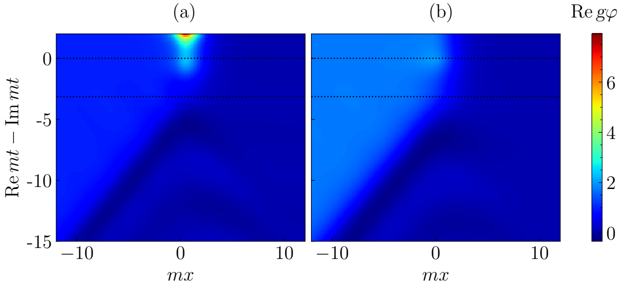


Vacuum transitions

The same recipe can be used to find solutions of the generalized problem in the case $T_\infty = 0$





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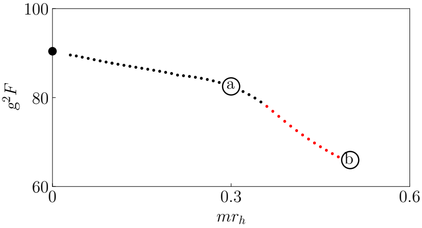
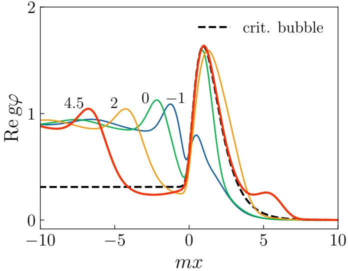
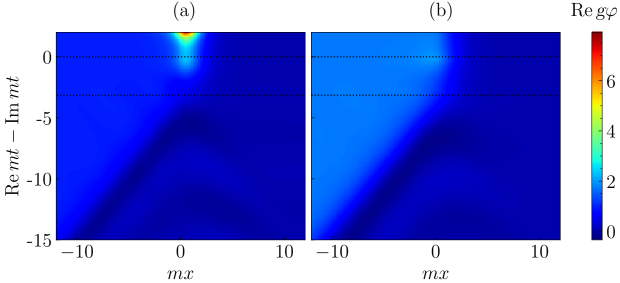
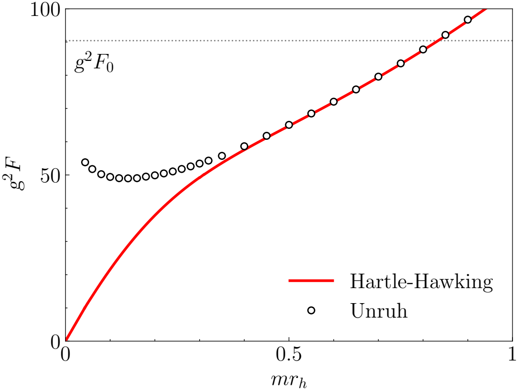






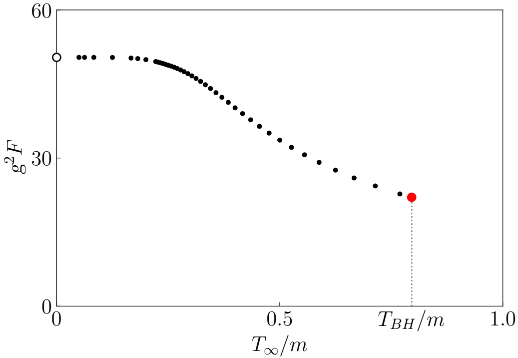
Vacuum transitions

Suppression as a function of horizon radius (Unruh and Hartle-Hawking vacua)



The suppression of Unruh vacuum stays exponentially suppressed in the limit of small r_h!

Solutions for Unruh and Hartle-hawking vacua are continuously connected with change of T_∞.



Conclusion

- In model ϕ^3, vacuum decay proceeds via a critical bubble for all BH sizes.
- In the limit of large r_h, the solutions and decay probabilities for the cases $T_\infty = 0$ and $T_\infty = T_{BH}$ coincide.
- Although BHs with $mr_h < 0.8$ enhance the Unruh vacuum decay probability, it remains exponentially suppressed and decreases in the limit of small r_h.

Thank you for your attention!

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