BRST Lagrangian formulation of partially massless bosonic fields

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based on

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Motivation

Higher spin theory is a actively developing area of modern theoretical and mathematical physics (see, e.g. the reviews [M.Vasiliev, 2004; D.Sorokin, 2005; X.Bekaert, S.Cnockaert, C.Iazeolla, M.Vasiliev, 2004; X.Bekaert, N.Boulanger, P.Sundell, 2012; V.Didenko, E.Skvortsov, 2014; M.Vasiliev, 2015; X.Bekaert, N.Boulanger, A.Campaneoli, et al, 2022; D.Ponomarev, 2022] and refs.therein).

Recently, there has been a surge of interest in studying the branch associated with the formulation of partially massless higher-spin fields.

Partially massless irreps are inherent in the (A)dS spaces, but not in flat space.

Pioneering papers on the study of such irreps are [S.Deser, A.Waldron, 2001; Yu.Zinoviev, 2001], taking into account the results obtained in [S.Deser, R.Nepomichie, 1984; A.Higuchi, 1987; L.Brink, R. Metsaev, M. Vasiliev, 2000].

To date, there have been many authors who have studied partially massless representations (see some of the latest articles [R. Metsaev, 2023; K.Hinterbichler, A.Joyce, 2025; Yu.Zinoviev, 2025, 2026] and refs. therein).

Conventional on-shell description of the $(A)dS_4$ irreducible representations with a given mass m and a given integer spin s is realized

(only fields with integer spins will be considered)

on the fields $\varphi_{\mu_1\ldots\mu_s}(x) = \varphi_{(\mu_1\ldots\mu_s)}(x) \equiv \varphi_{\mu(s)}(x)$, which are traceless,

$$g^{\mu_1\mu_2}\varphi_{\mu_1\mu_2\ldots\mu_s} = 0\,,$$

and subject to the conditions [C.Fronsdal, 1978; R.Metsaev, 2003]
(here $\Box = \nabla^\mu\nabla_\mu$)

$$\Big\{\Box + \varkappa\left(s^2 - 2s - 2\right) - m^2\Big\}\varphi_{\mu_1\ldots\mu_s} = 0\,, \qquad \nabla^{\mu_1}\varphi_{\mu_1\ldots\mu_s} = 0\,,$$

where $\varkappa$ is the constant in the curvature tensor $R_{\mu\nu}{}^{\lambda\rho} = \varkappa\left(\delta^\lambda_\mu\delta^\rho_\nu - \delta^\rho_\mu\delta^\lambda_\nu\right)$:

$\varkappa > 0$ for dS space and $\varkappa < 0$ for AdS space.

Similar to the flat case,

- at $m = 0$ we have massless representations with helicities $\pm s$

(in this case, the fields $\varphi_{\mu(s)}$ have gauge invariance $\delta\varphi_{\mu(s)} = \nabla_{(\mu}\lambda_{\mu(s-1))}$),

- whereas at $m \neq 0$ in general case the fields $\varphi_{\mu(s)}$ describe massive particles.

In contrast to flat case, other representations can be realized on fields in (A)dS.

When the mass parameter m^2 is not arbitrary but is equal for given s to one of the following values

$$m^2 = m_t^2\,, \qquad m_t^2 = \varkappa(s-t-1)(s+t)\,,$$

where the depth t takes one of the following values: $t=0$ or ... or $t=s-1$, basic equations are invariant with respect to gauge transformations

$$\delta\varphi_{\mu_1\dots\mu_s} = \underbrace{\nabla_{(\mu_1}\dots\nabla_{\mu_{s-t}}}_{s-t}\lambda_{\mu_{s-t+1}\dots\mu_s)}\,,$$

where local parameter is symmetric traceless field $\lambda_{\mu_1\dots\mu_t} = \lambda_{(\mu_1\dots\mu_t)}$, $g^{\mu_1\mu_2}\lambda_{\mu_1\mu_2\dots\mu_t} = 0$, obeying the equations

$$\Big\{\Box + \varkappa(s^2+s-t-2)\Big\}\lambda_{\mu_1\dots\mu_t} = 0\,, \qquad \nabla^{\mu_1}\lambda_{\mu_1\dots\mu_t} = 0\,.$$

When $t=s-1 \;\Rightarrow\; m_{s-1}=0$ – purely massless fields.

For all other values t, the fields describe partially massless representations, that envolve states with helicities $\pm s, \pm(s-1), \dots, \pm(t+2), \pm(t+1)$.

Note that gauge transformations of partially massless fields are always higher than first order in the covariant derivatives ∇_μ.

Note also that when $t=0$, the gauge parameter is a scalar: $\delta\varphi_{\mu(s)} = \nabla_{\mu(s)}\lambda$

We will focus on developing a general procedure for constructing Lagrangian description for the theory of partially massless bosonic fields.

In obtaining Lagrangian field theory, the methods of BFV-BRST approach [E.Fradkin, G.Vilkovisky, 1974; I.Batalin, G.Vilkovisky, 1977; E.Fradkin, T.Fradkina, 1978] are employed.

The advantage of the BRST approach is that it is based on the conditions defining the appropriate irreducible field representations as first-class constraints F_a, $[F_a, F_b] = 2f_{ab}{}^{c}F_c$ in some Fock space. All information about the system is encoded in the BRST charge (in the case of first-rank system)

$$Q = \eta^a F_a + f_{ab}{}^{c}\eta^a\eta^b\mathcal{P}_c\,, \qquad Q^2 = 0\,,$$

where η^a and $\mathcal{P}_a$ are the ghosts and their momenta.

It is important that the BRST equations of motion $Q\,|\Phi\rangle = 0$ and BRST-transformations $|\Phi'\rangle = |\Phi\rangle + Q\,|\Lambda\rangle$ follow from Lagrangian

$$\mathcal{L} \sim \langle\bar{\Phi}|Q|\Phi\rangle$$

The efficiency of using BVF-BRST methods to describe higher spin fields:

- There are methods for constructing a real free Lagrangian.
- The Lagrangian obtained in this way contains all the necessary auxiliary fields by construction.
- In BRST approach, the Lagrangian is gauge invariant.
- The BRST formulation, which is quite simple, provides opportunity for constructing a corresponding theory with interaction.

It is important: in this case, building the BRST formulations is not BRST quantization. In fact, such formulations utilize methods of BFV-BRST approach. The theories constructed are classical (not quantum) ones.

The Lagrangian construction of higher-spin fields in BFV-BRST approach was firstly presented in [A.Pashnev, 1989; I.Buchbinder, A.Pashnev, M.Tsulaia, 2001]. This approach was further developed by many authors in various directions of higher spin field theory.

Power of the BRST approach in formulating nonlinear equations for supersymmetric higher-spin theory was emphasized in [M.Vasiliev, 2025].

In the most of the papers, the Lagrangian BRST formulation uses the fields with only vector indices (plus one spinor index for fermions) and the necessary condition of tracelessness greatly complicates the design.

In [I.Buchbinder, V.Krychtin, H.Takata, 2018; I.Buchbinder, SF, A.Isaev, V.Krychtin, 2020, 2022, 2024] (BRST formulation of continuous spin fields) and in [I.Buchbinder, SF, A.Isaev, V.Krychtin, 2024; I.Buchbinder, SF, V.Krychtin, 2024] (BRST formulation of higher finite spin fields), the fields with Weyl indices were used.

We deal with spin-tensor fields (defined in the tangent space) of the form

$$\varphi_{\alpha_1\ldots\alpha_s}^{\dot\beta_1\ldots\dot\beta_s} = (\sigma^{m_1})_{\alpha_1}{}^{\dot\beta_1}\ldots(\sigma^{m_s})_{\alpha_s}{}^{\dot\beta_s} e_{m_1}^{\mu_1}\ldots e_{m_s}^{\mu_s}\varphi_{\mu_1\ldots\mu_s}$$

and their inverses. If this field is symmetric with respect to its spinor indices

$$\varphi_{\alpha_1\ldots\alpha_s}^{\dot\beta_1\ldots\dot\beta_s}(x) = \varphi_{(\alpha_1\ldots\alpha_s)}^{(\dot\beta_1\ldots\dot\beta_s)}(x) := \varphi_{\alpha(s)}^{\dot\beta(s)}(x)\,,$$

corresponding field $\varphi_{\mu_1\ldots\mu_s}$ is traceless.

Note that in the framework of the formulation in terms of the fields with vector indices, derivation of the invariance conditions and gauge parameter equations is technically an extremely cumbersome procedure.

Obtaining these relations is very simplified in the framework of the Fock space formulation of the theory under consideration.

Instead of a space with fields $\varphi^{\dot\beta(s)}_{\alpha(s)}(x)$ we consider the Fock space which is formed by ket- and their conjugate bra-vectors

$$|\varphi_s\rangle = \frac{1}{s!}\,\varphi^{\dot\alpha(s)}_{\alpha(s)}(x)\; c^{\alpha(s)}\,\bar c_{\dot\alpha(s)}|0\rangle\,, \qquad \langle\bar\varphi_s| = \frac{1}{s!}\,\langle 0|\,\bar a^{\dot\alpha(s)}\,a_{\alpha(s)}\;\bar\varphi^{\alpha(s)}_{\dot\alpha(s)}(x)\,,$$

where $c^{\alpha(s)} := c^{\alpha_1}\dots c^{\alpha_s}\,,\quad \bar c_{\dot\alpha(s)} := \bar c_{\dot\alpha_1}\dots\bar c_{\dot\alpha_s}\,,$ etc. and
a_α, $\bar a^{\dot\alpha}$ and $\bar c_{\dot\alpha} = (a_\alpha)^\dagger$, $c_\alpha = (\bar a_{\dot\alpha})^\dagger$ are annihilation and creation operators:

$$[a_\alpha, c^\beta] = \delta^\beta_\alpha\,, \qquad [\bar a^{\dot\alpha}, \bar c_{\dot\beta}] = \delta^{\dot\alpha}_{\dot\beta}\,.$$

Vacuum states $|0\rangle$ and $\langle 0| = (|0\rangle)^\dagger$, $\langle 0|0\rangle = 1$ is defined in standard way:

$$\langle 0|\bar c_{\dot\alpha} = \langle 0|c^\alpha = 0\,, \qquad \bar a^{\dot\alpha}|0\rangle = a_\alpha|0\rangle = 0\,.$$

The states $|\varphi_s\rangle$ are eigenvectors for the "particle number" operators

$$K = N + \bar N + 2\,,\quad N = c^\alpha a_\alpha\,,\quad \bar N = \bar c_{\dot\alpha}\bar a^{\dot\alpha}\ :$$

$$N|\varphi_s\rangle = s|\varphi_s\rangle\,, \qquad \bar N|\varphi_s\rangle = s|\varphi_s\rangle\,, \qquad K|\varphi_s\rangle = (2s+2)|\varphi_s\rangle\,.$$

On the vectors $|\varphi_s\rangle$ the on-shell equations are rewritten as follows:

$$\Big\{\ell_0 - 2\varkappa(2s+1)\Big\}|\varphi_s\rangle = 0\,, \qquad \ell|\varphi_s\rangle = 0\,.$$

The gauge transformations on the vectors $|\varphi_s\rangle$ take the form

$$\delta|\varphi_s\rangle = (\ell^+)^{s-t}|\lambda_t\rangle,$$

where the vector $|\lambda_t\rangle$ has the form $\qquad |\lambda_t\rangle = \frac{1}{t!}\,\lambda_{\alpha(t)}^{\dot\beta(t)}(x)\,c^{\alpha(t)}\,\bar c_{\dot\beta(t)}|0\rangle\,.$

Here, we use the set of operators closed under Hermitian conjugation

$$\ell_0 \;=\; D^2 \;+\; \varkappa\,(N\bar N + N + \bar N) - m_t^2\,,$$

$$\ell \;=\; (a\sigma^m\bar a)\,e_m^\mu D_\mu\,, \qquad \ell^+ \;=\; -\,(c\sigma^m\bar c)\,e_m^\mu D_\mu\,.$$

On the Fock space the covariant derivative operator has the form

$$D_\mu \;=\; \partial_\mu \;+\; \frac{1}{2}\,\omega_\mu{}^{mn}\,\mathcal{M}_{mn}\,,$$

where $\omega_\mu{}^{mn}(x)$ is the spin connection,
$\mathcal{M}_{mn} = c^\alpha\,(\sigma_{mn})_\alpha{}^\beta\,a_\beta + \bar c_{\dot\alpha}\,(\tilde\sigma_{mn})^{\dot\alpha}{}_{\dot\beta}\,\bar a^{\dot\beta}$ are the Lorentz algebra generators.

The operators ℓ_0, ℓ, ℓ^+ form the algebra:

$$\left[\ell^+, \ell\right] \;=\; K\,(\ell_0 + m^2)\,,$$

$$[\ell, \ell_0] = 2\varkappa\,(K+1)\,\ell\,, \qquad [\ell_0, \ell^+] = 2\varkappa\,(K-1)\,\ell^+\,.$$

This algebra is closed only in the purely massless case $m = 0$.

At $m = m_t \neq 0$ when $t \neq s-1$, in language of quantization of constrained system, this is typical algebra for systems with 2d-class constraints:
ℓ_0 is 1st-class constraint, whereas ℓ, ℓ^+ are 2nd-class ones.

It is important that as the constraint operator we must take precisely operator ℓ_0, which defines irreducible representations:
the shift $\ell_0 \to \ell_0 + m^2$ corresponds to the consideration of the massless case.

In standard procedure, BFV-BRST formulation uses only 1st-class constraints.

One of the ways to processing of 2nd-class constraints in the BFV-BRST formalism is the conversion procedure.

The conversion procedure is based on extending 2nd-class constraints by additional variables so that new constraints form 1st-class constraint system.
The number of added variables must be equal to the number of 2nd-class constraints being converted.

We add to the oscillators $\bar{c}_{\dot\alpha}$, c^α and $\bar{a}^{\dot\alpha}$ a_α the new bosonic oscillators b and $b^+ = (b)^+$ subject to the standard bosonic commutation relations:

$$[b, b^+] = 1\,.$$

Following the BFV prescription, we extend the Fock space by the fermionic ghost coordinate operators $\eta_0 = \eta_0^+$, η, $\eta^+ = (\eta)^+$ and the corresponding ghost momenta $\mathcal{P}_0 = \mathcal{P}_0^+$, $\mathcal{P}^+ = (\mathcal{P})^+$, $\mathcal{P}$:

$$\{\eta, \mathcal{P}^+\} \;=\; \{\mathcal{P}, \eta^+\} \;=\; \{\eta_0, \mathcal{P}_0\} \;=\; 1.$$

The extended vacuum $|0\rangle$ is defined by the conditions

$$b|0\rangle \;=\; 0\,, \qquad\qquad \mathcal{P}_0|0\rangle \;=\; \eta|0\rangle \;=\; \mathcal{P}|0\rangle \;=\; 0\,.$$

The BRST-vectors which have zero ghost number looks like:

$$|\Psi_s\rangle = |\Phi_s\rangle \;+\; \eta_0\mathcal{P}^+|\mathcal{Z}_{s-1}\rangle \;+\; \eta^+\mathcal{P}^+|\mathcal{X}_{s-2}\rangle\,.$$

The vectors $|\Psi_s\rangle$ is the eigenvector of the Hermitian spin operator:

$$\mathcal{S}|\Psi_s\rangle = s|\Psi_s\rangle\,,$$

where the spin operator in extended Fock space with ghosts has the form:

$$\mathcal{S} = \frac{1}{2}\left(N + \bar{N}\right) + \mathrm{n} + \eta^+\mathcal{P} + \mathcal{P}^+\eta\,, \qquad\qquad \mathrm{n} := b^+ b\,.$$

We take the following natural form of converted constraints:

$$L_0 = \ell_0 + A_0(\mathrm{n}), \qquad L = \ell + A(\mathrm{n})b, \qquad L^+ = \ell^+ + b^+A^+(\mathrm{n}).$$

where ℓ_0, ℓ, ℓ^+ are the initial constraints.

Additional parts $A_0(\mathrm{n})$, $A(\mathrm{n})b$, $b^+A^+(\mathrm{n})$ depend on b and b^+, where $A_0(\mathrm{n})$, $A(\mathrm{n})$ and $A^+(\mathrm{n})$ are some unknown yet functions of the operator $\mathrm{n} = b^+b$.

Explicit forms of $A_0(\mathrm{n})$ and $A(\mathrm{n})$ are determined by requirement that BRST operator is Hermitian and nilpotent on the states $|\Psi_s\rangle$.

To construct the BRST charge we use the analogy with pure massless theory ($t = s - 1$) in (A)dS space [I.Buchbinder, SF, A.Isaev, V.Krychtin, 2024]. Therefore, we consider the following desired algebra for converted constraints:

$$\left[L^+, L\right] = KL_0, \quad \left[L, L_0\right] = 2\varkappa\,(K+1)\,L, \quad \left[L_0, L^+\right] = 2\varkappa\,L^+\,(K+1),$$

which should be fulfilled in the space of the vectors $|\Psi_s\rangle$.

Taking into account this algebra, we get the Hermitian BRST charge:

$$\begin{aligned} Q \;\; = \;\; & \eta_0 L_0 + \eta^+ L + \eta\, L^+ + K\eta^+\eta\mathcal{P}_0 \\ & - 2\varkappa(K-1)\eta_0\eta\mathcal{P}^+ + 2\varkappa(K+1)\eta_0\eta^+\mathcal{P} - 4\varkappa\,\eta_0\eta^+\eta\,\mathcal{P}^+\mathcal{P}. \end{aligned}$$

When finding the nilpotent BRST charge, we obtain $A_0(\mathrm{n}) = 2\varkappa\,\mathrm{n}(2s-\mathrm{n})$ and the necessity of fulfilling the relation

$$A(\mathrm{n})A^+(\mathrm{n}) \;=\; (2s-\mathrm{n})\Big[m^2 - \varkappa\,\mathrm{n}\big(2s-\mathrm{n}-1\big)\Big].$$

The right-hand side must take non-negative values on the considered vectors.

In fact, it is a condition of unitarity of the partially massless theory.

In dS space with $\varkappa > 0$, this condition for fixed values of s and $t < s$ is satisfied for the following values of k in expansion for $|\Phi_s\rangle$:

$$k = t+1, t+2, \ldots, s-1, s\,.$$

Thus, in this case the expansions for $|\Phi_s\rangle$, $|\mathcal{Z}_{s-1}\rangle$ and $|\mathcal{X}_{s-2}\rangle$ have the form:

$$|\Phi_s\rangle = \sum_{k=t+1}^{s} (b^+)^{s-k}|\varphi_k\rangle\,,$$

$$|\mathcal{Z}_{s-1}\rangle = \sum_{k=t}^{s-1}(b^+)^{s-1-k}|\zeta_k\rangle\,, \qquad |\mathcal{X}_{s-2}\rangle = \sum_{k=t-1}^{s-2}(b^+)^{s-2-k}|\chi_k\rangle\,.$$

In AdS space with $\varkappa < 0$, the required condition is not satisfied at least for vectors with spin s. Thus, in this case the Lagrangian formulation for partially massless higher-spin fields is impossible in the BRST approach.

Equation of motion in the BRST approach has the form:

$$Q|\Psi_s\rangle = 0 .$$

Eliminating $|\mathcal{Z}_{s-1}\rangle$ algebraically with the help of the equation of motion, we get equations of motion of residual vectors $|\Phi_s\rangle$ and $|\mathcal{X}_{s-2}\rangle$:

$$\Big(L_0 - 2\varkappa(K-1) + L^+K^{-1}L\Big)|\Phi_s\rangle - L^+K^{-1}L^+|\mathcal{X}_{s-2}\rangle = 0 ,$$
$$\Big(L_0 + 2\varkappa(K+1) - LK^{-1}L^+\Big)|\mathcal{X}_{s-2}\rangle + LK^{-1}L|\Phi_s\rangle = 0 ,$$

which are obtained also from the Lagrangian

$$\begin{aligned}\mathcal{L}_{s,t} \;=\; & \langle\bar{\Phi}_s|\Big(L_0 - 2\varkappa(K-1) + L^+K^{-1}L\Big)|\Phi_s\rangle \\ & - \langle\bar{\mathcal{X}}_{s-2}|\Big(L_0 + 2\varkappa(K+1) - LK^{-1}L^+\Big)|\mathcal{X}_{s-2}\rangle \\ & - \langle\bar{\mathcal{X}}_{s-2}|\,LK^{-1}L\,|\Phi_s\rangle - \langle\bar{\Phi}_s|\,L^+K^{-1}L^+\,|\mathcal{X}_{s-2}\rangle .\end{aligned}$$

This Lagrangian are gauge invariant under transformations

$$\delta|\Phi_s\rangle = L^+|\Lambda_{s-1}\rangle , \qquad \delta|\mathcal{X}_{s-2}\rangle = L|\Lambda_{s-1}\rangle .$$

This gauge invariant Lagrangian describe partially massless fields.

Specifically, in the vector $|\Phi_s\rangle$ the states $|\varphi_{t+1}\rangle, \cdots, |\varphi_{s-1}\rangle$ are Zinoviev-like gauge ones (Stückelberg ones), and the vector $|\varphi_s\rangle$ describes physical helicities. All vectors in $|\mathcal{X}_{s-2}\rangle$ correspond to gauge and auxiliary degrees of freedom.

Demonstration of how gauge symmetries with higher derivatives are obtained:

In the de Donder gauge the gauge transformations have the form:

$$\begin{aligned} \delta|\varphi_s\rangle &= \ell^+|\lambda_{s-1}\rangle\,, \\ \delta|\varphi_k\rangle &= \ell^+|\lambda_{k-1}\rangle + A(s-k-1)\,|\lambda_k\rangle\,, \qquad t<k<s\,. \end{aligned}$$

We can put the gauges $|\varphi_k\rangle = 0\,, \quad t<k<s\,.$

After rescaling $|\lambda_t\rangle$, the remaining transformations take the required form:

$$\delta|\varphi_s\rangle \;=\; (\ell^+)^{s-t}|\lambda_t\rangle\,.$$

It is important that the structure $\delta|\varphi_s\rangle \sim (\ell^+)^{s-t}|\lambda_t\rangle$ arose due to the presence of the $(s-t-1)$ Stückelberg fields $|\varphi_{t+1}\rangle, |\varphi_{t+2}\rangle, \cdots, |\varphi_{s-2}\rangle, |\varphi_{s-1}\rangle$.

Helicity contents:

$$|\varphi_s\rangle \;= \underbrace{|\phi_s\rangle + \ldots + (\ell^+)^{s-t-1}|\phi_{t+1}\rangle}_{\text{physical states}} + \underbrace{(\ell^+)^{s-t}|\phi_t\rangle + \ldots + (\ell^+)^{s}|\phi_0\rangle}_{\text{gauge states}},$$

$$|\lambda_t\rangle \;=\; |\xi_t\rangle + \ell^+|\xi_{t-1}\rangle + \ldots + (\ell^+)^{t-1}|\xi_1\rangle + (\ell^+)^{t}|\xi_0\rangle\,.$$

Then, by using gauge transformations $\delta|\varphi_s\rangle = (\ell^+)^{s-t}|\lambda_t\rangle$

we remove the fields $|\phi_t\rangle, \ldots, |\phi_0\rangle$.

As a result, we are left with the fields $|\phi_s\rangle, |\phi_{s-1}\rangle, \ldots, |\phi_{t+2}\rangle, |\phi_{t+1}\rangle$, which describe the states with helicities $\pm s, \pm(s-1), \ldots, \pm(t+2), \pm(t+1)$.

Component Lagrangian

All relations are much simpler when we deal with the Fock space vectors. Moreover, conventional field theory is, in a certain sense, the same BRST theory just rewritten in “component form”.

Since the field formulation is often more commonly used, we will demonstrate here the transition from the formulation in terms of the Fock space vectors to the formulation in terms of multi-spinor fields.

For transition to the component field formulation, we use only the following obvious relations:

$$\langle 0|(b)^k (b^+)^n|0\rangle = \delta_{kn} k!\,,$$

$$\langle 0|a_{\alpha_1}\dots a_{\alpha_k}\bar{a}^{\dot\alpha_1}\dots\bar{a}^{\dot\alpha_k}c^{\beta_1}\dots c^{\beta_n}\bar{c}_{\dot\beta_1}\dots\bar{c}_{\dot\beta_n}|0\rangle = \delta_{kn}(k!)^2\delta^{\beta_1}_{(\alpha_1}\dots\delta^{\beta_k}_{\alpha_k)}\delta^{(\dot\alpha_1}_{\dot\beta_1}\dots\delta^{\dot\alpha_k)}_{\dot\beta_k}\,.$$

Taking into account these equalities we obtain that the the Fock space vector Lagrangian $\mathcal{L}_{s,t}$ in the action $S=\int d^4x\sqrt{-g}\,\mathcal{L}_{s,t}$ takes the following form:

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$$
\begin{aligned}
\mathcal{L}_{s,t} = & \sum_{k=t+1}^{s} (s-k)!\, \bar{\varphi}^{\alpha(k)}_{\dot\beta(k)} \left(\frac{2k+1}{k} \Box - m_t^2 + \varkappa \left(2s^2 + k \right) + \frac{(s-k)A^2(s-k-1)}{2k+2} \right) \varphi^{\dot\beta(k)}_{\alpha(k)} \\
& - \sum_{k=t+1}^{s} \frac{(s-k)!(k+1)^2}{2k} \bar{\varphi}^{(\alpha(k)}_{(\dot\beta(k)} \nabla^{\alpha)}_{\dot\beta)} \nabla^{(\dot\beta}_{(\alpha} \varphi^{\dot\beta(k))}_{\alpha(k))} \\
& + \sum_{k=t+1}^{s} \frac{(s-k+1)!}{2} A(s-k) \left(\bar{\varphi}^{\alpha(k)}_{\dot\beta(k)} \nabla^{(\dot\beta}_{(\alpha} \varphi^{\dot\beta(k-1))}_{\alpha(k-1))} - \bar{\varphi}^{(\alpha(k-1)}_{(\dot\beta(k-1)} \nabla^{\alpha)}_{\dot\beta)} \varphi^{\dot\beta(k)}_{\alpha(k)} \right) \\
& - \sum_{k=t-1}^{s-2} (s-k-2)!\, \bar{\chi}^{\alpha(k)}_{\dot\beta(k)} \left(\Box - m_t^2 + \varkappa \left[2s^2 - (k+1)^2 - 1 \right] + \frac{(s-k)A^2(s-k-1)}{2k+2} \right) \chi^{\dot\beta(k)}_{\alpha(k)} \\
& - \sum_{k=t-1}^{s-2} \frac{(s-k-2)!(k+1)^2}{2k} \bar{\chi}^{(\alpha(k)}_{(\dot\beta(k)} \nabla^{\alpha)}_{\dot\beta)} \nabla^{(\dot\beta}_{(\alpha} \chi^{\dot\beta(k))}_{\alpha(k))} \\
& + \sum_{k=t-1}^{s-2} \frac{k(s-k-1)!}{2k+2} A(s-k-2) \left(\bar{\chi}^{\alpha(k)}_{\dot\beta(k)} \nabla^{(\dot\beta}_{(\alpha} \chi^{\dot\beta(k-1))}_{\alpha(k-1))} - \bar{\chi}^{(\alpha(k-1)}_{(\dot\beta(k-1)} \nabla^{\alpha)}_{\dot\beta)} \chi^{\dot\beta(k)}_{\alpha(k)} \right) \\
& - \sum_{k=t+1}^{s} \frac{(k-1)(s-k)!}{2} \left(\bar{\varphi}^{\alpha(k)}_{\dot\beta(k)} \nabla^{(\dot\beta_1}_{(\alpha_1} \nabla^{\dot\beta_2}_{\alpha_2} \chi^{\dot\beta(k-2))}_{\alpha(k-2))} + \bar{\chi}^{(\alpha(k-2)}_{(\dot\beta(k-2)} \nabla^{\alpha_1}_{\dot\beta_1} \nabla^{\alpha_2)}_{\dot\beta_2)} \varphi^{\dot\beta(k)}_{\alpha(k)} \right) \\
& - \sum_{k=t+1}^{s-2} \frac{(s-k)!}{2k+2} A(s-k-1) A(s-k-2) \left(\bar{\chi}^{\alpha(k)}_{\dot\beta(k)} \varphi^{\dot\beta(k)}_{\alpha(k)} + \bar{\varphi}^{\alpha(k)}_{\dot\beta(k)} \chi^{\dot\beta(k)}_{\alpha(k)} \right) \\
& + \sum_{k=t+1}^{s-1} \frac{(2k+1)(s-k)!}{2(k+1)} A(s-k-1) \left(\bar{\chi}^{(\alpha(k-1)}_{(\dot\beta(k-1)} \nabla^{\alpha)}_{\dot\beta)} \varphi^{\dot\beta(k)}_{\alpha(k)} + \bar{\varphi}^{\alpha(k)}_{\dot\beta(k)} \nabla^{(\dot\beta}_{(\alpha} \chi^{\dot\beta(k-1))}_{\alpha(k-1))} \right) .
\end{aligned}
$$

Component field Lagrangian is invariant with respect to gauge transformations

$$
\begin{aligned}
\delta\varphi^{\dot\beta(s)}_{\alpha(s)} &= s\nabla^{(\dot\beta}_{(\alpha}\lambda^{\dot\beta(s-1))}_{\alpha(s-1))}\,,\\
\delta\varphi^{\dot\beta(k)}_{\alpha(k)} &= k\nabla^{(\dot\beta}_{(\alpha}\lambda^{\dot\beta(k-1))}_{\alpha(k-1))} + A(s-k-1)\,\lambda^{\dot\beta(k)}_{\alpha(k)}\,, \qquad t<k<s\,,
\end{aligned}
$$

$$
\delta\chi^{\dot\beta(k)}_{\alpha(k)} = -\frac{1}{k+1}\,\nabla^{\gamma}_{\dot\gamma}\lambda^{(\dot\gamma\dot\beta(k+1))}_{(\gamma\alpha(k+1))} + (s-k-1)A(s-k-2)\,\lambda^{\dot\beta(k)}_{\alpha(k)}\,, \quad (t-1)\le k\le(s-2)\,.
$$

Note that due to $A(s+t)=0\,,\ A(s-t-1)=0\,,$
some terms in these variations are zero.

Let us emphasize that the component Lagrangian, although it looks rather cumbersome, is nothing more than another form of a rather simple Lagrangian in terms of Fock space vectors.

Conclusion

The talk presented the BRST Lagrangian formulation for partially massless bosonic fields in four-dimensional space.

Main features and results of the approach under consideration:

- Mass-shell conditions are formulated in terms of the Weyl multi-spinor fields – the tracelessness condition is automatically fulfilled.
- Mass-shell conditions and gauge transformations are reformulated in terms of the operator equations for the Fock space vectors.
- In partially massless case, basic constraints correspond to the system with 2nd-class constraints. We develop conversion procedure to obtain the 1st-class constraints system. It is shown that the hermitianity and nilpotency of the BRST charge (unitary representation) lead to restrictions that are satisfied only for dS space.

- The hermitianity of the BRST charge in the theory under consideration requires that the vectors used have restricted expansions. Moreover, presence of $(s-t-1)$ Stückelberg fields leads, when they are eliminated, to gauge transformations of physical fields of the $(s-t)$ order in derivatives.

- Final Lagrangian describes on the mass-shell only the states with helicities $\pm s, \pm(s-1), \ldots, \pm(t+2), \pm(t+1)$. No other states remain in the theory after eliminating all auxiliary and gauge degrees of freedom. The resulting Lagrangian is rewritten in terms of conventional spin-tensor fields and is a generalization of the Fronsdal Lagrangian to the partially massless case.

Next possible tasks:

- Lagrangian formulation for fermionic partially massless fields in dS_4 space. Like in the bosonic case, the BRST approach can allow constructing the Lagrangian formulation only in terms of partially massless fields.

- Construction of cubic vertices for interacting partially massless fields among themselves as well as with conventional massless and massive higher spin fields within the BRST approach.

Thank you very much for your attention !