Symmetry breaking in the self-dual higher-spin theory

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QUARKS-2026
Based on arXiv:2509.01477 [hep-th]

Contens

Unfolded formulation

Higher-spin theory can be given via the *unfolded equations*:

$$\mathrm{d}_x\omega = -\omega\star\omega + \mathcal{V}(\omega,\omega,C) + \mathcal{V}(\omega,\omega,C,C) + \ldots \tag{1a}$$

$$\mathrm{d}_x C = -\omega\star C + C\star\pi(\omega) + \Upsilon(\omega,C,C) + \Upsilon(\omega,C,C,C) + \ldots \tag{1b}$$

- Fields are generating functions of auxiliary variables $Y = (y,\bar{y})$:

$$f(x|y,\bar{y}) = \sum_{m,n}\frac{1}{n!m!}f_{\alpha(m),\dot{\beta}(n)}(x)\;y^{\alpha(m)}\bar{y}^{\dot{\beta}(n)}. \tag{2}$$

  Where $y_\alpha, \bar{y}_{\dot{\beta}}$ are $sl(2,C)$ spinors. Indices are contracted using $\epsilon^{\alpha\beta}, \epsilon^{\dot{\alpha}\dot{\beta}}, \epsilon^{\alpha\dot{\beta}}$.
- *Star-product* governs the higher-spin algebra:

$$f(y,\bar{y})\star g(y,\bar{y}) = \int f(y+u,\bar{y}+\bar{u})g(y+v,\bar{y}+\bar{v})e^{iuv+i\bar{u}\bar{v}}. \tag{3}$$

- ω is a space-time 1-form which encodes gauge fields of spin $S \geqslant 1$.
- C is a space-time 0-form which consists of scalar and fermion $S = 1/2$ fields together with generalized Weyl tensors for all spins $S \geqslant 1$.

Self-duality

Unfolded equations must be consistent in a sense of $\mathrm{d}_x^2 = 0$. This constraint fixes *vertices* $\mathcal{V}, \Upsilon$ up to an exact form. To systematically restore all the vertices, Vasiliev proposed [Vasiliev, 1992] the generating system approach. This procedure will be presented on the next slide.

From Vasiliev equations it follows that the $4d$ theory has one free complex parameter η. Parameter enters vertices:

$$\begin{aligned} \mathcal{V}(\omega,\omega,C^n) &= \sum_{i+j=n} \eta^i \bar{\eta}^j \cdot \mathcal{V}_{i,j}(\omega,\omega,C^n) \\ \Upsilon(\omega,C^n) &= \sum_{i+j=n-1} \eta^i \bar{\eta}^j \cdot \Upsilon(\omega,C^n) \end{aligned} \tag{4}$$

Equations allow for consistent truncation: $\bar{\eta} = 0$. Truncated sector is called *self-dual.*

$$\mathcal{V}(\omega,\omega,C^n) = \sum_{i=n} \eta^i \cdot \mathcal{V}_{i,0}(\omega,\omega,C^n), \quad \Upsilon(\omega,C^n) = \sum_{i=n-1} \eta^i \cdot \Upsilon(\omega,C^n) \tag{5}$$

Generating system

In order to find self-dual vertices (5) one should enlarge the HS algebra $(y,\bar{y}) \to (y,\bar{y},z)$. We introduce z-dependent connection W that replace ω. Zero-form C does not depend on z and remains the same. Equations reads [Didenko, 2022]

$$\mathrm{d}_x W + W * W = 0\,, \tag{6a}$$

$$\mathrm{d}_z W + \{W,\Lambda\}_* + \mathrm{d}_x\Lambda = 0\,, \tag{6b}$$

$$\mathrm{d}_x C + \big(W(z';y,\bar{y}) * C - C * W(z';-y,\bar{y})\big)\Big|_{z'=-y} = 0\,. \tag{6c}$$

$$\Lambda = \mathrm{d}z^\alpha \int_0^1 \mathrm{d}\tau\tau z_\alpha C(-\tau z, y) e^{i\tau z y} \tag{7}$$

The vertices are perturbatively reconstructed in powers of C.

$$W(y,\bar{y},z) := \omega(y,\bar{y}) + W_1(y,\bar{y},z)[\omega,C] + W_2(y,\bar{y},z)[\omega,C,C] + ... \tag{8}$$

Substituting this into (6) restores explicit expressions for (5) [Didenko, Povarnin 2024]. Enlarged star-product:

$$(f * g)(z,y;\bar{y}) = \int f\left(z+u', y+u; \bar{y}\right) \star g\left(z-v, y+v+v'; \bar{y}\right) \exp(iu_\alpha v^\alpha + iu'_\alpha v'^\alpha)\,, \tag{9}$$

Trivial vacuum

Trivial vacuum is

$$\omega_0 = \omega_{AdS}\,,\;\; C_0 = 0. \tag{10}$$

One can linearize (1) or (6) for $\mathbf{w}, \mathcal{C}$. For trivial vacuum this procedure gives *free* higher-spin fields in AdS (Fronsdall equations). This result is known as *Central On-Mass-Shell Theorem*:

$$\mathrm{d}_x \mathbf{w} + \{\omega_{AdS}, \mathbf{w}\}_\star = \frac{1}{2}\mathbf{e}^{\alpha\dot\alpha} \wedge \mathbf{e}_\alpha{}^{\dot\beta}\bar\partial_{\dot\alpha}\bar\partial_{\dot\beta}\mathcal{C}(0,\bar y) \tag{11a}$$

$$\mathrm{d}_x \mathcal{C} = -\omega_{AdS} \star \mathcal{C} + \mathcal{C} \star \pi(\omega_{AdS}) \tag{11b}$$

Spin S modules are singled out by

$$(y\partial + \bar y\bar\partial)\mathbf{w} = 2(S-1)\mathbf{w}\,,\;\; (y\partial - \bar y\bar\partial)\mathcal{C} = \pm 2s\mathcal{C} \tag{12}$$

In what follows, we use the Poincaré coordinates.

$$ds^2 = \frac{1}{r^2}\left(\mathrm{d}r^2 + \mathrm{dx}_{\alpha\beta}\mathrm{dx}^{\alpha\beta}\right), \tag{13}$$

Here $\mathrm{x}^{\alpha\beta}$ are the boundary three-dimensional coordinates and the radial coordinate r.

Symmetries

Global symmetries ξ of generating equations are solutions of the following system

$$\delta_\xi W = \mathrm{d}_x \xi + [W, \xi]_* = 0 \tag{14a}$$

$$\delta_\xi \Lambda[C] = \mathrm{d}_z \xi + [\Lambda[C], \xi]_* = 0 \tag{14b}$$

$$\delta_\xi C = \Big(\xi(z; y, \bar{y}) *_y C - C *_y \xi(z; -y, \bar{y})\Big)\Big|_{z=-y} = 0 \tag{14c}$$

Consider trivial vacuum (10). Then we are left with

$$\mathrm{d}_x \xi + [\omega_{AdS}, \xi]_* = 0. \tag{15}$$

Spin-2 solutions are spanned with the following generators ($y^\pm := y \pm i\bar{y}$)

$$P_{\alpha\beta} = i y^+_\alpha y^+_\beta, \;\; L_{\alpha\beta} = i(y^+_\alpha y^-_\beta + y^-_\alpha y^+_\beta), \tag{16a}$$

$$K_{\alpha\beta} = i y^-_\alpha y^-_\beta, D = \frac{i}{8} y^+ y^-. \tag{16b}$$

which is $o(3,2)$. Commutation relations are obtained via star-product commutators. These are space-time global symmetries.

In what follows we are interested in non-trivial solutions that preserve AdS-background:

$$\begin{aligned} &W = \omega_{AdS} + W_1[\omega_{AdS}, C] + W_2[\omega_{AdS}, C, C] + ..., \;\; C \neq 0 \\ &W(y, \bar{y}, z)\Big|_{z=0} = \omega_{AdS} \end{aligned} \tag{17}$$

This leads to a symmetry breaking. We focus on solutions that has $3d$ Poincaré as leftover space-time symmetry. It is spanned via (16a):

$$P_{\alpha\beta} = i y^+_\alpha y^+_\beta, \;\; L_{\alpha\beta} = i(y^+_\alpha y^-_\beta + y^-_\alpha y^+_\beta). \tag{18}$$

We require C to be such that global symmetry equations (14a) have Poincaré (18) as a solution. Generator ξ has to be decomposed in powers of C

$$\xi = \xi_0 + \xi_1[C] + \xi_2[C, C] + ... \,. \tag{19}$$

Lowest order (14a) reads

$$\mathrm{d}_x\xi_0 + [\omega_{AdS}, \xi_0]_* = 0 \tag{20a}$$

$$\mathrm{d}_z\xi_0 = 0, \tag{20b}$$

$$\xi_0 * C - C * \pi(\xi_0) = 0. \tag{20c}$$

- (20a) contains Poincaré as for an empty AdS.
- (20c) for translations $P_{\alpha\beta}$ leads to a condition $\mathrm{d_x}C = 0$, where x is a boundary coordinate.
- (20c) for Lorentz generators $L_{\alpha\beta}$ leads to a spin constraint: $S = 0$.

That leaves us with the ansatz

$$C = C(r, y\bar{y}). \tag{21}$$

If there is a solution that realize symmetry breaking of AdS_{3+1} isometry down to a Poincaré in $3d$, it must have this form.

It is straightforward to obtain all solutions of the generating system (6) in the form of (21):

$$W_0 = \omega_{AdS} + \nu_2 w,\;\; C_0 = \nu_1 \cdot r e^{y\bar{y}} + \nu_2 \cdot r^2 (1 + y\bar{y}) e^{y\bar{y}},$$
$$w = \frac{ir}{2} \mathrm{dx}^{\alpha\beta} z_\alpha z_\beta \int_0^1 \mathrm{d}\tau\; \tau(1-\tau) e^{i\tau z y^+}. \tag{22}$$

- Vacuum (22) describes free scalar field in AdS.

$$\phi(x) := C_0(x|0,0) = \nu_1 \cdot r + \nu_2 \cdot r^2 \tag{23}$$

Rest of the components of C are *on-shell* derivatives of ϕ, i.e.

$$\partial_{\mu_1} ... \partial_{\mu_n} \phi = C_{\mu_1 ... \mu_n} \tag{24}$$

- $\nu_1,\; \nu_2$ are free parameters of deformation.
- (22) indeed has a Poincaré as a space-time global symmetry.

Linearized equations

Linear dynamics can be constructed using scalar vacuum. To simplify the problem we set $\nu_2 = 0$, i.e. we study only the first branch. Linear equations for fluctuations $\mathbf{w}$, $\mathcal{C}$ as follows

$$\mathrm{d}_x \mathbf{w} + \{\omega_{AdS}, \mathbf{w}\}_* = \frac{1}{2} \mathbf{e}^{\alpha\dot{\alpha}} \wedge \mathbf{e}_\alpha{}^{\dot{\beta}} \bar{\partial}_{\dot{\alpha}} \bar{\partial}_{\dot{\beta}} \mathcal{C}(0, \bar{y})\,, \tag{25a}$$

$$\mathrm{d}_x \mathcal{C} + \omega_{AdS} * \mathcal{C} - \mathcal{C} * \pi(\omega_{AdS}) = \Upsilon(\Omega, \mathcal{C}, C_0)\,. \tag{25b}$$

- Fluctuation of connection has a specific argument: $\mathbf{w} = \mathbf{w}(y^+)$. This leads to a vanishing of almost all the vertices in (25) and imposes a constraint on the $\mathcal{C}$

$$\mathcal{C}(0, \bar{y}) = \mathcal{C}_0 + \mathcal{C}_{\dot{\alpha}} \bar{y}^{\dot{\alpha}} + \frac{1}{2} \mathcal{C}_{\dot{\alpha}\dot{\alpha}} \bar{y}^{\dot{\alpha}} \bar{y}^{\dot{\alpha}} \tag{26}$$

- Equations (25a) and (25b) yield (26) independently. This is nontrivial result which allows further study of (25). General case is for the general $w(y^+, y^-)$.
- The source for the gauge fields on the right side of equation (25a) is zero for all spins except for $s = -1$. This means that symmetry breaking leads to a collapse of higher-spin states leaving us with only one gauge field of spin $s = -1$, along with matter fields of spins $s = 0$ and chiral $s = \frac{1}{2}$.

We introduce Vasiliev's current module T [Vasiliev, 2012]

$$\mathcal{C} = re^{y\bar{y}} \cdot T(w, \bar{w}), \quad w = \sqrt{r}y,\ \bar{w} = \sqrt{r}\bar{y} \tag{27}$$

Linear equations (25) become

$$\mathrm{d_x}T - \frac{i}{2}\mathrm{dx}^{\alpha\beta}\partial_\alpha\bar{\partial}_\beta T = \frac{i\nu}{2}\mathrm{dx}^{\alpha\beta}w_\alpha\bar{\partial}_\beta \int_0^1 d\tau(1-\tau)(T^- - T^+)\,, \tag{28a}$$

$$\frac{\partial}{\partial r}T - \frac{1}{2}\partial\bar{\partial}\,T = -\frac{\nu}{2}w^\alpha\bar{\partial}_\alpha \int_0^1 d\tau(1-\tau)(T^- - T^+)\,, \tag{28b}$$

where

$$T^\pm = T(\tau w, \bar{w} \pm i(1-\tau)w)\,. \tag{29}$$

Dual formulation

- Symmetry unbroken case arises when $\nu = 0$ [Vasiliev, 2012].

$$\mathrm{d_x} T - \frac{i}{2}\mathrm{dx}^{\alpha\beta}\partial_\alpha\bar{\partial}_\beta T = 0 \tag{30a}$$

$$\frac{\partial}{\partial r}T - \frac{1}{2}\partial\bar{\partial}\,T = 0 \tag{30b}$$

(30a) is an unfolded equation describing conserving currents of all spins

$$j(y) = T(y,0),\ \ \bar{j} = T(0,\bar{y}), j_0 = w\bar{w}\ \partial\bar{\partial}T(0,0) \tag{31}$$

(30b) gives the z-dependence. Equations are solved by the following T

$$T = C_+(y^+)\cdot C_-(y^-) \tag{32}$$

$$\mathrm{d_x} C_\pm \pm \frac{1}{2}\mathrm{dx}^{\alpha\beta}\partial_\alpha\partial_\beta C_\pm = 0 \tag{33}$$

Latter is nothing but unfolded equation for a massless scalar in $3d$ Minkowski. This is known as a Flato-Fronsdall decomposition.

Dual formulation

$$\mathrm{d_x}T - \frac{i}{2}\mathrm{dx}^{\alpha\beta}\partial_\alpha\bar{\partial}_\beta T = \frac{i\nu}{2}\mathrm{dx}^{\alpha\beta}w_\alpha\bar{\partial}_\beta\int_0^1 d\tau(1-\tau)(T^- - T^+)\,, \tag{34a}$$

$$\frac{\partial}{\partial r}T - \frac{1}{2}\partial\bar{\partial}T = -\frac{\nu}{2}w^\alpha\bar{\partial}_\alpha\int_0^1 d\tau(1-\tau)(T^- - T^+)\,, \tag{34b}$$

Symmetry broken case describes a set of non-conserved currents $s > 1$. Currents of spins $s = -1, \pm\frac{1}{2}, 0$ don't change and hence conserve, spin $s = 1$ current changes but remains conserved, and currents of $s < -1$ are prohibited as a consistency condition.

- We considered solutions that realize symmetry breaking of AdS_{3+1} isometry down to a Poincaré in $3d$. Global symmetry equations yield simple condition: $C = C(r, y\bar{y})$.
- Scalar solutions of the self-dual higher spin theory that breaks symmetry were obtained.
- Linear dynamics around the first branch were studied. The source for the gauge fields is zero for all helicities except for $s = -1$. Symmetry breaking leads to a collapse of higher-spin states leaving us with only one gauge field of helicity $s = -1$, along with matter fields of spins $s = 0$ and $s = \frac{1}{2}$.
- Holographic dual was examined. While undeformed system is dual to a set of conserved currents, in the symmetry broken phase higher-spin currents $(s > 1)$ do not conserve. Currents of low spins remain conserved, $s = 1$ current deforms and others are not.