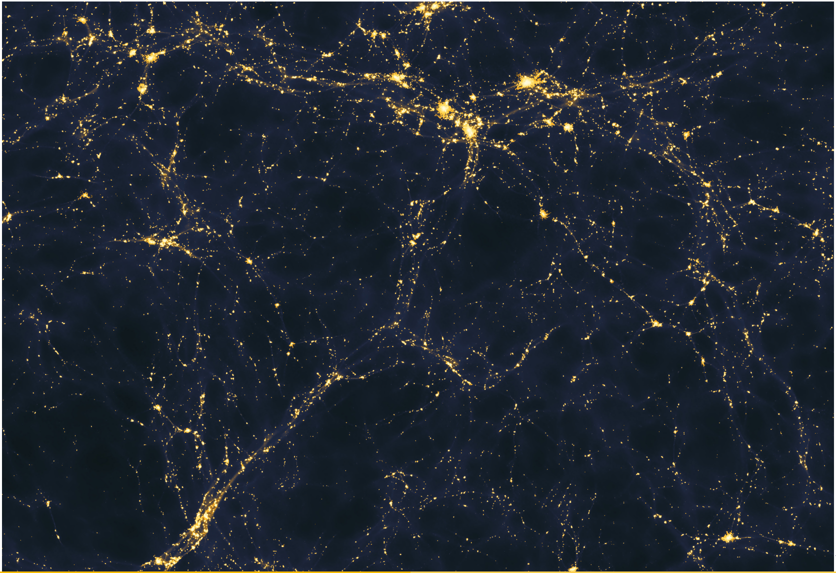
How empty are the voids?

Anton N. Baushev

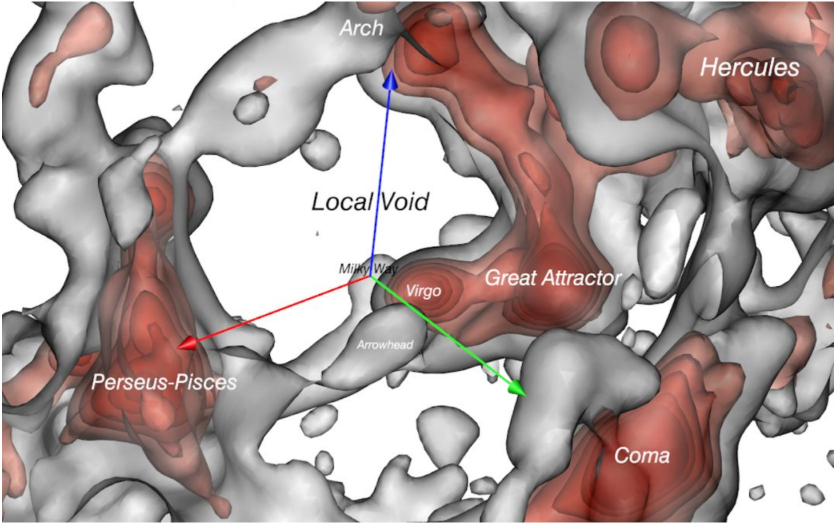
[1] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research Dubna, Moscow Region, Russia
[2] Department of Physics, University of Helsinki

arXiv:2409.02780

Voids



Voids



Arch
Hercules
Local Void
Milky Way
Virgo
Great Attractor
Arrowhead
Perseus-Pisces
Coma

Undisturbed Universe

$$\left(\frac{da}{adt}\right)^2 = \frac{8\pi}{3}G\left[\rho_{M,0}\left(\frac{a_0}{a}\right)^3 + \rho_{\gamma,0}\left(\frac{a_0}{a}\right)^4 + \rho_{\Lambda,0} + \rho_{a,0}\left(\frac{a_0}{a}\right)^2\right]$$

The curvature density $\frac{8\pi}{3c^2}G\rho_{a,0} = \frac{k}{a_0^2}$.

For our Universe $\Omega_{a,0} = 0$, $\Omega_{\gamma,0} \simeq 0$. Therefore, $\Omega_{\Lambda,0} + \Omega_{M,0} = 1$.

The age of the Universe

$$t_0 = H_0^{-1}\frac{2}{3\sqrt{\Omega_{\Lambda,0}}}\ln\left[\frac{\sqrt{1-\Omega_{M,0}}+1}{\sqrt{\Omega_{M,0}}}\right] = $$
$$= H_0^{-1}\frac{2}{3\sqrt{\Omega_{\Lambda,0}}}\,\mathrm{arcosh}\left(1/\sqrt{\Omega_{M,0}}\right)$$

Let us choose a time moment z_1 deeply at the matter-dominated phase of the Universe ($10 < z_1 < 100$). At $z = z_1$ the future void is only a shallow underdensity $|\delta_1| = |\Delta\rho_m/\rho_{m,1}| \ll 1$. The density contrast on the matter-dominated stage is proportional to $a(t)$, and it is more convenient to characterize the underdensity by

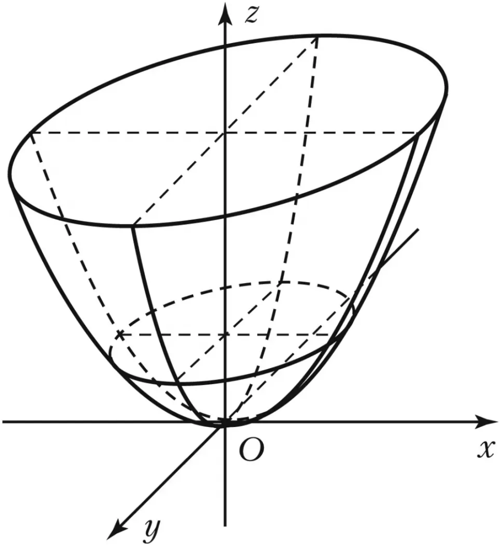
$$\aleph \equiv (z+1)\delta = \left(\frac{a_0}{a}\right)\delta; \quad \aleph = \left(\frac{a_0}{a_1}\right)\delta_1,$$

$\aleph$ remains constant on all the matter-dominated stage of the Universe. We can illustrate the physical meaning of $\aleph$ with the help of symmetry: an overdensity and an underdensity of the same size and absolute density contrast $|\delta_1|$ occur in the primordial perturbations with the same probability.

$$p(\aleph) = \frac{g_0}{\sigma_8\sqrt{2\pi}} \exp\left(-\frac{\aleph^2 g_0^2}{2\sigma_8^2}\right); \qquad \aleph = \frac{z_{form}+1}{g(z_{form})}$$

Since $\aleph$ is constant on the matter-dominated stage, it does not depend on the choice of z_1.

A spherical void



The evolution of the Universe inside R may be described by usual Friedman's equation, though its parameters should slightly differ from those of the undisturbed Universe. We denote the critical, matter, dark energy, and curvature densities of the undisturbed Universe by $\rho_c, \rho_m, \rho_\Lambda, \rho_a$, of the 'universe' inside R — by q_c, q_m, q_Λ, q_a, respectively. We denote the scale of the 'universe' inside R by $b(t)$, and the Hubble constant — by $Q \equiv \frac{db}{bdt}$.

$$H^2 = \frac{8\pi G}{3}\left[\rho_\Lambda + \rho_{m,1}\left(\frac{a_1}{a}\right)^3\right],$$

$$Q^2 = \frac{8\pi G}{3}\left[q_\Lambda + q_{m,1}\left(\frac{b_1}{b}\right)^3 + q_{a,1}\left(\frac{b_1}{b}\right)^2\right].$$

- Apparently, $q_\Lambda = \rho_\Lambda$.
- $q_{m,1} = (1-\delta_1)\rho_{m,1} = (1-\delta_1)\left(\frac{a_0}{a_1}\right)^3 \rho_{c,0}\Omega_{m,0}$
- $Q_1 R_1 = H_1 R_1 + v_1$
- $v_1 = \frac{1}{3}\delta_1 R_1 H_1$
- $(1+\frac{2}{3}\delta_1)(\rho_\Lambda + \rho_{m,1}) = q_\Lambda + q_{m,1} + q_{a,1}$
- $q_{a,1} = \frac{5}{3}\aleph\left(\frac{a_0}{a_1}\right)^2 \rho_{c,0}\Omega_{m,0}$

$$\boxed{Q^2 = H_0^2\left[\Omega_{\Lambda,0} + \Omega_{m,0}\left(\frac{a_0}{b}\right)^3 + \frac{5}{3}\aleph\,\Omega_{m,0}\left(\frac{a_0}{b}\right)^2\right]}$$

The final trick

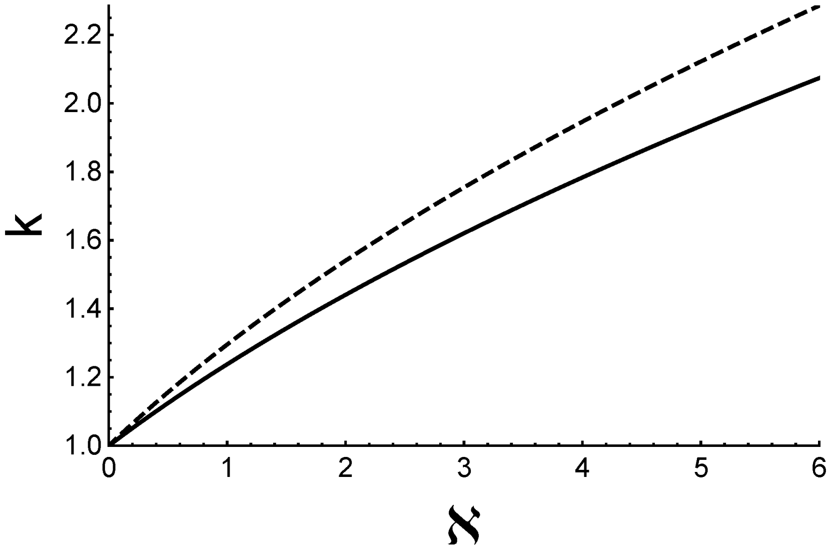
$$t_U = \int dt = \int_0^{a_0} \frac{da}{aH}, \qquad t_R = \int_0^{b_0} \frac{db}{bQ}, \qquad t_U \equiv t_R$$

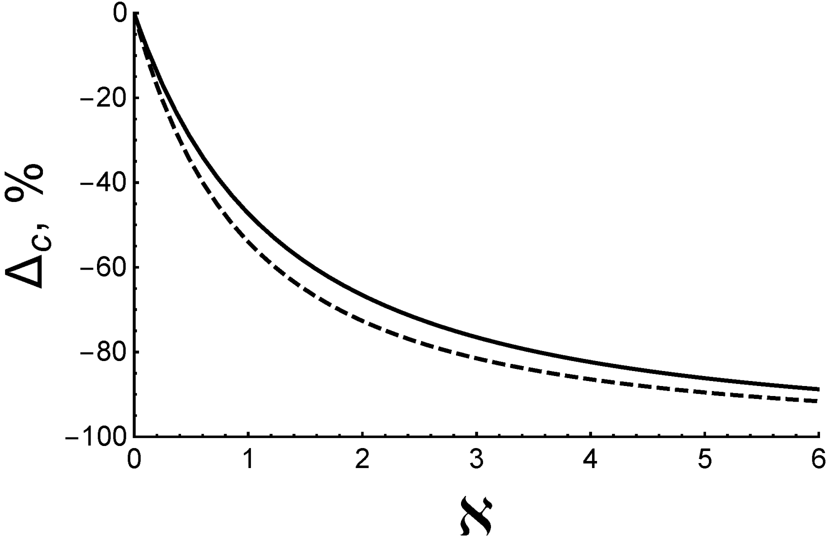
It is essential that $b_0 \neq a_0$: the void center expands much stronger than the Universe on average. Let us denote $k = b_0/a_0$. It is apparent that the central density of a void is $q_{m,0} = \rho_{m,0}/k^3$. As is customary, we will characterize voids by their present-day central underdensity

$$\Delta_c \equiv \frac{q_{m,0} - \rho_{m,0}}{\rho_{m,0}} = \frac{1}{k^3} - 1$$

The solution

$$\boxed{\frac{2}{3}\frac{\operatorname{arcosh}\left(1/\sqrt{\Omega_{m,0}}\right)}{\sqrt{1-\Omega_{m,0}}} = \int_0^k \frac{\sqrt{x}\,dx}{\sqrt{x^3 + \Omega_{m,0}\left(1 + \frac{5}{3}\aleph x - x^3\right)}}}$$


2.2
2.0
1.8
1.6
1.4
1.2
1.0
k
0
1
2
3
4
5
6
ℵ


0
−20
−40
−60
−80
−100
0
1
2
3
4
5
6
Δ_C, %
ℵ

A non-spherical void

$$\rho = \rho_c - \varrho\left(\frac{x^2}{e_1^2} + \frac{y^2}{e_2^2} + \frac{z^2}{e_3^2}\right) \qquad \frac{\dot{\rho}}{\rho} = -\operatorname{div}\vec{v}$$

$$\phi = \phi_u + \frac{1}{2}\left(\phi_{xx}x^2 + \phi_{yy}y^2 + \phi_{zz}z^2\right),$$

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 4\pi G\varrho(0) = 4\pi G(\rho - \rho_c).$$

On the stage when the perturbation is linear we may easily calculate the velocity field created by it. For instance, if we consider the x component, $dv_x/dt = x\phi_{xx}(t)$. At some previous moment of time, τ,

$$\frac{dv_x}{d\tau} = x\frac{a(\tau)}{a(t)}\phi_{xx}(\tau)$$

$$v_i = i\int_0^t \frac{a(\tau)}{a(t)}\phi_{ii}(\tau)d\tau$$

A non-spherical void

$$\frac{dv_x}{dx} + \frac{dv_y}{dy} + \frac{dv_z}{dz} = \int_0^t \frac{a(\tau)}{a(t)}(\phi_{xx} + \phi_{yy} + \phi_{zz})d\tau,$$

$$\mathrm{div}\,\vec{v} = 4\pi G \int_0^t \frac{a(\tau)}{a(t)}(\rho(\tau) - \rho_c(\tau))d\tau.$$

The velocity distribution is anisotropic in the case of an ellipsoidal perturbation: coefficients $\phi_{xx}, \phi_{yy}, \phi_{zz}$ are essentially different. The velocity components are not completely independent, however: they are implicitly bound by the Poisson's equation, and as a result the velocity divergence does not depend on the void ellipticity at all. The last equation shows that the central density evolution of an ellipsoidal void is exactly the same as in the spherically-symmetric case, at least, on the linear stage of the void formation.

Comparison with N-body simulations

N-body simulations (Lavaux & Wandelt, 2012)

$$\frac{\rho(r)}{\rho_{M,0}} = A_0 + A_3 \left(\frac{r}{R_V}\right)^3$$

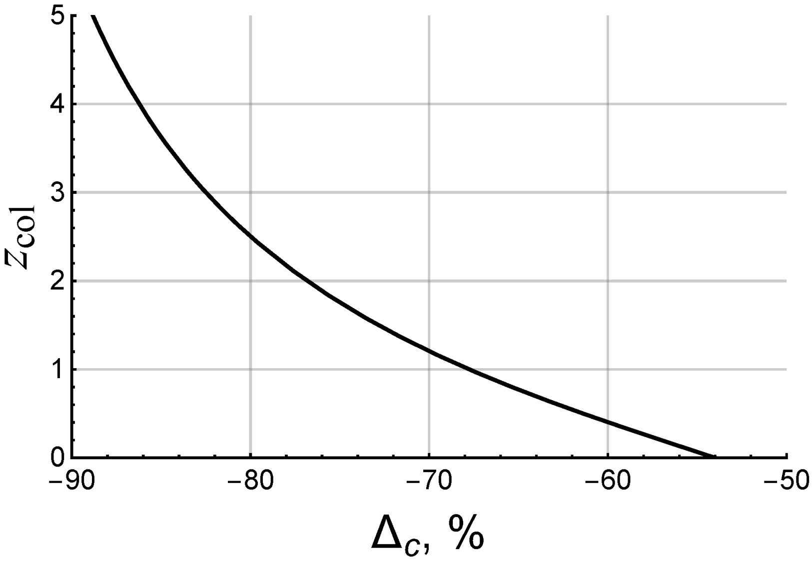
The best fit $A_0 = 0.13 \pm 0.01$, $A_3 = 0.70 \pm 0.03$ for voids of $R_V = 8h^{-1}$ Mpc. So $\Delta_c = -87\%$, the voids are almost empty.

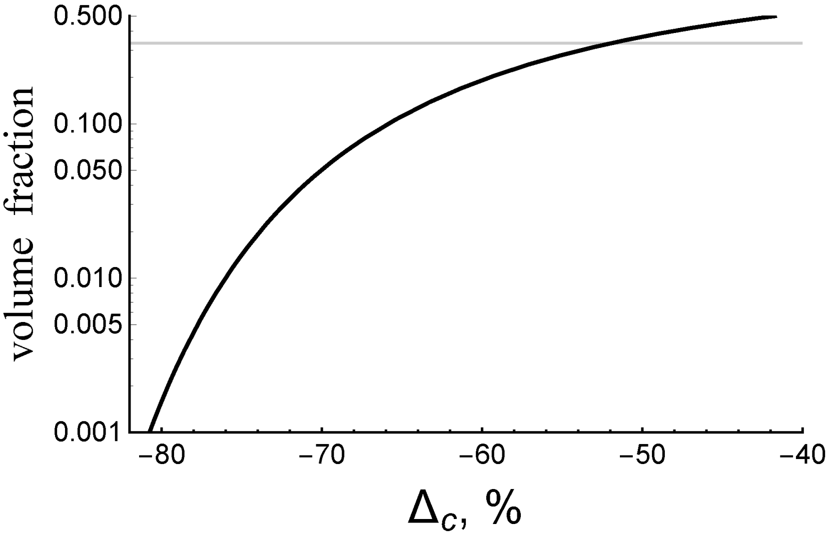
Gaussianity

$$p(\delta_L) = \frac{1}{\sigma_L\sqrt{2\pi}} \exp\left(-\frac{\delta_L^2}{2\sigma_L^2}\right), \qquad f(\aleph) = \frac{1}{\sigma_8\sqrt{2\pi}} \exp\left(-\frac{\aleph^2}{2\sigma_8^2}\right)$$

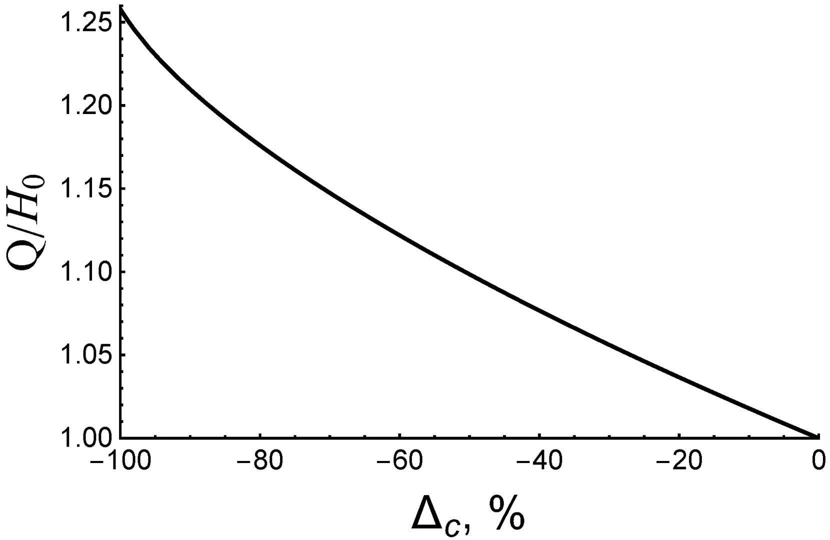
The volume $V(\Delta_c)$ occupied by the regions with matter density less than Δ_c

$$V(\Delta_c(\aleph)) \propto \frac{k^3(\aleph)}{\sigma_8\sqrt{2\pi}} \exp\left(-\frac{\aleph^2}{2\sigma_8^2}\right)$$


5
4
3
2
1
0
$z_{\rm col}$
−90
−80
−70
−60
−50
Δ_c, %



volume fraction
0.500
0.100
0.050
0.010
0.005
0.001
−80
−70
−60
−50
−40
Δ_c, %


1.25
1.20
1.15
1.10
1.05
1.00
Q/H_0
−100
−80
−60
−40
−20
0
Δ_c, %

Conclusions

1. The central density of a void is solely determined by the amplitude of the initial perturbation. The dark energy suppresses the voids, as well as all other structures.
2. N-body simulations somewhat overestimate the emptiness of voids: the majority of them should have the central underdensity $\delta_c \sim -50\%$, and there should be almost no voids with $\delta_c < -80\%$.
3. The central region of a void is a part of an open Friedmann's 'universe', and its evolution differs drastically from the Universe evolution: there is a long stage when the curvature term dominates, which prevents the formation of galaxy clusters and massive galaxies inside voids.
4. The density profile in the void center should be very flat.