Recent Progress in Holographic QCD

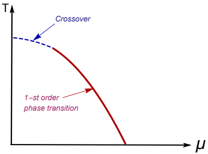
Irina Aref'eva

Steklov Mathematical Institute, RAS
MGU, Physical Department

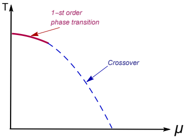
QUARKS-2026 *18 MAY - 23 MAY 2026*

Main Goal of HQCD - QCD Phase Diagram

- **What is new in the study of the QCD phase diagram using HQCD?**
  - **What we had**
    - **Separate phase diagrams for light and heavy quarks**



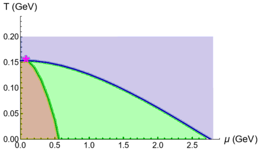
LQ



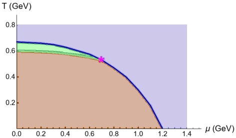
HQ

Main Goal of HQCD - QCD Phase Diagram

- **What is new in the study of the QCD phase diagram using HQCD?**
  - **What we had - Separate phase diagrams for light and heavy quarks**



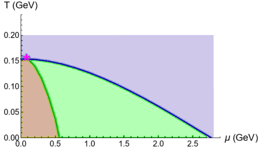
LQ



HQ

Main Goal of HQCD - QCD Phase Diagram

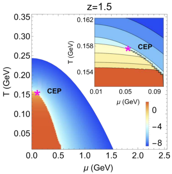
- What is new in the study of the QCD phase diagram using HQCD?
  - What we **had** - Separate phase diagrams for light and heavy quarks



LQ

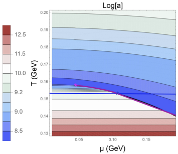
Signatures

- **Running coupling**
  I.A., Hajilou, Slepov, Usova, Phys.Rev.D 110 (2024) 12;
  I.A., Hajilou, Nicolaev, Slepov, Phys.Rev.D 110 (2024) 8
  Phys.Rev.D 111 (2025) 4;
  Eur.Phys.J.C 85 (2025) 1167;



Density plot of $\log \alpha(z;\mu,T)$

- **JQ**
  I.A., Hajilou, Nicolaev, Slepov, Phys.Rev.D 112 (2025) 126007;

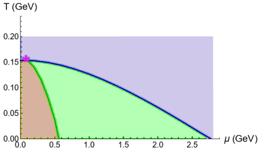


- **Energy lost**
  I.A., Hajilou, Rannu, Slepov, Phys.Rev.D 113 (2026) 10, 106004

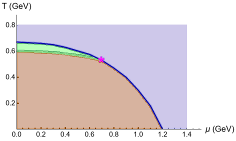
Next talk

Main Goal of HQCD - QCD Phase Diagram

- **What is new in the study of the QCD phase diagram using HQCD?**
  - **What we had - Separate phase diagrams for light and heavy quarks**



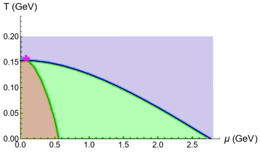
LQ



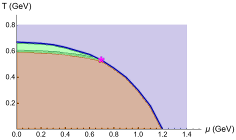
HQ

Main Goal of HQCD - QCD Phase Diagram

- **What is new in the study of the QCD phase diagram using HQCD?**
  - **What we had - Separate phase diagrams for light and heavy quarks**



LQ



HQ

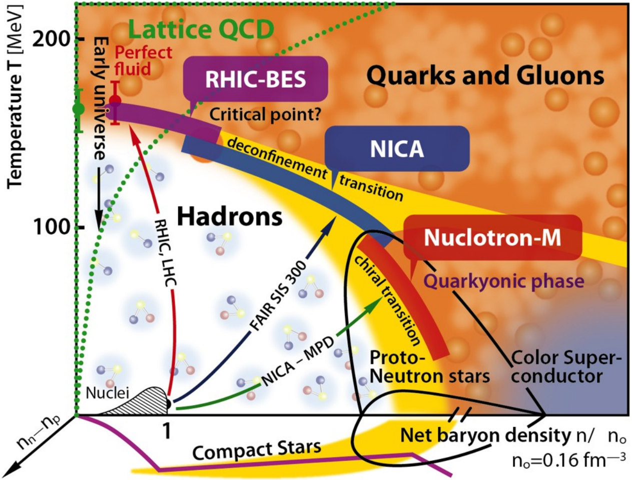
  - **What we have**
    - **Phase Diagram for light + heavy quarks [u, d, s]**
    - **toward the realistic model including all quarks [u, d, s, b, c, ~~t~~]**

- **New method(s) - HoloNet**
  - **main raison: too many parameters in conventional HQCD**
  - **Instead of pre-specifying functions for the holographic metric, the metric is learned by a neural network**

Outlook

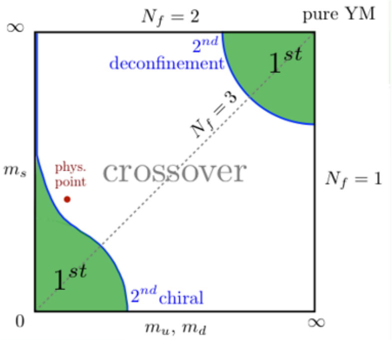
- **From HQCD to HoloNetQCD**

Studies of QCD Phase Diagram - the main goal of new facilities



From: https://nica.jinr.ru/physics.php

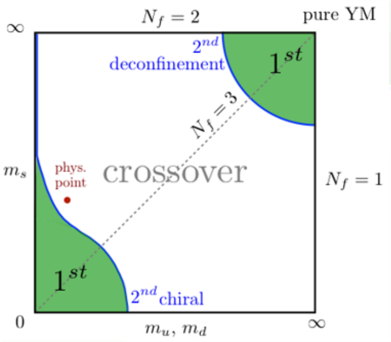
Dependence of phase diagram on quark mass



Brown et al., PRL 65 (1990) 2491

QCD Phase Diagram: Lattice

Dependence of phase diagram on quark mass



Brown et al., PRL 65 (1990) 2491

Как меняется порядок перехода при изменении масс кварков

- по горизонтали отложена масса двух лёгких кварков m_u, m_d
- по вертикали — масса странного кварка m_s;
- левая граница $m_u = m_d = 0$ соответствует двухфлейворному пределу $N_f = 2$
- диагональ $m_u = m_d = m_s$ соответствует трём вырожденным вкусам $N_f = 3$N;
- точка реального мира лежит где-то между этими предельными случаями: u,d очень лёгкие, а s заметно тяжелее.

QCD Phase Diagram.
2-nd order phase transition/crossover.

- **2nd-order phase transition** (critical point):
  - The order parameter (if any) changes continuously [as in $O(N)$ models],
  - **susceptibility diverges**
  - **correlation length becomes infinite**.

The system exhibits universal scaling, long-range fluctuations, and symmetry breaking (e.g., chiral symmetry restoration happens abruptly [внезапно] in the thermodynamic limit).

- **Crossover**:
  - The order parameter changes smoothly;
  - **no diverging susceptibility**,
  - **no true long-range correlations, and no sharp critical point**.

  Symmetry is never strictly restored at any single temperature; the transition is analytic.

- **In QCD** (with realistic quark masses) at zero baryon density, lattice simulations show a **crossover** (not a second-order transition).
- If a **critical** endpoint exists at nonzero density, it would be a **2nd-order** phase transition

QCD Phase Diagram – Experiment
Observables sensitive to the crossover

- At physical quark masses, the transition at small μ_B is a smooth crossover, so there is no single "smoking-gun" signal. The most informative quantities are those related to fluctuations of conserved charges.
- **Fluctuations** (cumulants) $\Longleftrightarrow$ **susceptibilities** (thermodynamics):

Cumulants are truncated correlators

"Truncation" means cutting off all contributions that can be expressed as products of lower-order correlators.

Cumulants of the net-proton number distribution N

Cumulants of N: $C_1 = \langle N \rangle$, $C_2 = \langle \delta N^2 \rangle$, $C_3 = \langle \delta N^3 \rangle$, and $C_4 = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle$, ...,
where $\delta N = N - \langle N \rangle$.

- The cumulants C_n are related to **susceptibilities**:

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}, \quad C_2/V \sim \chi_2^B, \quad C_3/C_2 \sim \chi_3^B/\chi_2^B, \quad C_4/C_2 \sim \chi_4^B/\chi_2^B, \quad B \to Q,\, S.$$

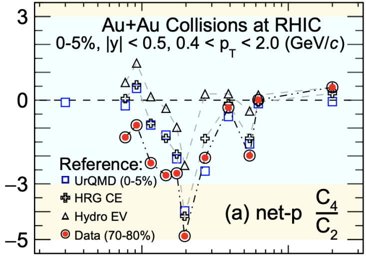
- At small baryon chemical potential, ratios of low-order cumulants, such as

$$R_{12}^X = \frac{\chi_1^X}{\chi_2^X}, \qquad R_{31}^X = \frac{\chi_3^X}{\chi_1^X}, \qquad X = B, Q, S,$$

are particularly useful, since they can be compared quantitatively with lattice-QCD calculations and employed to determine the chemical freeze-out parameters.

QCD Phase Diagram. Experiment.

- The ratio C_4/C_2 is often called the kurtosis: $C_4/C_2 \approx \kappa\sigma^2$, where κ is the kurtosis and σ^2 is the variance [2].
- A non-monotonic dependence on collision energy in the ratio C_4/C_2 is a signature of the CP [1].
- The sign of the kurtosis near the crossover area should be negative. This is a universal prediction that serves as a clear guideline for experimental searches.



Net-proton cumulant ratios C_4/C_2. From [3]=2504.00817

M. A. Stephanov, "On the sign of kurtosis near the QCD critical point," Phys. Rev. Lett. **107** (2011), 052301 doi:10.1103/PhysRevLett.107.052301 [arXiv:1104.1627 [hep-ph]].

M. Abdallah *et al.* [STAR], "Cumulants and correlation functions of net-proton, proton, and antiproton multiplicity distributions in Au+Au collisions at energies available at the BNL Relativistic Heavy Ion Collider," Phys. Rev. C **104** (2021) no.2, 024902 [arXiv:2101.12413 [nucl-ex]].

B. E. Aboona *et al.* [STAR], "Precision Measurement of Net-Proton-Number Fluctuations in Au+Au Collisions at RHIC," Phys. Rev. Lett. **135** (2025) no.14, 142301 [arXiv:2504.00817 [nucl-ex]].

QCD Phase Diagram. Experiment.

- **The critical point** marks the boundary between a crossover quark-hadron transition at low baryochemical potential (μ_B) and a possible first-order transition at higher μ_B [1, 2, 3].
- While first-principle calculations based on lattice QCD have established the crossover at small μ_B [4], the extension of theoretical methods to enable calculations at large μ_B remains a challenge.
- Extensive efforts are underway to search for a possible critical point in the QCD phase diagram.

K. Rajagopal and F. Wilczek, "The Condensed matter physics of QCD," doi:10.1142/9789812810458_0043 [arXiv:hep-ph/0011333 [hep-ph]].

A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov and N. Xu, "Mapping the Phases of Quantum Chromodynamics with Beam Energy Scan," Phys. Rept. **853** (2020), 1-87 doi:10.1016/j.physrep.2020.01.005 [arXiv:1906.00936 [nucl-th]].

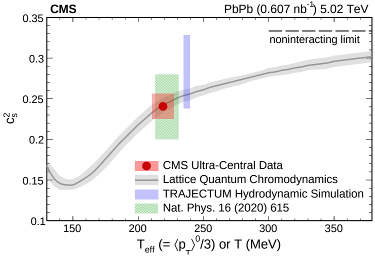
A. Pandav, D. Mallick and B. Mohanty, "Search for the QCD critical point in high energy nuclear collisions," Prog. Part. Nucl. Phys. **125** (2022), 103960 doi:10.1016/j.ppnp.2022.103960 [arXiv:2203.07817 [nucl-ex]].

Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, "The Order of the quantum chromodynamics transition predicted by the standard model of particle physics," Nature **443** (2006), 675-678 doi:10.1038/nature05120 [arXiv:hep-lat/0611014 [hep-lat]].

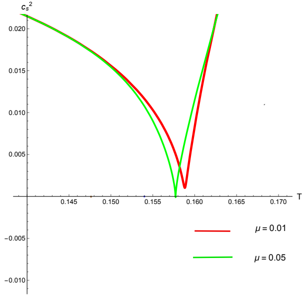
QCD Phase Diagram. Speed of sound. Experiment-Lattice-HQCD.

- Speed of sound, c_s^2 as a function of temperature, (T_{eff}).

With the CMS data point obtained from ultra-central PbPb collision data at $\sqrt{s_{NN}} = 5.02\text{TeV}$, from [1]



With HQCD for light quarks only [2]



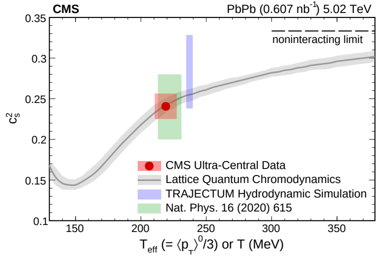
A. Hayrapetyan *et al.* [CMS], "Extracting the speed of sound in quark–gluon plasma with ultrarelativistic lead-lead collisions at the LHC," Rept. Prog. Phys. **87** (2024) 077801

I.A, "Phase diagram structure of QCD under critical conditions", ВЕСТНИК МГУ, СЕРИЯ 3. Suppl.2, 2025, S924–S946

QCD Phase Diagram. Speed of sound. Experiment-Lattice-HQCD.

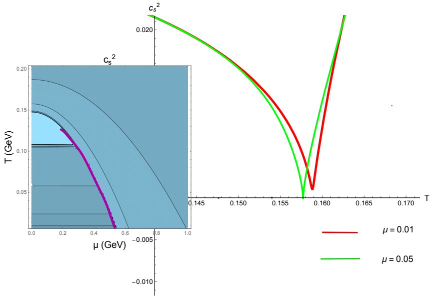
- Speed of sound, c_s^2 as a function of temperature, (T_{eff}).

With the CMS data point obtained from ultra-central PbPb collision data at $\sqrt{s_{NN}} = 5.02$TeV, from [1]



A. Hayrapetyan *et al.* [CMS], "Extracting the speed of sound in quark–gluon plasma with ultrarelativistic lead-lead collisions at the LHC," Rept. Prog. Phys. **87** (2024) 077801

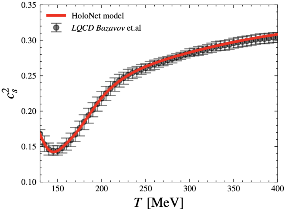
With HQCD for light quarks only [2]



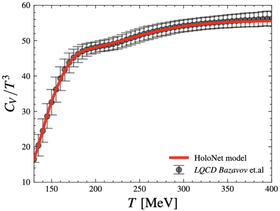
I.A, "Phase diagram structure of QCD under critical conditions", ВЕСТНИК МГУ, СЕРИЯ 3. Suppl.2, 2025, S924–S946

QCD Phase Diagram. Speed of sound. Lattice-HoloNet

- Speed of sound c_s^2



- Specific heat



HoloNet results from 2512.06044 and the error bars indicating the lattice data from

QCD Phase Diagram: Experiments + Lattice, 1-nd order phase transition

- Main experimental method to find 1-st order phase transition:
  - Beam Energy Scan Method
  - We propose **a new method** related with study of scattering amplitudes of particles created in colliding heavy ions beams

EMD Holographic models

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S=\int d^5x\,\sqrt{-g}\left[R-\frac{f_1(\phi)}{4}\,F_{(1)}^2-\frac{f_B(\phi)}{4}\,F_{(B)}^2-\frac{1}{2}\,\partial_M\phi\partial^M\phi-V(\phi)\right]$$
$$ds^2=\frac{L^2}{z^2}\,\mathfrak{b}(z)\left[-\,g(z)\,dt^2+dx^2+dy_1^2+e^{c_B z^2}dy_2^2+\frac{dz^2}{g(z)}\right]$$

$A_{(1),m}=A_t(z)\delta_m^0,\ A_t(0)=\mu,\ F_{(B)}=dx\wedge dy^1$ ϕ - dilaton, $\alpha(z)=e^{\phi(z)}$ - **running** coupling
Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$\mathfrak{b}(\mathbf{z})=\mathbf{e}^{\mathbf{2}\mathcal{A}(\mathbf{z})}$ $\Longleftrightarrow$ quarks mass **"Bottom-up approach"**

Heavy quarks (c, b, t):
$\mathcal{A}(z)=-cz^2/4$ *Andreev, Zakharov'06*

$\mathcal{A}(z)=-cz^2/4+pz^4$ *IA, Hajilou, Rannu, Slepov, EPJ C (2023)83*

Light quarks (d, u, s)
$\mathcal{A}(z)=-\,a\ln(bz^2+1)$ *Li, Yang, Yuan'17*

EMD Holographic models

I.A, K. Rannu, P.Slepov, JHEP, 2021

$$S=\int d^5x\,\sqrt{-g}\left[R-\frac{f_1(\phi)}{4}F_{(1)}^2-\frac{f_B(\phi)}{4}F_{(B)}^2-\frac{1}{2}\partial_M\phi\partial^M\phi-V(\phi)\right]$$

$$ds^2=\frac{L^2}{z^2}\mathfrak{b}(z)\left[-g(z)\,dt^2+dx^2+dy_1^2+e^{c_Bz^2}dy_2^2+\frac{dz^2}{g(z)}\right]$$

$A_{(1),m}=A_t(z)\delta_m^0,\ A_t(0)=\mu,\ F_{(B)}=dx\wedge dy^1$ ϕ - dilaton, $\alpha(z)=e^{\phi(z)}$ - **running** coupling

Giataganas'13; IA, Golubtsova'14; Gürsoy, Järvinen '19; Dudal et al.'19

$$\mathfrak{b}(z)=e^{2\mathcal{A}(z)}\iff$$ quarks mass **"Bottom-up approach"**

Heavy quarks (c, b, t):

$\mathcal{A}(z)=-cz^2/4$ *Andreev, Zakharov'06*

$\mathcal{A}(z)=-cz^2/4+pz^4$ *IA, Hajilou, Rannu, Slepov, EPJ C (2023)83*

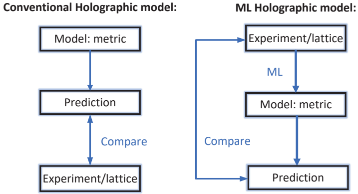
Light quarks (d, u, s)

$\mathcal{A}(z)=-\,a\ln(bz^2+1)$ *Li, Yang, Yuan'17*

Realistic model - recent progress with ML (see next slides), *H.Mei's group*

Few comments on applications of machine leaning for fitting parameters in HQCD

- The parameter space is high-dimensional and non-linear, making traditional fitting methods less effective.
- Goal: an effectively automatic way through machine learning to determine the parameters of HQCD
- The sketch of holographic QCD and machine learning.



From X. Chen and M. Huang, JHEP **02** (2025), 123

What is Machine Learning?

Definition (Arthur Samuel, 1959)

"Field of study that gives computers the ability to learn without being explicitly programmed."

Modern definition (Tom Mitchell, 1997)[1]

A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E.

[1] T.Mitchell, Machine Learning, NY, 1997.

Why Machine Learning Now?

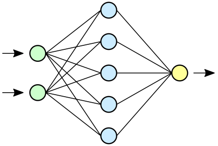
- **Data explosion**: Internet, sensors, scientific instruments generate massive datasets.
- **Computational power**: GPUs, TPUs, cloud computing enable training large models.
- **Algorithmic advances**: Deep learning, reinforcement learning, transformers.
- **Automation**: Extract patterns and insights without manual rule crafting

Neural Networks and Deep Learning

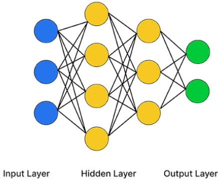
- Composed of layers of neurons (units) with non-linear activations.

A neural network is a mathematical model together with its software implementation, built on the principle of the organization of networks of nerve cells (neurons) in a living organism (W. McCulloch, W. Pitts, 1943)

- **Input layer** → hidden layers → **output layer**.
- Learning via **backpropagation and gradient descent**.
- **Deep learning**: many hidden layers.



Green input neurons, blue — hidden neurons, yellow — output neuron



Key architectures

Convolutional Neural Networks (CNNs) for images, Recurrent Neural Networks (RNNs)/Transformers for sequences, etc.

The Learning Process: Typical Workflow

1. **Split** data into training, validation, and test sets.
2. **Choose model** and loss function.
3. **Optimize** via gradient descent.

Gradient descent is the primary optimization algorithm used to train ML models. It iteratively adjusts the model's parameters to minimize a loss function $L(\theta)$, which measures how wrong the model's predictions are.

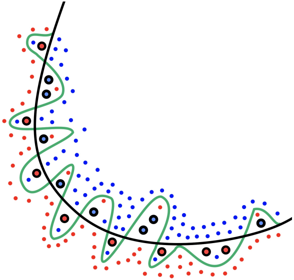
4. **Validate** hyperparameters «Валидация гиперпараметров» — это процесс выбора оптимальных

"Hyperparameters" are choised before training begins. These hyperparameters control the learning process and model structure. Common hyperparameters include:

- Learning rate – step size for gradient descent.
- Architecture – number of layers, neurons per layer, activation functions.
- Regularization – strength of L1/L2 penalties, dropout rate.

5. **Evaluate** on test set.

Overfitting and Regularization



Green line — overfitted model
black line — properly trained model

Overfitting: "Overfitting" — is when a model memorizes the training examples too well, including their noise and errors. As a result, it becomes unable to correctly predict answers for any other data.

Regularization techniques:

- L1/L2 weight decay [adds a penalty to the loss function for large weight values]
- Dropout [randomly sets a certain fraction of neurons to zero at each step. This forces the network to learn more robust features without relying on individual neurons]
- Early stopping [stops training when the validation set performance stops improving / starts worsening for a given number of epochs]
- Data augmentation
- Cross-validation

Evaluation Metrics

Metrics in machine learning are quantitative measures of model performance, used to evaluate prediction quality, compare algorithms, and select the optimal model.

Task	**Common Metrics**
Regression	MSE, MAE, R^2
Classification	Accuracy, Precision, Recall, F1-score, ROC-AUC
Clustering	Silhouette score, Davies–Bouldin index
Ranking	Precision@k, NDCG

Regression

— a type of supervised learning task where the model predicts a continuous numerical value

Classification

— a supervised learning task where the model predicts a discrete label for an input object

Clustering

— an unsupervised learning task. The model finds hidden groups (clusters) in the data without any hints or correct answers. Silhouette score is a clustering quality metric

Key metrics by task type

For regression (predicting a continuous number)

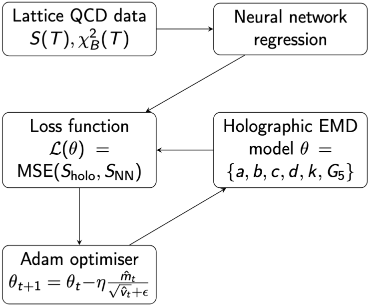
Metric	**Formula**
MSE (Mean Squared Error)	$\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
MAE (Mean Absolute Error)	$\frac{1}{n}\sum_{i=1}^{n}\lvert y_i - \hat{y}_i\rvert$
RMSE (Root MSE)	$\sqrt{\text{MSE}}$
R^2 (coefficient of determination)	$1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$

Motivation for HoloNet

- Lattice QCD (LQCD) is the first-principles method for QCD at zero baryon density, but breaks down at finite μ_B (sign problem).
- HQCD maps QCD to a 5D gravity theory – can explore finite μ_B.
- Traditional holographic models rely on *ad hoc* ansätze for the dilaton potential and metric.
- **Key idea**: Use machine learning to directly learn the holographic bulk functions from LQCD data, then extrapolate to finite μ_B.

Two-Stage ML Pipeline

Две стадии - выход 1-го этапа - вход для 2-го.



From: 2405.06179 by X.Chen, M.Huang

1-st ML: NN as differentiable interpolant of LQCD

Trained on LQCD entropy $S(T)/T^3$ and baryon susceptibility $\chi_B^2(T)$.

Second ML: Adam optimisation of 6 parameters of EMD model.

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{f(\phi)}{4} F^2 \right]$$

Metric function: $A(z) = d \ln(az^2 + 1) + d \ln(bz^4 + 1)$.
Parameters $\theta = \{a, b, c, d, k, G_5\}$ minimising:

$$\mathcal{L} = \sum_i (S_{\text{hol}}(T_i; \theta) - S_{\text{NN}}(T_i))^2 + \lambda \sum_j (\chi_{2,\text{hol}} - \chi_{2,\text{NN}})^2$$

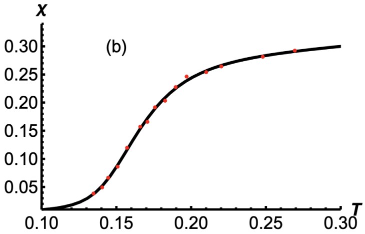
Adam* =Adaptive Moment Estimation - алгоритм оптимизации, используемый для обновления весов NN во время обучения

1. The metric $A(z)$ is learned solely from the equation of state (EoS) – entropy density, speed of sound, etc. – obtained from LQCD.
2. With $A(z)$ fixed, the coupling $f(z)$ is optimized using LQCD data for the baryon number susceptibility χ_B^2 —

EMD HQCD Model. ML Results

First ML: neural network regression

Trained on LQCD entropy $S(T)/T^3$ and baryon susceptibility $\chi_B^2(T)$.



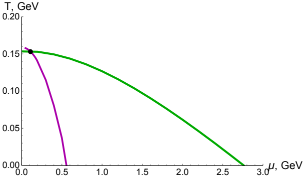
The baryon number susceptibility as a function of temperature. The dots represent the results obtained from lattice simulations, while the black line indicates the prediction made by the neural network. The unit of T is in GeV. From: 2405.06179

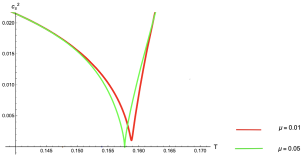
Second ML: parameter optimisation

	a	b	c	d	k	G_5	T_c (GeV)	CEP (GeV)
$N_f = 0$	0	0.072	0	-0.584	0	1.326	0.265	/
$N_f = 2$	0.067	0.023	-0.377	-0.382	0	0.885	0.189	(μ_B^c=0.46, T^c=0.147)
$N_f = 2+1$	0.204	0.013	-0.264	-0.173	-0.824	0.400	0.128	(μ_B^c=0.74, T^c=0.094)
$N_f = 2+1+1$	0.196	0.014	-0.362	-0.171	-0.735	0.391	0.131	(μ_B^c=0.87, T^c=0.108)

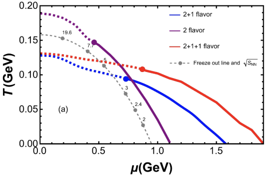
LQ HQCD Model vs. More Realistic ML Results

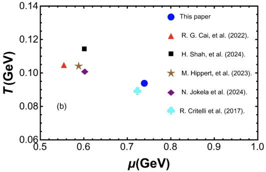
- LQ, I.A,A.Ermakov,K.Runnu,P.Slepov, EPJC'23 — IA, MGU Phys.Bull'25

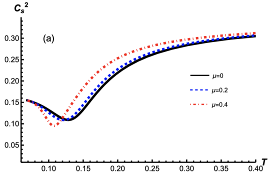




- $N_f = 2, 2+1, 2+1+1$, — Chen, Huang, JHEP'24







Real case: $N_f = 2+1+1+1$ (u, d, s, c, b). Currently, there **are no lattice simulations** providing susceptibility data for this configuration.

HoloNet

- 2405.06179 – ML is used simply to find the best parameters $\theta = \{a, b, c, d, k, G_5\}$ for the ansatz

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1), \quad f(z) = e^{cz^2 - A(z) + k},$$

  that match lattice data for the baryon number susceptibility as a function of temperature.

- 2512.06044 – HoloNet determines $A(z)$ and $f(z)$ as the curves that best fit the experimental points – an example of a neural network operator that completely replaces traditional analytical approximations by learning directly to reconstruct unknown functions of the theory.
  Two-stage optimisation:
  1. Learn metric $A(z)$ from LQCD equation of state.
  2. With $A(z)$ fixed, learn coupling $f(z)$ from baryon susceptibility.

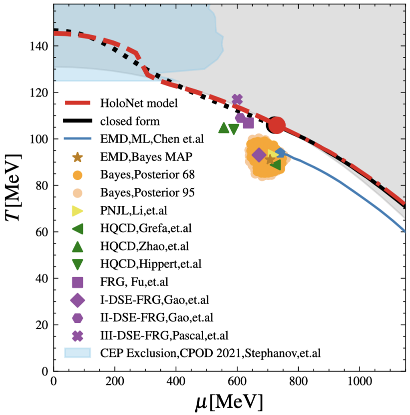
H. A. Zeng, L. Wang and M. Huang, "HoloNet: Toward a Unified Einstein-Maxwell-Dilaton Framework of QCD," [arXiv:2512.06044 [hep-lat]].

HoloNet

- 2405.06179

  $$A(z) = d\ln(az^2+1) + d\ln(bz^4+1)$$

- 2512.06044, NeroNet



Confinement in QCD

- **Wilson criterion (Wilson '74):**

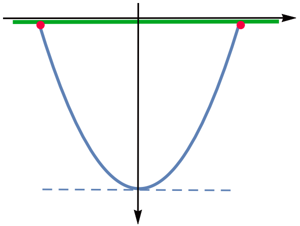
$$< W(C) >_{vac} \equiv < Pe^{g\int A_\mu dx^\mu} > \sim e^{-\sigma_{string} S(C)}$$

- **D=4**
  - **according to pert. theory - not satisfied**
  - **on a lattice - satisfied (in strong coupling, Wilson '74); record calculations $96 \cdot 48^3$**
  - **D=2 QCD_2 - is satisfied ('t Hooft, Gross)**
- **On the lattice the main question $\mu \neq 0$ -** $\mu_B = \frac{1}{3}[\mu_u + \mu_d + \mu_s]$
- **The problem of the sign [Fugacity expansion]**

Time-like Wilson loops in a deformed metric

O.Andreev, V.Zakharov, PRD'07, I.A., K.Rannu, JHEP'18 + P.Slepov, PLB'19.

$$ds^2_{Einstein\ Frame} = \frac{\mathfrak{b}(z)}{z^2}\left(-g(z)dt^2 + d\vec{x}^2 + \frac{1}{g(z)}dz^2\right)$$ in "string frame" $\mathfrak{b}_s = e^{\sqrt{\frac{2}{3}}\phi}\mathfrak{b}$



String action "on the barn":

$$S = \mathfrak{t}\int_{-\ell/2}^{\ell/2} d\xi\ M(z(\xi))\sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}$$

$$M(z) = \frac{\mathfrak{b}_s(z)}{z^2} \quad \mathcal{F}(z) = g(z) \quad \mathcal{V}(z) = M(z)\sqrt{\mathcal{F}(z)}$$

$$\frac{\ell}{2} = \int_0^{z_*} \frac{1}{\sqrt{\mathcal{F}(z)}}\ \frac{dz}{\sqrt{\frac{\mathcal{V}^2(z)}{\mathcal{V}^2(z_*)} - 1}}$$

Lemma. If there is a solution to the equation defining **the dynamic wall** ($z = z_{DW}$)

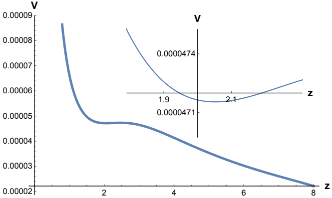
$$\mathcal{V}'\Big|_{z=z_{DW}} = 0 \quad \Longleftrightarrow \quad \frac{M'(z)}{M(z)} + \frac{1}{2}\frac{\mathcal{F}'(z)}{\mathcal{F}(z)}\Big|_{z=z_{DW}} = 0,$$

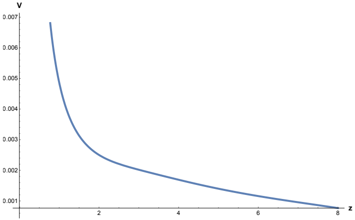
then when $\ell \to \infty$ $\quad S \sim \sigma_{DW}\ \ell$, where $\quad \sigma_{DW} = \frac{\mathfrak{b}(z_{DW})}{(z_{DW})^2}\sqrt{g(z_{DW})}$

this is confinement.

Wilson loops & Cornell potential in ML's metrics
Our calculations

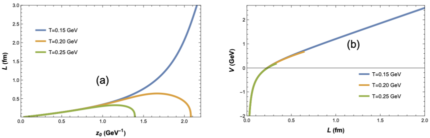
- $N_f = 2$ $\qquad$ $N_f = 2+1$ $\qquad$ IA, P.Slepov

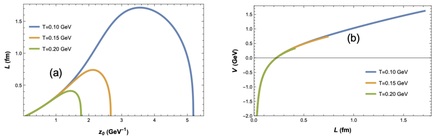




Wilson loops & Cornell potential in ML's metrics

- $N_f = 2$ $\qquad\qquad$ $N_f = 2+1$ $\qquad\qquad$ 2406.04650





X. Guo, X. Chen, D. Xiang, M. A. Martin Contreras and X. H. Li, "Potential energy of heavy quarkonium in flavor-dependent systems from a holographic model," Phys. Rev. D **110** (2024) 046014 [arXiv:2406.04650]

Linear confinement: Lattice & Experiment Indications

1. Hadron spectroscopy (most direct indication)
   - Upon excitation of a hadron, the mass M and spin J are related.
   - For a linear potential, $J \propto M^2$ — **Regge trajectories**.
   - Experimental observation of linear trajectories for light mesons, baryons, etc., is one of the most convincing pieces of evidence.

2. Potential models for heavy quarkonia
   - Systems such as $c\bar{c}$ (charmonium) and $b\bar{b}$ (bottomonium) are described by the Schrödinger equation.
   - To agree with experiment, the potential **must include a linear term** — the Cornell potential.

- Latest experiments with quantum simulators: people have learned to create artificial "strings" in controlled quantum systems and observed them breaking. For the first time, it has become possible to track the dynamics of color string breaking in real time.

D.Gonzalez-Cuadra, Lukin,... Zoller, Observation of string breaking on a (2 + 1)d Rydberg quantum simulator, Nature (2025), 2410.16558

M. John et al "Non-Abelian String-Breaking Dynamics on a Qudit [Multi-level] Quantum Compute 2605.05841

- Mesons spectrum & HoloNet

Chemical freeze-out and confinement

At $\mu_B = 0$:

- Lattice QCD shows that the chiral/deconfinement transition is a smooth crossover around

$$T_c \sim 155\text{–}160 \text{ MeV}.$$

- Statistical hadronization fits to heavy-ion data give a chemical freeze-out temperature

$$T_{\rm fo} \sim 155 \text{ MeV},$$

  essentially coincident with the crossover region.

Thus, at $\mu_B \approx 0$, freeze-out and hadronization occur almost simultaneously.
At finite μ_B:
One commonly finds

$$T_{\rm fo}(\mu_B) < T_c(\mu_B)$$

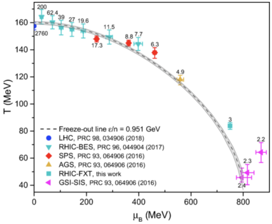
for moderate baryon density.
Physically, this means:

1. The system hadronizes near the phase boundary.
2. It then spends some time as an interacting hadron gas.
3. Inelastic reactions cease later at a slightly lower temperature, producing chemical freeze-out.

Chemical freeze-out and confinement

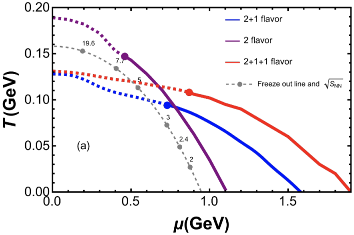
[noframenumbering]

- Chemical freeze-out data points



From: PhysRevC.111.054903. Different symbol styles represent different experiments with the labels indicating $\sqrt{s}_{NN}$ [GeV] for each point.

- HoloNet



From: 2401.06417

Conclusion

- **Holographic QCD:**
  - **Reproduces the expected phase transition (PT) structure in QCD in (T, μ, B, ν)-space.**
  - **Provides methods for calculating observable signatures of PTs.**

- **HoloNet models:**
  - **Learn at $\mu = 0$ and predict at large μ.**
    - **Models with explicit 5D geometry: train on lattice $\mathrm{EoS}|_{\mu=0}$ data, construct the EoS at $\mu > 0$**
    - **Models with 5D geometry encoded in a neural network: train on LQCD_{2+1} data at $\mu = 0$ and predict for $\mu > \mu_{\mathrm{CEP}}$.**
- **Next steps:**
  - **Compute signatures of phase transitions using realistic HoloNet models.**