Holographic equation of state with multiple quantum nubmers for numerical simulation of relativistic heavy ion collisions

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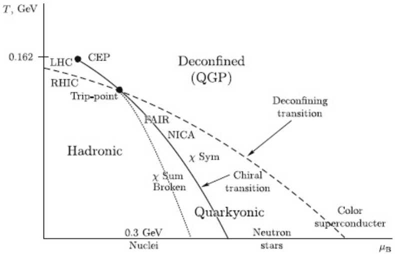
Open problems of the QCD phase diagram research

The aims of the previous study

1. To propose **a method for setting up holographic equations of state (EoS)** for simulations of real experiments with heavy ions for detailed study of the phase diagram at $\mu_B > 0$.
2. To implement holographic equations of state **into a software package** for solving equations of relativistic hydrodynamics
3. To demonstrate **the results of a multi-stage simulation of a heavy ion collision experiment using a holographic EoS**



Pic.: I. Ya. Aref'eva, Theoret. and Math. Phys., 217, 3 (2023)

Lattice QCD results:

- The **great** results for $\mu_b \approx 0$.
- Area $\mu_b > 0$ is unavailable because of **the sign problem**.

AdS/CFT duality

Gerard 't Hooft - the Holographic principle (1993):

Information about matter in space is a "flat hologram"at the boundary of this space (no more than one degree of freedom per Planck area).

Maldacena - AdS/CFT - duality (1998)J. Maldacena // Adv.Theor.Math.Phys.2:231-252,1998] :

Under certain boundary conditions, there is a duality of QCD and Einstein's theory in the low-energy limit - AdS/QCD

Thermodynamic characteristics of quark-gluon plasma:

T, ε, P, μ_b

EoS: $P = P(\varepsilon)$

Parameters of the AdS_5 space deformation:

$T(z_h)$, $S(z_h)$, where z_h is the characteristic of the gravitational horizon, T, S are the temperature and entropy of the corresponding black brane

Holographic EoS of QGP within the framework of the model with a double dilaton field

An ansatz proposed is[I. Aref'eva et al. // JHEP 06, 090 (2021)] :

$$ds^2 = \frac{L^2}{z^2} b(z) \left[-g(z)dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} (dy_1^2 + dy_2^2) + \frac{dz^2}{g(z)} \right],$$

A deforming factor $b(z) = e^{2A(z)}$ restore the lattice QCD,
L is the radius of AdS, g(z) is a thermodynamic blackening function

Advantages of the approach:

- The possibility of studying the properties of nuclear matter at sufficiently large μ_b
- One of the model parameters is selected in accordance with the Regge spectra of ρ mesons

The experimental dependence of the multiplicity density on energy

$M \propto s_{NN}^{0.15}$,

[K. Aamodt, et al. // Phys. Rev. Lett. 105 252301 (2010)]

The result within isotropic holographic models

$M \propto s_{NN}^{\frac{1}{3}}, \nu = 1$

[S. Gubser et al. // Phys. Rev. D 78 066014 (2008]

The result of Arefieva's group (the anisotropic case)

$M \propto s_{NN}^{0.16}, \nu = 4.5$

[I. Aref'eva, A. Golubtsova // JHEP 04 011 (2015)]

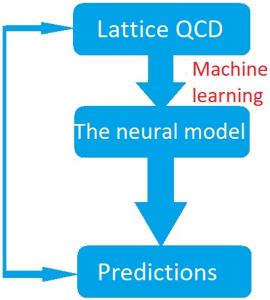
The model for light quarks (A.Anufriev, V.Kovalenko; arXiv:2510.03157)

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1);\ a, b \geq 0, d \leq 0$$

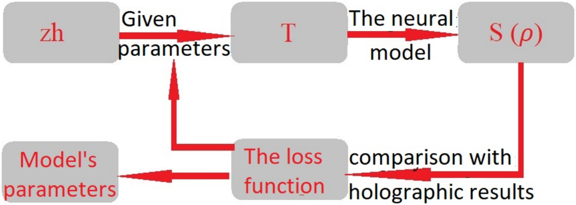
from [X. Chen, M. Huang // J. High Energ. Phys. 2025, 123 (2025)]

Machine learning application to the model calibration:

Task 1:
A model for testing



Task 2:
The optimization of parameters



Implementation of the holographic EoS in the packages of relativistic hydrodynamics (A.Anufriev, V.Kovalenko; arXiv:2504.20207)

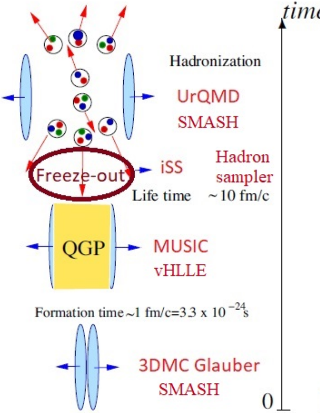
A program code was written based on the built-in MUSIC (https://github.com/MUSIC-fluid/MUSIC.git) and vHLLE (https://github.com/yukarpenko/vhlle.git) methods for reading tables constructed on lattice QCD results:

1. A two-dimensional table of thermodynamic quantities (T, p, μ_b) is being constructed based on the holographic formulae.
2. One uses the initial profile of energy and baryon densities to interpolate the values of thermodynamical quantities at each point of the grid initialized.
3. Evolution stops when the energy density reaches a certain critical value - freeze-out energy (preset)
4. Provides a special parameter of the MUSIC input file for quick table switching from the isotropic model to the anisotropic case.

Evolution of the QGP within the iEBE-MUSIC(https://github.com/chunshen1987/iEBE-MUSIC.git) and vHLLE-SMASH(https://github.com/smash-transport/smash-vhlle-hybrid.git) packages

The structure of the QGP evolution:

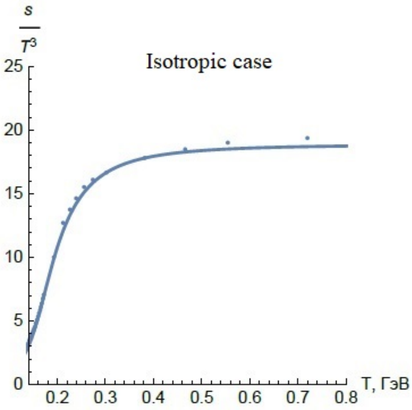
1. An initial profile of energy and baryon density is calculated using the 3D Monte Carlo Glauber (or SMASH) model.
2. The MUSIC (vHLLE) package (modified with a holographic equation of state) accepts this profile as an input and performs the QGP evolution.
3. The iSS (SMASH hadronic sampler) package performs Monte Carlo sampling to obtain a finite set of particles.
4. Transport model UrQMD (SMASH-afterburner) allows one to get the final spectrum of hadrons

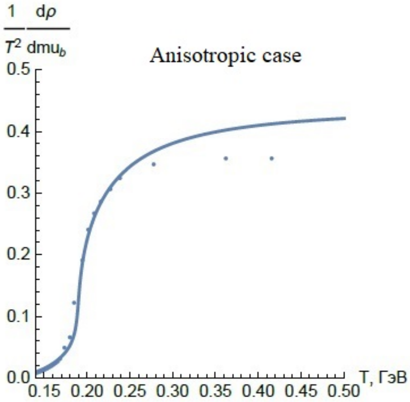


A holographic equation with the HRG matching

Matching at $\mu_b = 0$

The points for T<0.18 GeV are taken from SMASH HRG EoS
[A. Schafer et al. // Eur. Phys. J. A 58, 230 (2022).]





Matching at the nonzero baryonic potentials (A.Anufriev, V.Kovalenko; arXiv:2510.27541)

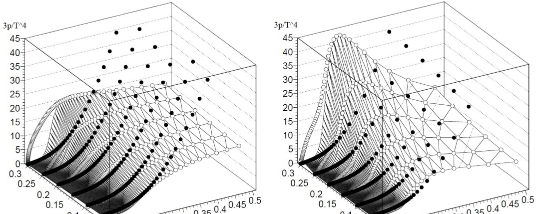
The main thermodynamic variables (P, s, ρ_b) can be expressed in the following form:

$$P = \frac{1}{2}(1 - f(T,\mu_b))P_{had}(T,\mu_b) + \frac{1}{2}(1 + f(T,\mu_b))P_{holo}(T,\mu_b),$$

Here $f(T,\mu_b)$ is an "switching"function and

$$f(T,\mu_b) = tanh(\frac{T - T_c(\mu_b)}{\Delta T_c}), \ \ \Delta T_c = 0.1T_c(0).$$

Connecting temperature T_c is obtained within the results of the chemical freeze-out study from [J. Cleymans, et al. // Phys. Rev. C73, 034905 (2006)].



Implementation of the stangeness potential into holographic approach (A. Anufriev, V. Kovalenko; TMP, to appear)

Hadron Gas Hypothesis

The full chemical potential is equal to linear combination of baryonic and strangeness potentials: $\mu = a_1\mu_B + a_2\mu_S$

From the study of hadron gas behavior for the main thermodynamic variables (p, ϵ, ρ_B) with the neutrality condition on the net strangeness density within the Themal-FIST package [V. Vovchenko, H. Stoecker // Comput. Phys. Commun. 244, 295 (2019)]:

$$\rho_S = 0, \ \ \mu_S \approx 0.2\mu_B, \ \ \mu \approx 0.95\mu_B - 0.3\mu_S$$

Quark Gas Hypothesis

Based on the results of [K.-il Kim, Y. Kim, S. H. Lee, // Journal of the Korean Physical Society, 55 (2009)], the full chemical potential is equal to linear combination of potential for light quarks and for strange one: $\mu = b_1\mu_q + b_2\mu_s$, where

$$\mu_q = \frac{\mu_B}{3}, \ \ \mu_s = \mu_q - \mu_S, \ \ \rho_f = \frac{p_F, f^3}{\pi^2} (\text{where } f = [u, d, s])$$

$$p_F, f = \sqrt{(\mu_f - 6G_V\rho_B)^2 - (m_f - 4G_S\langle\bar\psi\psi\rangle_f + 2K\langle\bar\psi\psi\rangle_j\langle\bar\psi\psi\rangle_k)^2}$$

from the Nambu-Jona-Lasinio approach https://arxiv.org/pdf/hep-ph/0402234

Implementation of the stangeness and charge potential into holographic approach

Hadron Gas Hypothesis

The full chemical potential is equal to linear combination of different potentials: $\mu = a_1\mu_B + a_2\mu_S + a_3\mu_Q$

From the study of hadron gas behavior for the main thermodynamic variables (p, ϵ, ρ_B) with the neutrality condition on the net strangeness density within the Themal-FIST package [V. Vovchenko, H. Stoecker // Comput. Phys. Commun. 244, 295 (2019)]:

$$\rho_S = 0,\ \ \rho_Q = 0.4\rho_B,\ \ \mu_S \approx 0.2\mu_B,\ \ \mu \approx 0.99\mu_B - 0.13\mu_S + 0.02\mu_Q$$

Quark Gas Hypothesis

Based on the results of [K.-il Kim, Y. Kim, S. H. Lee, // Journal of the Korean Physical Society, 55 (2009)], the full chemical potential is equal to linear combination of potential for light quarks (different flavors) and for strange one: $\mu = b_1\mu_u + b_2\mu_d + b_3\mu_s$, where

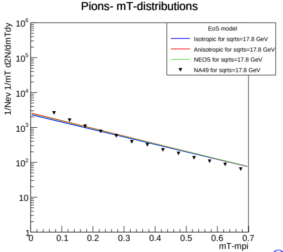
$$\mu_B = \mu_u + 2\mu_d,\ \mu_Q = \mu_u - \mu_d,\ \mu_S = \mu_d - \mu_s,$$

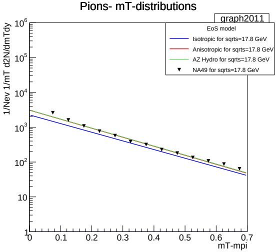
$$\rho_u = \frac{\mu_u^3}{\pi^3},\ \rho_d = \frac{\mu_d^3}{\pi^3},\ \rho_s = \frac{(\mu_s^2 - m_s^2)^{\frac{3}{2}}}{\pi^2}$$

The comparison of π^- spectra by transverse mass with the results NA49 for the holographical EoS with multiple quantum numbers

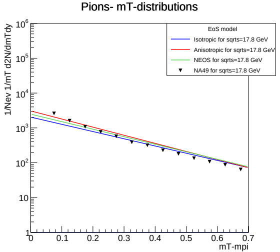
Pb-Pb collisions, $|y| < 2.6$,$\sqrt{s}$=17.8 GeV, b<2.5 fm

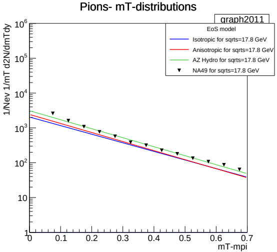
Hadron Gas hypothesis





Quark Gas hypothesis

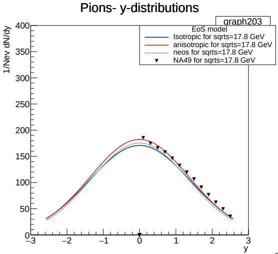


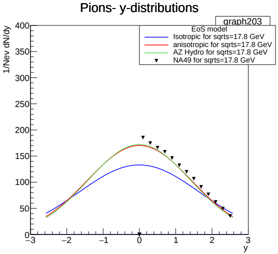


The comparison of π^- spectra by rapidity with the results NA49 for the holographical EoS with multiple quantum numbers

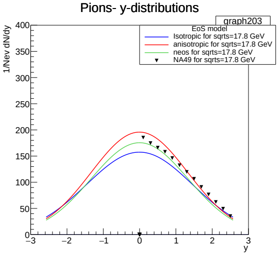
Pb-Pb collisions, $|y| < 2.6$,$\sqrt{s}$=17.8 GeV, b<2.5 fm

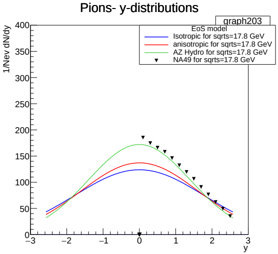
Hadron Gas hypothesis





Quark Gas hypothesis





Conclusion and potential for further development

Results

1. A method is proposed for setting up holographic models with **HRG matching and multiple quantum numbers**.

2.**Holographic equations of state fully are implemented into the software packages of relativistic hydrodynamics**.

3. The results of calculations in the iEBE-MUSIC and vHLLE-SMASH packages for **multi-stage simulation** of the QGP dynamics are considered.

4. A **model dependence of the results on the applied equations of state** is shown.

Future plans and prospects

1. We schould find a holographic model for the physical masses of quarks **("hybrid model")**, preserving all the advantages of the approach - the need for a correct deforming factor

2. It is possible to **account for the secondary magnetic field** generated by the outgoing charged particles

3. Using the approaches described, it is planned to study **fluctuations and long-range correlations as tools for studying the phase diagram**

Thank you for your attention!

Backup Slides

Tensorflow input

Task 1:

- The neural network trained is four fully connected layers on scheme 64-128-64-1.
- Activation functions scheme is relu-relu-relu-linear.
- Adam algorithm is used as an optimizer with MSE loss function.
- Learning rate=0.001

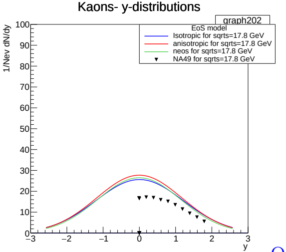
Task 2:

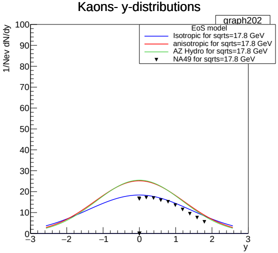
- A gradient descent algorithm to find the optimal parameter values.
- The loss function is the difference between the predictions of the holographic model and the values obtained from the neural network model
- The least squares method is employed to minimize the loss function.
- The "Adam" optimizer is used for training with constraints on paranmeters from slide № 9.
- In every training epoch, the loss function is calculated first and then the gradients should be computed. The parameter values are updated afterwise.

The comparison of K^- spectra by rapidity with the results NA49 for the holographical EoS with multiple quantum numbers

Pb-Pb collisions, $|y| < 2.6$,$\sqrt{s}$=17.8 GeV, b<2.5 fm

Hadron Gas hypothesis





Quark Gas hypothesis