Astrophysical positronium and Dicke superradiance

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How much positronium is there?

- Despite 90 years passed since Mohorovičić [1] suggested the possibility of the existence of astrophysical positronium, none of its astrophysical recombination lines has been detected.

- These lines, particularly the optical Lyman-alpha line (2431 Å) and radio recombination lines, offer a unique way to study positronium formation and annihilation processes in astrophysical environments.

- Observation of such recombination lines is particularly important given the half-century-long mystery of the origin of galactic 511 keV gamma rays measured by various instruments since their first observation in pioneering balloon experiments [2,3].

- Moreover, the morphology of the 511 keV signal, emanating from the Galactic bulge, center, and disk with a bulge-to-disk luminosity ratio of ~ 1, is unique and does not resemble any other distribution of astrophysical sources [2].

[1] S. Mohorovičić, Astron. Nachr. 253, 93 (1934)

[2] T. Siegert, Astrophys. Space Sci. 368, 27 (2023)

[3] N. Prantzos *et al.*, Rev. Mod. Phys. 83, 1001 (2011)

How much positronium is there?

- Positrons in space can be produced by a variety of mechanisms: β^+ decay of unstable nuclei produced in stellar explosions, such as ^{56}Ni, ^{22}Na, ^{44}Ti, and ^{26}Al; inelastic collisions of relativistic cosmic ray protons with interstellar gas and the production of secondary positrons in decays of positively charged mesons; pair production in $\gamma - \gamma$ interactions, and other mechanisms [see our published paper].

- About 5×10^{43} positrons annihilate every second near the Galactic Center in the Milky Way [1].

- How much positronium are there?

  - A certain fraction of the astrophysical positrons produced, f_{Ps}, forms positronium either through radiative recombination or in an endoenergetic charge exchange reaction [2,3].

  - This fraction can be estimated from the observed intensities of the two photon 511 keV line and the continuous emission of three photons with energies below 511 keV as follows [3,4].

[1] T. Siegert, Astrophys. Space Sci. 368, 27 (2023)

[2] S. C. Ellis *et al.*, Eur. Phys. J. D 72, 18 (2018)

[3] N. Prantzos *et al.*, Rev. Mod. Phys. 83, 1001 (2011)

[4] M. Leventhal, Am. J. Phys. 60, 856 (1992)

How much positronium is there?

Estimation of the f_{Ps}:

- Positronium will be formed one-quarter of the time in the spin-singlet state of para-positronium and three-quarters of the time in the spin-triplet state of ortho-positronium.
- Ortho-positronium decays almost exclusively into three photons and thus produces a continuous spectrum of gamma rays with intensity $I_{cont} \sim 3\frac{3}{4}f_{Ps} = \frac{9}{4}f_{Ps}$.
- Para-positronium decays almost exclusively into two photons. However, another process of two photons production involves the direct annihilation of positrons with electrons when some $1 - f_{Ps}$ positrons annihilate, without producing positronium. Therefore, the intensity of the 511 keV line is $I_{511} \sim 2\left(1 - f_{Ps} + \frac{1}{4}f_{Ps}\right) = 2\left(1 - \frac{3}{4}f_{Ps}\right)$.

  which implies

$$f_{Ps} = \frac{8}{6 + 9\,\frac{I_{511}}{I_{cont}}}.$$

In fact, the observed intensities I_{511} and I_{cont} indicate that almost all positrons annihilate through the formation of positronium [1].

[1] T. Siegert, Astrophys. Space Sci. 368, 27 (2023)

Dicke superradiance: an overview - 1

Definition & Requirement:
Cooperative coherent spontaneous emission by a large number of atoms.
In order to occur, atoms must evolve into a phase correlated state.

Intensity:
Scales as N^2 (coherent) rather than N (incoherent); N is number of atoms.

Characteristic Timescales:

- Superradiance puls duration – T_{SR}
- Delay time (building coherence) – T_D
- Dephasing time – T_{dph}

where a necessary condition is $T_D < T_{dph}$.

*In some regimes (in our work) we use τ and t_0 for T_{SR} and T_D, respectively.

Dicke superradiance: an overview - 2

A fairly detailed review is available in the published paper (covers following regimes):

- Classical analog superradiance: (where Λ is the shape factor, a coherence parameter)

$$I(t) = -\frac{d}{dt}(2\mu B W) = \frac{N^2\Lambda}{2\tau_0}\mu B\,\mathrm{sech}^2\left[\frac{N\Lambda}{2\tau_0}(t-t_0)\right], \quad \tau = \frac{\tau_0}{N\Lambda}, \quad t_0 = \frac{\ln N\Lambda}{N\Lambda}\tau_0$$

- Small-sample $L \ll \lambda$ (quantum consideration): similar to classical regime when $\Lambda = 1$.

- Large-sample $L \geq \lambda$ (semiclassical approach):
  - Rough estimate (based on a possible interpolation between the quantum incoherent exponential decay law and the classical coherent radiation pulse):

$$I(t) = -\hbar\omega\frac{dW}{dt} = \hbar\omega\frac{(N\Lambda+1)^2}{4\Lambda\tau_0}\,\mathrm{sech}^2\left(\frac{N\Lambda+1}{2\tau_0}(t-t_0)\right), \quad T_{SR} = \frac{\tau_0}{N\Lambda+1}, \quad t_0 = T_{SR}\ln(N\Lambda)$$

  - Rigorous calculation requires solving the Burnham-Chiao equation.

- Small-sample $L \ll \lambda$ (semiclassical approach):
  Rigorous form as a limiting case of the large sample approach (Maxwell-Bloch equation):

$$T_{SR} = \frac{4\pi S}{3N\lambda_\omega^2}\tau_0, \quad where: \quad \lambda_\omega = \frac{2\pi c}{\omega} \quad \& \quad \Lambda = \frac{3}{4\pi}\frac{\lambda_\omega^2}{S} \tag{1}$$

This agrees with quantum consideration when $\Lambda = 1$.

The problem & what to estimate?

Astrophysical Motivation:

- Despite the early theoretical proposal of superradiance by Dicke in 1954 [1] and its subsequent experimental confirmation [2] and the intensive research carried out in the quantum optics community following that, this phenomenon has until recently been largely ignored in the astrophysical community.
- The discovery of astronomical masers in the interstellar medium [3] indicates the possibility of such phenomena in an astrophysical context.
- Superradiance was used to explain the observations of intensity bursts in maser lines 1612 MHz OH, 6.7 GHz CH_3OH, and 22 GHz H_2O for some astronomical sources [refs in pub].
- There is a region in the Galaxy where positronium is present in large quantities.

Possible application?

Can superradiance occur in the spin-flip lines of astrophysical Hydrogen and Positronium?

Goals:

To speculate on this possibility, we estimate sample lengths L and r (for a cylindrical sample), and, for atomic density n, the time scales T_R, T_D and T_{dph} for a superradiant pulse in space.

[1] R. H. Dicke, Phys. Rev. 93, 99 (1954)

[2] N. Skribanowitz *et al.*, Phys. Rev. Lett. 30, 309 (1973)

[3] H. Weaver *et al.*, Nature (London) 208, 29 (1965)

The approach (1): the Arecchi-Courtens criterion

- **Causality Limit:**
  Cooperating atoms must be within a distance cT_R to communicate.

- **Criterion:**
  $L \leq cT_R = 2cT_{SR}$. Assuming Fresnel number ≈ 1:

$$N \leq \frac{4}{3}\omega\tau_0 = \begin{cases} 4.2\cdot 10^{24} & \text{(Hydrogen)} \\ 5.1\cdot 10^{19} & \text{(Positronium)} \end{cases} \qquad (2)$$

- **Note:**
  - For hydrogen, this limitation on the number of atoms applies only when all atoms are inverted simultaneously (inverting radiation propagates transversely).
  - If inverting radiation propagates longitudinaly through the sample the situation will be different.

The approach (2): Maxwell-Bloch equations

Governing Equation:

- Astrophysical application requires the semiclassical approach (for large sample).
- The dynamics of Dicke superradiance of a large sample can be conveniently described by the semiclassical Maxwell-Bloch equations (full review is available in the published paper).
- After a lengthy derivation, the large-sample superradiance problem reduces to the Burnham-Chiao equation for the Bloch angle θ:

$$\frac{d^2\theta(q)}{dq^2}+\frac{1}{q}\frac{d\theta(q)}{dq}=B\sin\theta(q), \quad where: \quad q=2\sqrt{\frac{1}{LT_R}\,z\left(t-\frac{z}{c}\right)} \tag{3}$$

with initial values:

$$\theta(0)=\theta_0=\frac{2}{\sqrt{N}},\quad \frac{d\theta}{dq}(0)=0 \tag{4}$$

Initial Conditions in (H) & (Ps) problem: θ_0 corresponding to eq(2)

$$\theta_0=\begin{cases}9.8\cdot 10^{-13} & \text{(Hydrogen)}\\ 2.8\cdot 10^{-10} & \text{(Positronium)}\end{cases} \tag{5}$$

Numerical solution: Using (5) in (4), we solve the Burnham-Chiao equation by the Runge–Kutta–Nyström method (using DRKNYS routine from the CERNLIB library).

Solution and estimates

- Solution is obtained for a cylindrical sample of length L and cross-section S that can emit a superradiant pulse of characteristic time of the order of T_R.
- For that, causality requires that the emitting atoms be within cT_R distance of each other $L \leq cT_R = 2cT_{SR}$, where T_{SR} is the duration of the superradiant peak (For large samples the duration of the peak scales as $2\sqrt{T_R T_D}$ when dephasing is negligible [1]).
- Delay time T_D can be obtained by defining it as the interval from the moment of population inversion of the sample to the moment of the appearance of the first peak of the emitted radiation [1] so one can determine the corresponding q.
- Dephasing time can be obtained by taking nonideality effects (like: incoherent decay of the inverted population difference and the violation of phase synchronization) into account phenomenologically by adding appropriate terms to the Maxwell-Bloch equations [2,3]. Solution of the modified equation accounts for non-ideality.

[1] J. MacGillivray *et al.*, Phys. Rev. A 14, 1169 (1976)

[2] F. Rajabi *et al.*, Astrophys. J. 826, 216 (2016)

[3] F. Rajabi *et al.*, Astrophys. J. 828, 57 (2016)

Radiance of ideal samples

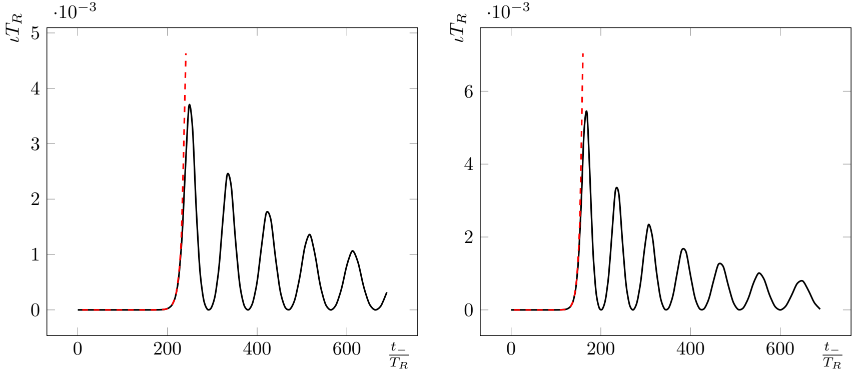

Figure: Dimensionless radiance $\imath T_R$ of ideal samples of atoms of hydrogen (left) and of positronium (right) as a function of dimensionless retarded time t_-/T_R. The number of atoms is maximal and corresponds to (2). Red dashed line – small Bloch angle approximation $\imath T_R \approx \theta_0^2 I_1(q)^2/q^2$.

$$\imath T_R = \frac{1}{q^2}\left(\frac{d\theta}{dq}\right)^2, \quad T_D \approx \frac{T_R}{4}\ln^2\left(\frac{\sqrt{2\pi}}{\theta_0}\sqrt{\ln\frac{\sqrt{2\pi}}{\theta_0}}\right). \tag{6}$$

where $q = 2\sqrt{\frac{t_-}{T_R}}$ when $z = L$; $t_- = t - \frac{z}{c}$ is retarded time.

- θ_0-s from (5) gives: $T_D \approx 228.7\, T_R$ (hydrogen) & $T_D \approx 149.8\, T_R$ (positronium).
- **Ideal ringing:** Maximum number of rings per time with strongest bursts. Energy stored in the inverted system can be emitted in successive bursts of decreasing amplitude.

Radiance of nonideal samples

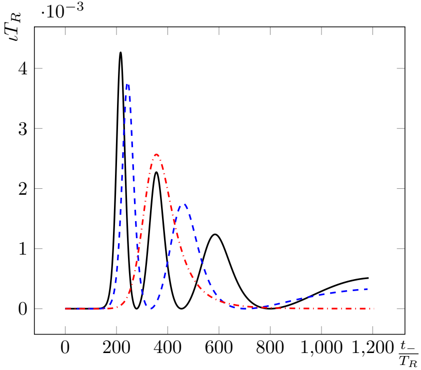

Figure: Dimensionless radiance ιT_R of a non-ideal sample of positronium atoms as a function of dimensionless retarded time t_-/T_R. The number of atoms is maximal and corresponds to (2), but dephasing effects with a common time scale T_{dph} are present. The solid curve corresponds to $T_{dph} = 400\,T_R$, the dashed blue curve corresponds to $T_{dph} = 300\,T_R$, and the dashed red curve corresponds to $T_{dph} = 200\,T_R$.

$$\frac{t_-}{T_R} = -\frac{T_{dph}}{T_R} \ln\left(1 - \frac{q^2}{4}\frac{T_R}{T_{dph}}\right), \quad where: \quad q \in [0, 2\sqrt{T_{dph}/T_R}]. \tag{7}$$

- Necessary condition for the occurrence of superradiance: $T_D < T_{dph}$.
- Also, number of participating atoms ηN can be less than the maximum from eq(2).
- **Nonideal ringing:** the shorter is the T_{dph} the less is the number of the rings per time and the weaker are the bursts.

Conditions for astrophysical superradiance

1. **Population Inversion:** η is the inversion fraction ($N_{inv} = \eta N$). Triggered by:
   - Hydrogen: changing the hyperfine state (Wouthuysen-Field effect).
   - Positronium: huge lifetime difference between ortho- and parapositronium.
2. **Velocity Coherence:** Required to overcome Doppler broadening.
3. **Time threshold:** $T_D < T_{dph}$.

- **Dephasing Estimate:** $T_{dph} = 1/(n\sigma v)$. Where v is impact velocity (average relative velocity of colliding particles, electrons/atoms, at local temperature T).

Table: Dephasing estimates for hydrogen and positronium

	T [K]	v [cm/s]	σ [cm^2]	**n** [cm^{-3}]	T_{dph} [sec]
(H)	≈ 100	$\approx 1.3\times10^5$	$\approx 2\times10^{-14}$ [1]	$< 2\times10^{10}$	$\approx \frac{3\times10^8}{n}$ *
(Ps)	≈ 8000	$\approx 5\times10^7$	$\approx 1.85\times10^{-14}$ [2]	$< 10^8$	$\approx \frac{10^6}{n}$ *

* There can be other effects that shorten the dephasing time (that requires more attention).

[1]- E. M. Purcell, Astrophys. J. 124, 542 (1956)

[2]- W. Mitchell, J. Phys. B 58, 075203 (2025)

Rough numerical estimates of superradiance for astrophysical Hydrogen & Positronium

Without dephasing effects, using eq(1) for $T_{SR} = T_R/2$ in the condition $L = \eta\, c\, T_R$ and assuming a unit Fresnel number, we obtain for a sample in the form of a long cylinder of radius r and length L, $N = \frac{4}{3}\eta\,\omega\tau_0 = \eta\, N_{max}$ and

for **Hydrogen sample:**

$$3 \times 10^6 \text{ cm} < L < \frac{4.4 \times 10^{11}}{\sqrt{n}} \text{ cm}, \quad n < 2 \times 10^{10} \text{ cm}^{-3}, \quad T_R > 10^{-4} \text{ s} \tag{8}$$

for **Positronium sample:**

$$1.7 \times 10^6 \text{ cm} < L < \frac{1.86 \times 10^{10}}{\sqrt{n}} \text{ cm}, \quad n < 10^8 \text{ cm}^{-3}, \quad T_R > 6 \times 10^{-5} \text{ s} \tag{9}$$

Possible examples for superradiant astrophysical-samples of Hydrogen & Positronium

Hydrogen examples (H), $N_{hyd} \leq 4.2 \cdot 10^{24}$
For $n = 1\,\mathrm{cm}^{-3}$

η	T_R [s]	T_D [min]	L [km]	r [m]
1	14.8	56.5	4400000.0	23000.0
0.1	46.8	165.4	1391402.0	12933.9
0.05	66.2	227.9	983869.9	10876.0
0.01	148.0	481.0	440000.0	7273.2
0.001	468.0	1396.3	139140.2	4090.0

Also, for $n = 1\,\mathrm{cm}^{-3}$ and
$T = 100\mathrm{k},\ v \approx 1.3 \times 10^5\,\mathrm{cm/s},\ \sigma \approx 2 \times 10^{-14}\,\mathrm{cm}^2$
$T_{dph} \approx 3 \times 10^8\,\mathrm{s}$

Positronium examples (Ps), $N_{pos} \leq 5.1 \cdot 10^{19}$
For $n = 10\,\mathrm{cm}^{-3}$

η	T_R [s]	T_D [min]	L [km]	r [m]
1	0.2	0.5	58818.4	218.8
0.1	0.6	1.4	18600.0	123.0
0.05	0.9	1.9	13152.2	103.4
0.01	2.0	4.0	5881.8	69.2
0.001	6.2	11.4	1860.0	38.9

Also, for $n = 10\,\mathrm{cm}^{-3}$ and
$T = 8000\mathrm{k},\ v \approx 5 \times 10^7\,\mathrm{cm/s},\ \sigma \approx 1.85 \times 10^{-14}\,\mathrm{cm}^2$
$T_{dph} \approx 10^5\,\mathrm{s}$

The estimated sample lengths L and r, density n, and time scales T_R, T_D and T_{dph}, for both the (H) and the (Ps), do not seem physically unrealistic for the interstellar medium.

- The combination (N, n, η, L, r, T, v, σ, T_R, T_D, T_{dph}, and other factors) determines the possibility of the occurrence of superradiance, but it varies from system to system (neutron stars, microquasars, around Galactic supermassive black holes, etc.), and so an accurate estimation and prediction must depend on the observational data.
- Note that in thermally relaxed regions Dicke superradiance is unlikely to develop.

Hydrogen & Positronium spin-flip lines

Table: Triplet-singlet spin-flip transitions in Hydrogen & Positronium

	Hydrogen	**Positronium**
Hyperfine splitting	$\Delta E_{hyd} \approx 5.884 \cdot 10^{-6}$ eV	$\Delta E_{pos} \approx 8.45 \cdot 10^{-4}$ eV
Transition frequency	$\nu_{hyd} \approx 1420$ MHz	$\nu_{pos} \approx 203.4$ GHz
Line (wavelength)	$\lambda_{hyd} \approx 21$ cm	$\lambda_{pos} \approx 1.474$ mm
Decay width (rate)	$\gamma_{hyd} \approx 2.88 \cdot 10^{-15}\ \mathrm{s}^{-1}$	$\gamma_{pos} \approx 3.37 \cdot 10^{-8}\ \mathrm{s}^{-1}$
Lifetime of excited state	$\tau_{hyd} = \frac{1}{\gamma_{hyd}} \approx 3.5 \cdot 10^{14}\ \mathrm{s}$	$\tau_{pos} = \frac{1}{\gamma_{pos}} \approx 3 \cdot 10^{7}\ \mathrm{s}$

Summary & concluding remarks

- The search for astrophysical manifestations of positronium continues to this day.
- Our results (8) and (9) indicate that Dicke superradiance is possible for the spin-flip lines of hydrogen and positronium under astrophysical conditions (and for hydrogen we confirm the findings of the pioneering paper [1]).
- Hydrogen vs positronium:
    - Population inversion requires different mechanisms for the 1420 MHz hydrogen line (Wouthuysen-Field effect) and the 203 GHz positronium line (huge lifetime difference between ortho and para).
    - The former is stable, while the latter annihilates very quickly. However, we assume that the continuous process of positronium production helps to maintain a nearly constant positronium density for a sufficiently long time.
- Dicke superradiance offers a potential explanation for intense bursts of a collective and coherent phenomenon that standard maser theory cannot accommodate.
- Interestingly, considering the hyperfine splitting in hydrogen ≈ 1420 MHz, it may be that this hydrogen spin-flip line superradiance forms the source of the (SETI) 1977 Wow! signal, which has never been found, despite numerous attempts to detect it, and remains a mystery to this day [2,3]).
- The same is possible for the positronium spin-flip line superradiance, which also represents a very interesting observational problem in millimeter-wave radio astronomy.
- We hope that this work will help to attract the attention of the astrophysical community to the exciting possibility of astrophysical superradiance and stimulate both theoretical and observational activity in this new emerging field.

[1]- F. Rajabi *et al.*, Astrophys. J. 826, 216 (2016)
[2]- A. Méndez *et al.*, arXiv:2408.08513.
[3]- A. Méndez *et al.*, arXiv:2508.10657.

Thanks for attention!

Acknowledgement

Alizzi would like to thank Professor Zurab Silagadze for his guidance and support.

Maxwell-Bloch equations

- Maxwell-Bloch equations for an ideal sample:

$$\begin{aligned}\frac{\partial R}{\partial t} &= -i\frac{d}{\hbar}EZ,\\ \frac{\partial Z}{\partial t} &= i\frac{d}{2\hbar}(ER^* - E^*R),\\ \frac{\partial E}{\partial z}+\frac{1}{c}\frac{\partial E}{\partial t} &= i\frac{2\pi\omega\, n}{c}d\,R.\end{aligned} \tag{10}$$

- Maxwell-Bloch equations for a non-ideal sample:

$$\begin{aligned}\frac{\partial R}{\partial t} &= -i\frac{d}{\hbar}EZ - \frac{R}{T_2},\\ \frac{\partial Z}{\partial t} &= i\frac{d}{2\hbar}(ER^* - E^*R) - \frac{Z}{T_1},\\ \frac{\partial E}{\partial z}+\frac{1}{c}\frac{\partial E}{\partial t} &= i\frac{2\pi\omega\, n}{c}d\,R.\end{aligned} \tag{11}$$