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Theoretical & Observational Probes of Extended Gravities

S.Alexeyev, O.Zenin, R.Stamov, E.Plyzshnikov, S.Kuzmin
Sternberg Astronomical Institute & Dep. QT HEP & Space Res. Fac.
Lomonosov Moscow State University

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Horndeski theory:

$$L =+ G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - \phi_{\mu\nu}\phi^{\mu\nu}\right]$$
$$+ G_5(\phi, X)G^{\mu\nu}\phi_{\mu\nu} - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\Box\phi\phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\nu\lambda}\phi^{\mu}_{\lambda}\right]$$

Fab Four

Horndeski theory:

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Limitations from GW170817: FabFour is cancelled

$$-3 \times 10^{-15} < c_{GW} < 7 \times 10^{-16} \Rightarrow \ldots G_5 = 0$$

Fab Four

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Limitations from GW170817: FabFour is cancelled

Modified FabFour*:

$$S = \int \sqrt{-g}\left[+ \left(\frac{2}{\kappa^2} + \alpha\phi^2\right)R + \kappa^2\beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{3!}\lambda\phi^3 - \frac{1}{4!}\tilde{g}\phi^4\right] d^4x$$

c_{gw} lies inside experimental constraints

$$-3 \times 10^{-15} \leq 256\beta\left(\pi G\dot{\phi}\right) \leq 7 \times 10^{-16}$$

* B.Latosh, EPJ C 78, 991 (2018)

Field equations

Klein-Gordon: $$-\frac{1}{2!}\lambda\phi^2 - \frac{1}{3!}\tilde{g}\phi^3 + \Box\phi + 2\alpha\phi R - 2\kappa^2\beta G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = 0.$$

Einstein:

$$\mathcal{G}_{\mu\nu} = \frac{1}{16\pi G_{\text{eff}}}G_{\mu\nu} - (\nabla_{\mu\nu} - g_{\mu\nu}\Box)\alpha\phi^2 - \frac{1}{2}\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\left(\frac{1}{2}(\nabla\phi)^2 + \frac{1}{3!}\lambda\phi^3 + \frac{1}{4!}\tilde{g}\phi^4\right)$$

$$-\kappa^2\beta\Bigg(-\nabla_\lambda\nabla_\mu\phi\nabla^\lambda\nabla_\nu\phi + \nabla_\mu\nabla_\nu\phi\Box\phi + R_{\alpha\mu\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi - \frac{1}{2}\left[\nabla_\mu\phi G_{\nu\lambda}\nabla^\lambda\phi + \nabla_\nu\phi G_{\mu\lambda}\nabla^\lambda\phi\right] - \frac{1}{2}\Big[\nabla_\mu\phi R_{\nu\lambda}\nabla^\lambda\phi$$

$$+\nabla_\nu\phi R_{\mu\lambda}\nabla^\lambda\phi\Big] + \frac{1}{2}G_{\mu\nu}\nabla^\lambda\phi\nabla_\lambda\phi + g_{\mu\nu}\left[R^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - \frac{1}{2}(\Box\phi)^2 + \frac{1}{2}(\nabla_{\alpha\beta}\phi)^2\right]\Bigg) = \frac{1}{2}T_{\mu\nu}$$

Cosmology

FRW Metric: $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$

Klein-Gordon: $\ddot{\phi} = 12\alpha\phi\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) - \beta\kappa^2\left(\frac{\dot{a}^2}{a^2}\ddot{\phi} + 2\frac{\dot{a}}{a}\frac{\ddot{a}}{a}\dot{\phi} + \frac{\dot{a}^3}{a^3}\dot{\phi}\right) - 3\frac{\dot{a}}{a}\dot{\phi} + \frac{1}{2}\lambda\phi^2 + \frac{1}{6}\tilde{g}\phi^3 = 0$

Einstein:

$$G_{00} = 3\frac{\dot{a}^2}{a^2}\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) + 6\alpha\frac{\dot{a}}{a}\phi\dot{\phi} - \frac{1}{4}\dot{\phi}^2 - \frac{9}{2}\beta\kappa^2\frac{\dot{a}^2}{a^2}\dot{\phi}^2 + \frac{1}{12}\lambda\phi^3 + \frac{1}{48}\tilde{g}\phi^4 = 0,$$

$$G_{ii} = \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\left(\frac{2}{\kappa^2} + \alpha\phi^2\right) + 2\alpha\left(\dot{\phi}^2 + \phi\ddot{\phi} + 2\frac{\dot{a}}{a}\phi\dot{\phi}\right) + \frac{1}{4}\dot{\phi}^2 - \beta\kappa^2\left(\frac{\ddot{a}}{a}\dot{\phi}^2 + 2\frac{\dot{a}}{a}\dot{\phi}\ddot{\phi} + \frac{1}{2}\frac{\dot{a}^2}{a^2}\dot{\phi}^2\right) + \frac{1}{12}\lambda\phi^3 + \frac{1}{48}\tilde{g}\phi^4 = 0$$

In Early universe regime model contains:

1. **Bounce*** $\lambda > 0, \alpha > 0 \Rightarrow \phi > 0, \ddot{\phi} < 0,$ $\lambda < 0, \alpha > 0 \Rightarrow \phi < 0, \ddot{\phi} > 0$

2. **Galileon Genesis**** $0 < \alpha < \frac{1}{8}\frac{\tilde{g}^2}{\lambda^2\kappa^2}$ $\alpha > \frac{1}{8}\frac{\tilde{g}^2}{\lambda^2\kappa^2}$

3. **Inflation** (including source terms for it)** $-\alpha \gg 3\kappa^2\beta,$

==> **Bounce + genesis & inflation are realized**

* S.A., A.Nemtinova, O.Zenin, A.Baiderin, JETP 167, 45 (2025)
** O.Zenin, R.Stamov, S.Kuzmin, SA, EPJ C 86, 471 (2026)

Wilczec model

Einstein-Hilbert $S_{EH} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(R-2\Lambda)$

no torsion $\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$

SO(3,1) connections ω_μ^{ab} **, tetrad** e_μ^a

==>

$$S_{EH} = \frac{1}{64\pi G}\int d^4x\, \epsilon_{abcd}\left(R^{ab}_{\mu\nu}\, e^c_\rho\, e^d_\sigma - \frac{\Lambda}{3}\, e^a_\mu\, e^b_\nu\, e^c_\rho\, e^d_\sigma\right)\epsilon^{\mu\nu\rho\sigma}$$

where

$$R_{\mu\nu}{}^{ab} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + \omega_{\mu}{}^{a}{}_{c}\, \omega_\nu^{cb} - \omega_{\nu}{}^{a}{}_{c}\, \omega_\mu^{cb},$$

$$T^a_{\mu\nu} = \mathcal{D}^\omega_\mu e^a_\nu - \mathcal{D}^\omega_\nu e^a_\mu,$$

Wilczec model: $\mathcal{L}_W = \kappa_3\, \epsilon^{\alpha\beta\gamma\delta}\epsilon_{ABCDE}\, F^{AB}_{\alpha\beta}\, \nabla_\gamma\phi^C\, \nabla_\delta\phi^D\, \phi^E$

where $\nabla_\gamma\phi^C = \partial_\gamma\phi^C + A^C_{\gamma F}\phi^F$ **SO(4,1)**

$$F^{AB}_{\alpha\beta} = \partial_\alpha A^{AB}_\beta - \partial_\beta A^{AB}_\alpha + f^{AB}{}_{CDEF}A^{CD}_\alpha A^{EF}_\beta$$

$$f^{AB\,LM\,PQ} = \eta^{BL}\eta^{AP}\eta^{MQ} - \eta^{AL}\eta^{BP}\eta^{MQ} - \eta^{BM}\eta^{AP}\eta^{LQ} + \eta^{AM}\eta^{BP}\eta^{LQ}$$

Potential: $\mathcal{L}_1 = \kappa_1\left(\eta_{AB}\phi^A\phi^B - \nu^2\right)^2$

minimum: $\phi^A = 0$ or when $|\phi| = \nu$. $\phi^A = \delta^A_5\nu$

==> spontaneus symmetry breaking

==> $\mathcal{L}_2 = \kappa_2\,(J-\omega)^2$

where $J = \epsilon^{\alpha\beta\gamma\delta}\epsilon_{ABCDE}\,\phi^E\nabla_\alpha\phi^A\nabla_\beta\phi^B\nabla_\gamma\phi^C\nabla_\delta\phi^D$

==> $\mathcal{L}_W = \kappa_2\,(J-\omega)^2 + \kappa_1\left(\eta_{AB}\Phi^A\Phi^B - \nu^2\right)^2 + \kappa_3\,\epsilon^{\alpha\beta\gamma\delta}\epsilon_{ABCDE}\,F^{AB}_{\alpha\beta}\,\nabla_\gamma\phi^C\,\nabla_\delta\phi^D\,\phi^E$ **==>**

$$\mathcal{L}_W = \kappa_3\,\nu^3\,\epsilon^{\alpha\beta\gamma\delta}\epsilon_{abcd}\left[\left(\partial_\alpha\omega^{ab}_\beta - \partial_\beta\omega^{ab}_\beta + f^{ab}_{cdef}\,\omega^{cd}_\alpha\,\omega^{ef}_\beta\right) - \Lambda\, e^a_\alpha e^b_\beta\right]e^c_\gamma e^d_\delta$$
$$= \kappa_3\,\nu^3\,\epsilon^{\alpha\beta\gamma\delta}\epsilon_{abcd}\left[F^{ab}_{\alpha\beta} - \Lambda\, e^a_\alpha e^b_\beta\right]e^c_\gamma e^d_\delta.$$

A.Addazi,P.Chen,F.Fabrocini,C.Fields,E.Greco,M.Lulli,A.Marciano,P.Pasechnik, Front. Astron. Space Sci., 8, 563450 (2021)

Gauss-Bonnet with non-minimal coupling generation (1)

BF theory for 4D space-time and Lorentz Lie algebra SO(3,1)

$$S_{BF} = \int_M \mathrm{tr}(B \wedge F) = \int_M B^{ab} \wedge F^{cd} \epsilon_{abcd}$$

Equations of motion: $d_A B = 0, \qquad F = 0$

Adding constraints: $S = \int \mathrm{tr}(B \wedge F) + \frac{1}{2}\theta_{abcd} B^{ab} \wedge B^{cd} + \mu H(\theta)$

$H(\theta) = a_1 \theta^{ab}_{ab} + a_2 \epsilon_{abcd} \theta^{abcd} \Rightarrow B = \alpha e \wedge e + \beta \star (e \wedge e)$ **no torsion ==> B=0**

E.Pluzshnikov, SA, submitted to EPJ C

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String-inspired action (after moduli compactification and supersymmetry breaking):

$$S_{ST} = \int d^4x \sqrt{-g} \Big[\xi(\phi, X) R + \eta(\phi, X) \mathcal{G} + \zeta(\phi, X) \Big]$$

where $X = -\frac{1}{2} g^{\mu\nu} (\nabla_\mu \phi)(\nabla_\nu \phi)$

E.Pluzshnikov, SA, submitted to EPJ C

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Typical BF action: $S = S^{TQFT}[B, F] + S^{Constr}[\lambda_i, B, F] + S^{Dyn}[A]$

$S^{TQFT}[B, F] = B \wedge F$,

2nd: constraints depending on Lagrange multipliers Λ_i,

3rd: generates the dynamics of other fields

E.Pluzshnikov, SA, submitted to EPJ C

Gauss-Bonnet with non-minimal coupling generation (2)

$$S = S^{TQFT}[B,F] + S^{Constr}[\lambda_i, B, F] + S^{Dyn}[A]$$

1. The action is effective. Thus, the two fields of **BF** theory are considered on-shell. The on-shell consideration can be proceeded by adding the constraints from the parent theory

2. The action describes an interaction between **B** and **F** mediated by a scalar field and has a `*BF theory over BF theory*` form.

3. The result should represent the most general well-defined mixing for fields B and F analogously to mixing matrices of Standard Model. Therefore a linear operator acting on a doublet **(B, F)** should appear. Such transformation is natural in supergravity.

Gauss-Bonnet with non-minimal coupling generation (3)

Effective action $S = \frac{1}{3}\int \mathrm{tr}\Big[\hat{B}\wedge\hat{F} - \lambda_i U^i(\hat{B},\hat{F})|_{B,F} + \mu_i V^i(\phi, X, \lambda_i)\Big]$

here $U^1 = \hat{B}\wedge F, \qquad U^2 = \hat{B}\wedge\star B, \qquad U^3 = B\wedge\hat{F}, \qquad U^4 = \star F\wedge\hat{F}$

Assume: $V^i(\phi, X, \lambda) = \lambda_i - f_i(\phi, X)$

A nonlinearity in dependence $V^i = V^i(\phi, X, \lambda_i)$ **with** $\partial^2 V^i/\partial\lambda_i^2 \neq 0$ **relates** Λ_i **to** ϕ **by few multiple branches.**

The aim is to obtain a single well-defined scalar-tensor theory ==> the linear ansatz is the minimal choice.

The equations $\delta\mu_i \; : \; \lambda_i = f_i(\phi, X)$ **==>**

$$\delta\hat{B} \;\; : \hat{F} = \lambda_1 F + \lambda_2 \star B,$$
$$\delta\hat{F} \;\; : \hat{B} = \lambda_3 B + \lambda_4 \star F.$$

==>

$$\delta\lambda_1 \; : \; \hat{B}\wedge F = \mu_1, \qquad \delta\lambda_3 \; : \; B\wedge\hat{F} = \mu_3, \qquad \delta\lambda_2 \; : \; \hat{B}\wedge\star B = \mu_2, \qquad \delta\lambda_4 \; : \; \star F\wedge\hat{F} = \mu_4.$$

Gauss-Bonnet with non-minimal coupling generation (4)

==>
$$\mu_1 = f_3(\phi, X) B \wedge F + f_4(\phi, X) \star F \wedge F,$$
$$\mu_2 = f_3(\phi, X) B \wedge \star B + f_4(\phi, X) \star F \wedge \star B,$$
$$\mu_3 = f_1(\phi, X) B \wedge F + f_2(\phi, X) B \wedge \star B,$$
$$\mu_4 = f_1(\phi, X) \star F \wedge F + f_2(\phi, X) \star F \wedge \star B.$$

==> the action reduces to $S = \frac{1}{3} \int \mathrm{tr}(\hat{B} \wedge \hat{F} + f_i(\phi, X) \mu^i)$

==> The resulting effective theory is equivalent to one obtained by applying the following transformation of the on-shell forms **B** and **F**

$$g_{\mu\nu} \mapsto \Phi(\phi, X) g_{\mu\nu} + \Psi(\phi, X) \nabla_\mu \phi \nabla_\nu \phi$$

The effective action transforms as:
$$B \mapsto \hat{B} = \Psi_1(\phi, X) B + \Phi_1(\phi, X) \star F,$$
$$F \mapsto \hat{F} = \Phi_2(\phi, X) F + \Psi_2(\phi, X) \star B.$$

==>

Gauss-Bonnet with non-minimal coupling generation (5)

$$S_{\text{on-shell}} = \int \mathrm{tr}\left[B \wedge F\right] \quad \longrightarrow \quad S = \int \mathrm{tr}\left[\hat{B} \wedge \hat{F}\right]$$

$$\begin{aligned} S = \int_M \mathrm{tr}(\hat{B} \wedge \hat{F}) &= \int_M \mathrm{tr}\left((\Psi_1 B + \Phi_1 \star F) \wedge (\Phi_2 F + \Psi_2 \star B)\right) \\ &= \int_M \mathrm{tr}\Big(\Phi_1 \Phi_2 F \wedge \star F + (\Psi_1 \Phi_2 - \Phi_1 \Psi_2) B \wedge F + B \wedge \star B\Big). \end{aligned}$$

Gauss-Bonnet with non-minimal coupling generation (6)

Consider the following mixing functions:

$$\Phi_1 = -\frac{1}{2}\chi(\phi), \quad \Phi_2 = 1 \qquad \Psi_1 = \frac{1 \mp \sqrt{1 - 2\chi(\phi)(X + V(\phi))}}{2}, \quad \Psi_2 = \frac{1 \pm \sqrt{1 - 2\chi(\phi)(X + V(\phi))}}{\chi(\phi)}$$

==>
$$\begin{aligned} S &= \int \mathrm{tr}(\hat{B} \wedge \hat{F}) \\ &= \int \left[(e^a \wedge e^b \wedge R^{cd} - (X + V)e^a \wedge e^b \wedge e^c \wedge e^d - \frac{1}{2}\chi R^{ab} \wedge R^{cd})\right]\epsilon_{abcd} \\ &= \int d^4x\sqrt{-g}\left(R + X - V(\phi) - \frac{1}{2}\chi(\phi)\mathcal{G}\right), \end{aligned}$$

where

$$\mathcal{G} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Gauss-Bonnet with non-minimal coupling generation (7)

$$S = \int d^4x\sqrt{-g}\left(R + X - V(\phi) - \frac{1}{2}\chi(\phi)\mathcal{G}\right)$$

1. $V(\phi) = \dfrac{M(\kappa\phi)^2}{d + (k\phi)} \Rightarrow n_S = 0.973871, r = 0.0112445, |c_T^2 - 1| = 9.12899 \times 10^{-16}, \mathcal{P}_\zeta(k_*) = 2.196 \times 10^{-9}$

2. $V(\phi) = M(1 - \dfrac{\delta}{\kappa\phi})^2 \Rightarrow n_S = 0.975688, r = 0.00130174, |c_T^2 - 1| = 1.05845 \times 10^{-17}, \mathcal{P}_\zeta(k_*) = 2.19599 \times 10^{-9}$

3. $V(\phi) = \dfrac{3}{4\beta\kappa^{\nu+4}\phi^\nu + 3\gamma\kappa^4} \Rightarrow n_S = 0.973503, r = 0.0287288, n_T = 0.0442981, \mathcal{P}_\zeta(k) = 2.19 \times 10^{-9}$

==> compatible with ACT data

How to catch

1. Gravitational Wave Astronomy*:

$$S = \mathrm{const} \int d^4x \sqrt{-g}\left[R - \nabla_\mu\phi\nabla^\mu\phi + \frac{m^2\phi^2}{2} + L_{matter}\right] \Rightarrow \left(\Box - m^2\right)\phi = T$$

==> + additional scalar mode ==> 3 polarizations.

The situation in Horndeski (including FabFour) is the same.

2. Black hole shadows & accetion disks & black hole mergers (to be fulfilled)

3. Turnaround radius (to be fulfilled)

…

***S.A., S.Kuzmin, E.Plyzshnikov, R.Stamov, I.Chekh, accepted to UFN (2026)**

Thank you for your attention!